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Vibration of an orthotropic nanoplate resting on a viscoelastic foundation: nonlocal and fractional derivative viscoelasticity approach

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Abstract

Here, we investigate the free vibration behavior of a nanoplate resting on a foundation with viscoelastic properties using nonlocal elasticity and fractional viscoelasticity approach. Nanoplate is modeled using nonlocal and fractional viscoelastic constitutive equation and orthotropic Kirchhoff-Love plate theory. Viscoelastic foundation is represented by the viscoelastic model with fractional derivative operator. Governing equation is derived using D'Alambert's principle and solution is assumed in terms of Fourier series using separation of variables method and satisfying the simply supported boundary conditions for nanoplate. Fractional differential equation is solved using the Laplace and Mellin-Fourier transforms and residue theory. Complex poles of unknown function are determined by finding the roots of the characteristic equation using technique that is available in the literature. In order to show the effect of fractional derivative parameters, damping coefficients and nonlocal parameter on complex roots i.e. damped frequency and damping ratio as well as on nanoplate's displacement, few numerical examples are given.

Key words: nonlocal elasticity, nanoplate, fractional viscoelasticity, damped vibration

1. Introduction

Recent advances in nanotechnologies increased a number of theoretical studies developing the corresponding mathematical models to describe mechanical behavior and small-scale effects of nanobjects. Materials based on nanostructures [1] are having improved thermal, mechanical and electrical properties compared to conventional materials and can be applied in different nano-electromechanical systems (NEMS), opto-mechanical or nanoresonator devices [2], etc.

Nonlocal theory of Eringen [3] receive an intensive application in recent time, especially in theoretical studies of the mechanical behavior of nanostructures such as carbon, boron-nitride and zinc-oxide nanotubes, graphene, gold and silver nanosheets etc. Models based on this theory are originally introduced for nonlocal elastic bodies in order to catch small-scale effects such as forces between atoms or long range interactions that can be significant in small size objects and structures. These effects are taken into account via single parameter called nonlocal or small-scale parameter. Beside this, there are other continuum-based theories such as strain gradient, couple-stress and their modifications or recently developed space fractional derivative models [4, 5] that can catch size effects as well. Many authors proved the suitability of nonlocal theory in modelling of the dynamic behavior or stability problems of nanostructures [6, 7]. For certain problems, an excellent agreement of the results obtained by using the nonlocal theory and molecular dynamics simulations was confirmed in the literature [8]. In addition, combination of nonlocal elasticity [6] and fractional viscoelasticity models [9] shown to be a promising subject for future research. Inclusions of both theories can significantly contribute in accurate modeling of size and dissipation effects in nanostructures [10-12].

In this study, we adopted the nonlocal and modified fractional Kelvin-Voigt viscoelastic constitutive equation to model an orthotropic nanoplate structure. Viscoelastic foundation is represented into the model using the force-displacement relationship with fractional operator. Solution of the governing equation in time domain is obtained using the Laplace and Mellin-Fourier transforms and residue theory. Behavior of complex roots of the characteristic equation is examined in the parametric study. Effects of fractional derivative parameters on nanoplate's displacement in time are examined for few different values of fractional parameters.

2. Preliminaries

2.1 Fractional derivative viscoelasticity

In this study, we will consider only the Riemann-Liouville's definition of fractional derivative [5], which is given as

$$ {}_0D_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha}} d\tau \,, \;\; t \in [0, \infty). $$

where $0 < \alpha < 1$. Usually, fractional derivatives are used for accurate modelling in rheology as well as in structural mechanics to describe vibration damping. In some cases, application of fractional derivative models is necessary since it was shown [9] that classical viscoelastic models failed to describe the mechanical properties of viscoelastic solids. Further, reliability of fractional derivative models is confirmed with molecular theories describing the viscoelastic behavior. In follow, we give a constitutive relation of one dimensional isothermal viscoelastic body with so called modified fractional Kelvin-Voigt type model [9] as

$$t_{xx} = E_0 \left(\varepsilon_{xx}(t) + \tau_\sigma^\alpha \, {}_0D_t^\alpha \varepsilon_{xx}(t) + \tau_\sigma^\beta \, {}_0D_t^\beta \varepsilon_{xx}(t) \right), \tag{1}$$

where t_{xx} is the stress, ε_{xx} is the strain, E_0 is prolonged modulus of elasticity, τ_σ is retardation time and ${}_0D_t^\alpha$ and ${}_0D_t^\beta$ are operators denoting the Riemann-Liouville's derivative of real order α and β, respectively for $0 < \alpha, \beta < 1$. In addition, we give the definition of the model with fractional operator as

$$t_{xx} = E_\infty \left[\varepsilon_{xx}(t) - \nu \left(1 + \tau_\varepsilon^{\gamma_1} \, {}_0D_t^{\gamma_1} \right)^{-\gamma_2} \varepsilon_{xx}(t) \right], \tag{2}$$

where E_∞ is instantaneous modulus of elasticity, $\nu = (E_\infty - E_0)/E_\infty$, τ_ε is the relaxation time and γ_1, γ_2 are fractional parameters , where $0 < \gamma_1, \gamma_2 < 1$.

2.1 Nonlocal theory

In the nonlocal elasticity theory the stress at a point x is a function of the strains at all other points of the elastic body. Eringen [3] proposed a differential form of constitutive relation in the form

$$(1 - \epsilon^2 l^2 \nabla^2)\sigma_{ij} = t_{ij} \,, \tag{3}$$

where σ_{ij} is the nonlocal stress tensor, t_{ij} is the local or classical stress tensor at a point x'. Further, $\epsilon = (e_0\kappa)/l$ denotes the nonlocal parameter that incorporates nonlocal effects into the constitutive equation, where l is an external characteristic length, κ is an internal characteristic length and e_0 is a material constant that can be determined from molecular dynamics simulations or by using dispersive curve of the Born-Karman model of lattice dynamics. Based on Hooke's law for one dimensional case, local stress t_{xx} at a point x' is related to the strain at that point as

$$t_{xx}(x') = E\varepsilon_{xx}(x'), \tag{4}$$

where E denotes the elastic modulus. Based on Eqs. (3) and (4) we can write nonlocal constitutive equation for one-dimensional elastic body as

$$\sigma_{xx} - \mu \frac{d^2\sigma_{xx}}{dx^2} = E\varepsilon_{xx}, \tag{5}$$

where $\mu = (e_0\kappa)^2$ is the nonlocal parameter and σ_{xx} is the nonlocal stress. In order to obtain constitutive relation for a nonlocal viscoelastic body we can combine elasticity and viscoelasticity theory [10-12].

2. Governing equation

Let as consider the free vibration of an orthotropic nanoplate resting on viscoelastic foundation (see Fig.1). Nanoplate is of the length a, width b, height h and density ρ.

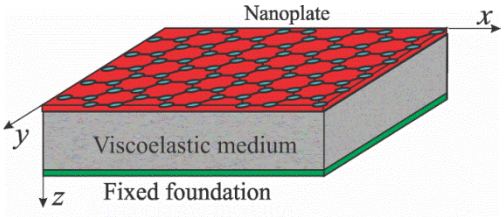

Figure 1. Nanoplate resting on viscoelastic foundation

Here, Eringen's differential form of nonlocal elasticity constitutive equation Eq. (5) in combination with fractional viscoelastic constitutive equation Eq. (1) is employed to consider nonlocal and dissipation effects in nanoplate. Constitutive equation for two-dimensional orthotropic body is given in the form

$$(1-\mu\,\nabla^2)\begin{pmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{pmatrix}=\begin{bmatrix}\frac{E_{01}\left(1+\tau_p^\alpha D^\alpha+\tau_p^\beta D^\beta\right)}{1-\vartheta_{12}\vartheta_{21}} & \frac{\vartheta_{12}E_{01}\left(1+\tau_p^\alpha D^\alpha+\tau_p^\beta D^\beta\right)}{1-\vartheta_{12}\vartheta_{21}} & 0\\ \frac{\vartheta_{12}E_{02}\left(1+\tau_p^\alpha D^\alpha+\tau_p^\beta D^\beta\right)}{1-\vartheta_{12}\vartheta_{21}} & \frac{E_{02}\left(1+\tau_p^\alpha D^\alpha+\tau_p^\beta D^\beta\right)}{1-\vartheta_{12}\vartheta_{21}} & 0\\ 0 & 0 & G_{12}\left(1+\tau_p^\alpha D^\alpha+\tau_p^\beta D^\beta\right)\end{bmatrix}\begin{pmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{pmatrix}, \quad (6)$$

where $\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$ are normal and shear stresses, $\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{xy}$ are strains, E_{01}, E_{02} and G_{12} are prolonged modules of elasticity, τ_p is the retardation time of nanoplate, α, β are fractional parameters and D^α, D^β are operators of Riemann-Liouville's fractional derivatives corresponding to modified fractional Kelvin-Voigt constitutive equation and $\vartheta_{12}, \vartheta_{21}$ are Poisson's ratios.

Governing equation for the free transverse vibration of an orthotropic nanoplate is derived by using the D'Alembert's principle and taking into account constitutive equation (6) using the similar methodology as in [7], which yields

$$\rho h\frac{\partial^2 w}{\partial t^2}+q+D_{011}\left(1+\tau_p^\alpha D^\alpha+\tau_p^\beta D^\beta\right)\frac{\partial^4 w}{\partial x^4}+D_{022}\left(1+\tau_p^\alpha D^\alpha+\tau_p^\beta D^\beta\right)\frac{\partial^4 w}{\partial y^4}+$$
$$+2(D_{012}+2D_{066})\left(1+\tau_p^\alpha D^\alpha+\tau_p^\beta D^\beta\right)\frac{\partial^2 w}{\partial x^2\partial y^2}=\mu\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left[\rho h\frac{\partial^2 w}{\partial t^2}+q\right], \quad (7)$$

where

$$D_{011}=\frac{E_{01}h^3}{12(1-\vartheta_{12}\vartheta_{21})},\ D_{012}=\frac{\vartheta_{12}E_{02}h^3}{12(1-\vartheta_{12}\vartheta_{21})},$$
$$D_{022}=\frac{E_{02}h^3}{12(1-\vartheta_{12}\vartheta_{21})},\ D_{066}=\frac{G_{012}h^3}{12}.$$

In Eq. (7) q denotes an external force from viscoelastic foundation acting on nanoplate, which is represented by the force-displacement relation similar to the constitutive equation (3) with fractional operator as

$$q=\lambda_\infty\left[1-\nu\left(1+\tau_\lambda^{\gamma_1}D^{\gamma_1}\right)^{-\gamma_2}\right]w,\ \ \nu=\frac{\lambda_\infty-\lambda_0}{\lambda_\infty},$$

where λ_∞ and λ_0 are instantaneous and prolonged magnitudes of compliance of the viscoelastic foundation , γ_1, γ_2 are fractional parameters and τ_λ is the relaxation time regarding to a viscoelastic foundation.

2.1 Solution of the governing equation

In order to find the solution of equation (7), we will employ the method described in [9], where the solution is proposed in terms of Fourier series and separation of variables. For the simply supported nanoplate, we write the following boundary conditions

$$w(x,y,t)|_{x,y=0} = w(x,y,t)|_{x,y=a,b} = 0,$$
$$M\,|_{x,y=0} = M\,|_{x,y=a,b} = 0, \tag{8}$$

and initial conditions in the form

$$w(x,y,t)|_{t=0} = 0,\ \dot{w}(a/2,b/2,t)|_{t=0} = \dot{w}_0, \tag{9}$$

After applying the Laplace transformation over Eq. (7), taking into account initial conditions (9) and rearranging some terms, we obtain the following equation

$$p^2\left[1-\mu\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\right]\bar{w}+\frac{\lambda_\infty}{\rho h}\left[1-\nu(1+(p\,\tau_\lambda)^{\gamma_1})^{-\gamma_2}\right]\left[1-\mu\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\right]\bar{w}$$
$$+\left(\bar{w}-\mu\frac{\partial^2\bar{w}}{\partial x^2}\right)+\frac{1}{\rho h}\left[1+\left(p\tau_p\right)^\alpha+\left(p\tau_p\right)^\beta\right]\left[D_{011}\frac{\partial^4\bar{w}}{\partial x^4}+2(D_{012}+2D_{066})\frac{\partial^4\bar{w}}{\partial x^2\partial y^2}+\right.$$
$$\left.D_{022}\frac{\partial^4\bar{w}}{\partial y^4}\right]=\dot{w}_0\left(\delta(x-a/2\,,y-b/2)+\mu\frac{\delta(x-a/2,y-b/2)}{\partial x^2}\right), \tag{10}$$

Here, we take that fractional order initial conditions obtained from Laplace transform of Riemann-Liouville's derivative of a function are equal to zero when function is bounded at zero. Further, we assume the solution in the form

$$\bar{w}(x,y,p)=\sum_{\substack{n=1\\m=1}}^{\infty}\bar{T}_{nm}\sin(k_n x)\sin(k_m y), \tag{11}$$

where $k_n = n\pi/a$, $k_m = m\pi/b$ and $\bar{T}_{nm}(p)$ is unknown function.

After taking into account assumed solution Eq. (11), orthogonality conditions and properties of Dirac delta function we obtain the following equation

$$p^2\bar{T}_{nm}+\frac{\lambda_\infty}{\rho h}\left[1-\nu(1+(p\,\tau_\lambda)^{\gamma_1})^{-\gamma_2}\right]\bar{T}_{nm}+\frac{\bar{A}}{\eta}\left[1+\left(p\tau_p\right)^\alpha+\left(p\tau_p\right)^\beta\right]\bar{T}_{nm}$$
$$=\frac{4\dot{w}_0}{ab}\sin\left(\frac{n\pi}{2}\right)\sin\left(\frac{m\pi}{2}\right), \tag{12}$$

where

$$\eta = 1+\mu(k_n^2+k_m^2)\,,\quad \bar{A}=\frac{1}{\rho h}\left[D_{011}k_n^4+2k_n^2k_m^2(D_{012}+2D_{066})+D_{022}k_m^4\right].$$

Finally, we obtain the solution for unknown functions T_{nm} in Laplace domain as

$$\bar{T}_{nm}(p)=\frac{R}{f_{nm}(p)}\sin\left(\frac{n\pi}{2}\right)\sin\left(\frac{m\pi}{2}\right), \tag{13}$$

where

$$R=\frac{4\dot{w}_0}{ab}\,,\ f_{nm}(p)=p^2+\frac{\lambda_\infty}{\rho h}\left[1-\nu(1+(p\,\tau_\lambda)^{\gamma_1})^{-\gamma_2}\right]+\frac{\bar{A}}{\eta}\left[1+\left(p\tau_p\right)^\alpha+\left(p\tau_p\right)^\beta\right],$$

The solution (13) can be written in time domain by using the Mellin-Fourier inversion formula in the form

$$T_{nm}(t)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\bar{T}_{nm}(p)e^{pt}dp, \tag{14}$$

To calculate the integral one first need to determine all singular points of the complex function (13). Function T_{nm} has two branch points $p=0$ and $p=\infty$ and simple poles which are the roots of the characteristic equation

$$f_{nm}(p) = 0, \tag{15}$$

Integral in Eq. (14) is calculated using the same closed contour as in [9]. Applying the residue theory, we can write Eq. (14) in the following form

$$T_{nm}(t) = T_{nm}^{\text{drift}}(t) + T_{nm}^{\text{vibr}}(t), \tag{16}$$

where

$$T_{nm}^{\text{drift}}(t) = \frac{1}{2\pi i}\int_0^\infty \left[\bar{T}_{nm}\left(se^{-i\pi}\right) - \bar{T}_{nm}\left(se^{i\pi}\right)\right]e^{-st}dp, \tag{17}$$

$$T_{nm}^{\text{vibr}}(t) = \sum_j \text{res}\left[\bar{T}_{nm}(p)e^{p_j t}\right]. \tag{18}$$

In the above equations $T_n^{\text{drift}}(t)$ denotes the drift part of the solution and $T_n^{\text{vibr}}(t)$ denotes the vibration part. In Eq. (18) summation is taken over all isolated singular points i.e. poles $p = p_j$. Poles can be determined by finding the roots of the characteristic equation (15), having two retardation times and four fractional parameters, which can be done following the procedure described in [9]. First, we put $p = re^{i\psi}$ in Eq. (15) as

$$r^2 e^{i2\psi} + \omega_\infty^2\left[1 - \nu\left(1 + (r\tau_\lambda)^{\gamma_1}e^{i\gamma_1\psi}\right)^{-\gamma_2}\right] + \omega_{0nm}^2\left[1 + \left(r\tau_p\right)^\alpha e^{i\alpha\psi} + \left(r\tau_p\right)^\beta e^{i\beta\psi}\right] = 0, \tag{19}$$

where

$$\omega_\infty^2 = \frac{\lambda_\infty}{\rho h},\ \ \omega_{0nm}^2 = \frac{\bar{A}}{\eta}.$$

After using Euler formula and separating real and imaginary parts we can write

$$r^2\cos(2\psi) - \omega_\infty^2 R_{\gamma_1}^{-\gamma_2}\nu\cos\left(\gamma_2\Phi_{\gamma_1}\right) + \ \omega_{0nm}^2 R_{\alpha\beta}\cos\left(\Phi_{\alpha\beta}\right) + \omega_\infty^2 + \omega_{0nm}^2 = 0, \tag{20}$$

$$r^2\sin(2\psi) + \omega_\infty^2 R_{\gamma_1}^{-\gamma_2}\sin\left(\gamma_2\Phi_{\gamma_1}\right) + \ \omega_{0nm}^2 R_{\alpha\beta}\sin\left(\Phi_{\alpha\beta}\right) = 0, \tag{21}$$

where

$$R_{\gamma_1} = \sqrt{1 + 2\left(r\tau_\lambda\right)^{\gamma_1}\cos(\gamma_1\psi) + \left(r\tau_\lambda\right)^{2\gamma_1}},$$

$$\tan\left(\Phi_{\gamma_1}\right) = \frac{\left(r\tau_\lambda\right)^{\gamma_1}\sin(\gamma_1\psi)}{1+\left(r\tau_\lambda\right)^{\gamma_1}\cos(\gamma_1\psi)},$$

$$R_{\alpha\beta} = \sqrt{\left(r\tau_p\right)^{2\alpha} + 2\left(r\tau_p\right)^\alpha\left(r\tau_p\right)^\beta\cos\left(\psi(\alpha-\beta)\right) + \left(r\tau_p\right)^{2\beta}},$$

$$\tan\left(\Phi_{\alpha\beta}\right) = \frac{\left(r\tau_p\right)^\alpha\sin(\alpha\psi)+\left(r\tau_p\right)^\beta\sin(\beta\psi)}{\left(r\tau_p\right)^\alpha\cos(\alpha\psi)+\left(r\tau_p\right)^\beta\cos(\beta\psi)}.$$

are parameters obtained by making the following replacements $R_{\alpha\beta}e^{i\Phi_{\alpha\beta}}$ and $R_{\gamma_1}e^{i\Phi_{\gamma_1}}$ for the bracket terms in Eq. (19). The system of equations (20) and (21) is rootless for $0 < |\psi| < \pi/2$, [9]. Considering the next replacements $X_1 = \left(r\tau_p\right)$ and $X_2 = \left(r\tau_\lambda\right)$ in Eqs. (20) and (21), where X_1 and X_2 can take values from 0 to ∞, we can search the unknown angle ψ and parameter r. Using Eqs. (20) and (21) and eliminating r^2, we can determine the value of angle ψ from the following transcendental equation

$$\tan(2\psi) = \frac{\omega_\infty^2 R_{\gamma_1}^{-\gamma_2}\nu\sin\left(\gamma_2\Phi_{\gamma_1}\right)+\omega_{0nm}^2 R_{\alpha\beta}\sin\left(\Phi_{\alpha\beta}\right)}{\omega_{0nm}^2 R_{\alpha\beta}\cos\left(\Phi_{\alpha\beta}\right)-\omega_\infty^2 R_{\gamma_1}^{-\gamma_2}\nu\cos\left(\gamma_2\Phi_{\gamma_1}\right)+\omega_\infty^2+\omega_0^2}, \tag{22}$$

and the value of r from Eq. (21) as

$$r^2 = -\frac{\omega_\infty^2 R_{\gamma_1}^{-\gamma_2} \nu \sin(\gamma_2 \Phi_{\gamma_1}) + \omega_{0nm}^2 R_{\alpha\beta} \sin(\Phi_{\alpha\beta})}{\sin(2\psi)}, \tag{23}$$

Found values of ψ and r completely determines one pair of complex roots of characteristic equation (15), where by changing ψ to $-\psi$ yields complex conjugate value of the root. Thus, for each pair of fixed values of $X_i, i = 1,2$ and within the half plane $\pi/2 < |\psi| < \pi$, characteristic equation possess two complex conjugate roots

$$p_{1,2(nm)} = re^{\pm i\psi} = -\xi \pm i\Omega\,, \tag{24}$$

Another step is calculation of the drift of the equilibrium position of our nanobeam system that is governed by retardation process. Using Eqs. (15) and (17) we can calculate $T_{nm}^{\text{drift}}(t)$ as

$$T_{nm}^{\text{drift}}(t) = \frac{R_2}{\pi} \int_0^\infty \mathrm{D}_{nm}^{-1} \sin(\mathrm{d}_{nm})\, e^{-st} dp, \tag{25}$$

where

$$R_2 = \frac{4\dot{w}_0}{ab} \sin\left(\frac{n\pi}{2}\right) \sin\left(\frac{m\pi}{2}\right).$$
$$\mathrm{D}_{nm} = \sqrt{\Re^2\{\cdot\} + \Im^2\{\cdot\}},\ \ \mathrm{d}_{nm} = \Im\{\cdot\}/\Re\{\cdot\}$$
$$\Re\{\cdot\} = \mathrm{Re}\{f_{nm}(se^{\pm i\pi})\},\ \Im\{\cdot\} = \mathrm{Im}\{f_{nm}(se^{\pm i\pi})\}$$

Further, based on two complex conjugate poles $p_{1,2(nm)} = -\xi \pm i\Omega$ calculated from Eq. (24) and using formula (18) we have

$$T_{nm}^{\text{vibr}}(t) = \mathrm{res}\left[\bar{T}_{nm}(p_{1(nm)}) e^{p_{1(nm)}t}\right] + \mathrm{res}\left[\bar{T}_{nm}(p_{2(nm)}) e^{p_{2(nm)}t}\right] = Ae^{-\xi t} \sin(\Omega t + \varphi), \tag{26}$$

where

$$A = 2R_2 \left\{ \left[\Re \frac{\partial f_n(re^{i\psi})}{\partial p}\right]^2 + \left[\Im \frac{\partial f_n(re^{i\psi})}{\partial p}\right]^2 \right\}^{-1/2},$$
$$\tan(\varphi) = \left[\Im \frac{\partial f_n(re^{i\psi})}{\partial p}\right] \left[\Re \frac{\partial f_n(re^{i\psi})}{\partial p}\right]^{-1},$$
$$\frac{\partial f_{nm}(p)}{\partial p} = 2p + \omega_\infty^2 \nu \gamma_1 \gamma_2 \tau_\lambda (p\tau_\lambda)^{\gamma_1 - 1} \left(1 + (p\tau_p)^{\gamma_1}\right)^{-1-\gamma_2} + \omega_{0nm}^2 \left(\alpha \tau_p (p\tau_p)^{\alpha-1} + \beta \tau_\varepsilon (p\tau_p)^{\beta-1}\right).$$

In the above equations $\Re$ and $\Im$ denotes the real and the imaginary parts of the first derivative of the characteristic equation, respectively.

3. Numerical results

From the literature, it is well known that that an orthotropic nanoplate model can represent graphene sheet nanostructure [7]. Dissipation of the mechanical energy in such structures can be described using some rheological models. We used values of parameters corresponding to graphene sheet nanostructure in order to investigate the effects of different model parameters on damped frequency and displacement. The following values of parameters in numerical simulations are adopted from [7]: Young's modules $E_{01} =$ 2.434 [TPa], $E_{02} = 2.473$ [TPa], shear modulus $G_{12} = 1.039$ [Pa], $\rho = 6316$ [kg/m^3], Poisson's ratios $\vartheta_{12} = \vartheta_{21} = 0.197$, length of nanoplate $a = 9.519$ [nm], width

$a = 4.844$ [nm], height $h = 0.129$ [nm], characteristic length $\kappa = 1.5$ [nm], nonlocal parameter $e_0 = 1$, and parameters of viscoelastic foundation $\lambda_\infty = 26$ [GPa] and $\nu = 9/10$. All simulations are performed for the mode numbers $n = m = 1$.

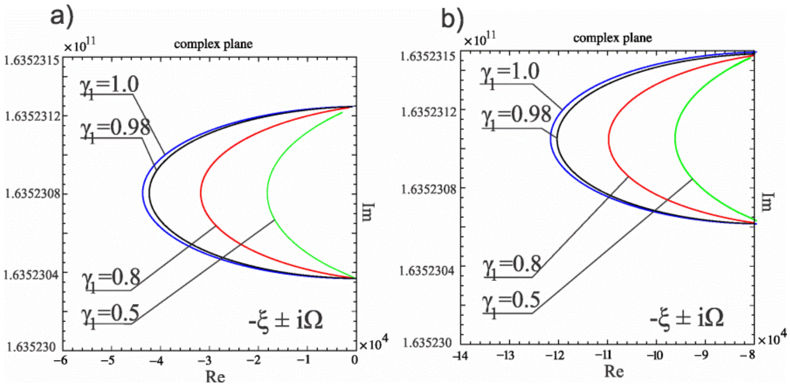

Figure 2. Complex roots of the characteristic equation as functions of X_2 for $\alpha, \beta = 0.9$, $\gamma_2 = 0.98$ and a) $X_1 = 1 \times 10^{-9}$ and b) $X_1 = 1 \times 10^{-6}$

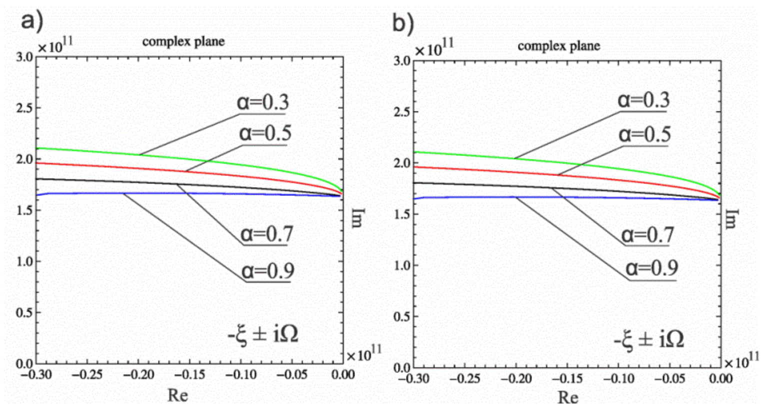

Figure 3. Complex roots of the characteristic equation as functions of X_1 for $\gamma_1, \gamma_2 = 0.9$, $\beta = 0.9$ and a) $X_2 = 0.001$ and b) $X_2 = 10$

Fig. 2 a) and b) shows the behavior of complex roots, where imaginary part represents damped frequency and real part is damping ratio, as a function of the parameter $X_2 = r\tau_\lambda$ and fixed values of other parameters. One can notice that change of frequency and damping ratio are small since the effect of damping parameter X_2 i.e. relaxation time τ_λ of viscoelastic foundation in the model is weak. A decrease of fractional parameter γ_1 reduces the value of damping ratio but not the frequency. In addtion, one can notice significant influence of damping parameter $X_1 = r\tau_p$, whose increase shifts the starting values of damping ratio as well as frequency to the left side of complex plane. Obtained behavior of complex roots is similar to the behavior of the fractional operator model found in the literature [9]. Fig. 3 a) and b) shows the behavior of complex roots as function of parameter X_1 and fixed values of other parameters. One can notice that changes of the frequency and damping ratio are large for an increase of X_1, which is the nature of diffusion type models [9] such as modified fractional Kelvin-Voigt. Characteristics of such damping models are increase of frequency above the frequency of elastic system due to an increase of damping parameter i.e. retardation time. However, usual behaviour of decresing frequency can be noticed for the higher values of fractional parameter. Comparing Figs. 3 a) and b), one can clearly notice weak influence of parameter X_2. Fig. 4 shows the influence of nonlocal parameter on damped frequency (imaginary parts of complex roots) and damping ratio (real parts of complex roots). As expected, frequency of the system decreases for an increase of nonlocal parameter, which is in line with other results from the literature. Finally, Fig. 5 shows the influence of fractional derivative parameters on nanoplate's displacement. One can notice different time dependent behaviors of the model for changes of fractional derivative parameters. When calculating the displacement in time (Fig. 5 b)), drift part of the solution and the phase angle are neglected since they are small.

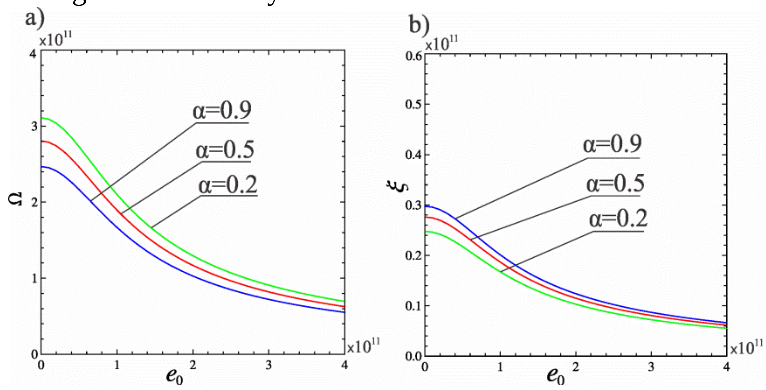

Figure 4. Influence of nonlocal parameter e_0 for $X_1 = X_2 = 0.1, \gamma_1, \gamma_2 = 0.5$ and $\beta = 0.9$ on complex roots $p_1 = -\xi + i\Omega$ a) damped frequency and b) damping ratio.

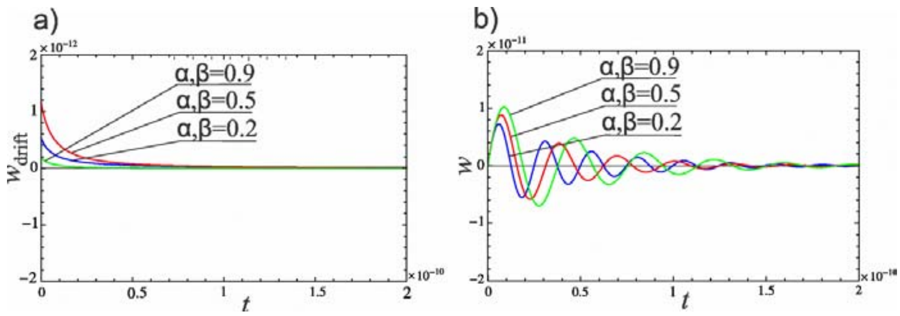

Figure 5. Nanoplate's displacement of the midpoint in time for $X_1 = X_2 = 0.1, \gamma_1, \gamma_2 = 0.9$ a) drift part of the solution b) displacement in time.

4. Conclusions

In this paper, we investigated the free vibration behavior of an orthotropic nanoplate resting on the viscoelastic foundation. Nonlocal and fractional viscoelastic constitutive equation is used for nanoplate to describe structural damping while viscoelastic model with fractional operator is used for a foundation. Governing equation is derived and solved using separation of variables and Laplace transform method. Solution in time domain is obtained using Mellin-Fourier transform and residue theory. Complex roots of the characteristic equation are found using the method from the literature. In the parametric study, effects of different parameters on complex roots i.e. damped frequency and ratio as well as on nanoplate's displacement are investigated through several numerical examples. According to the obtained results, both complex and time domain analysis show to be very important for interpretation of the results and should be used in future analyses of similar models.

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