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Open-closed-loop fractional-order iterative learning control for singular fractional-order system

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Abstract
An open-closed-loop P/PDalpha type iterative learning control (ILC) of fractional-order singular system is investigated. In particular, we discuss fractional-order linear singular systems in pseudo state space form. The sufficient conditions for the convergence in time domain of the proposed fractional-order ILC for a class of fractional-order singular system are defined by the corresponding theorem together with its proof. Finally, a numerical example is presented to illustrate the performance of the proposed factional order ILC.

Key words: iterative learning control, open-closed-loop, singular system, fractional calculus, convergence analysis

1. Introduction

Iterative learning control (ILC) is one of the recent topics in control theories and it is a powerful intelligent control concept that iteratively improves the behavior of processes that are repetitive in nature [1,2,3]. Since the early 80's, ILC [4,5] has been one of the very effective control strategies in dealing with repeated tracking control with the aim of improving tracking performance for the systems that work in a repetitive mode. For the purpose of emulating human learning, ILC uses knowledge obtained from the previous trial to adjust the control input for the current trial so that a better performance can be achieved. Namely, ILC is a trajectory tracking improvement technique for control systems, which can perform the same task repetitively in a finite time interval to improve the transient response of a system using the previous motion. Therefore, ILC requires less a priori knowledge about the controlled system in the controller design phase and also less computational effort than many other kinds of control. Besides, in terms of how to use tracking error signal of the previous iteration to form the control signal of the current iteration, ILC updating schemes can be classified as P-type, D-type, PD-type, and PID type. A typical ILC in the time domain is a simple open-loop control (off-line ILC) and it cannot suppress the unanticipated, non-repeating disturbances. In real application, to overcome such drawbacks, an ILC scheme is usually performed together with a proper feedback controller for compensation [6], where we often design a learning operator for the closed-loop (on-line ILC) systems that have achieved a good performance. Since the theories and learning algorithms on ILC were firstly proposed, ILC has attracted considerable interests [3] due to its simplicity and effectiveness of the learning algorithm, and its ability to deal with the problems associated with nonlinear, time-delay, uncertainties, and, recently, singular systems. During the past years, singular systems have attracted attention of a lot of researchers from the mathematics and control communities due to the fact that singular systems can describe the behavior of some physical systems better than regular systems such as: electrical network models [7], mechanical models [8,9], etc. Naturally, many theoretical results for regular systems have been extended to singular cases. It is well known that issues of concern for singular systems are much more complicated than those for regular systems, because for singular systems we need to consider not only stability, but also regularity and the absence of impulses at the same time [10]. Actually, elimination of algebraic constraints needs a suitable feedback control [11]. From the control point of view, it is also necessary to study the ILC for singular systems. Until now, there are few results reported on introducing ILC methods to studying of tracking control for singular systems [12,13]. Recently, increasing attentions are paid to fractional differential equations and their applications in various science and engineering fields [14,15]. Moreover, an increasing attention has been paid to fractional calculus (FC) and its application in control and modeling of fractional-order singular systems [16,17]. It is not difficult to conclude that other dynamic systems (robotic systems of fractional-order, etc.) [18] can be displayed in the singular form, especially in realization of various robotic tasks.

Recently, the application of ILC to the fractional-order systems has become a new topic [19-22]. For the first time, in a paper [23] is considered a robust iterative learning feedback control of the second-order for fractional-order singular systems. The present paper considers open-closed-loop iterative learning control for given fractional-order singular systems described in the form of state space and output equations. The sufficient convergent conditions of the proposed ILC will be derived in time-domain and formulated by a theorem. Finally, the simulation results are presented to illustrate the performance of the proposed P/PDalpha ILC scheme.

2. Preliminaries

2.1 The λ-norm, maximum norm, induced norm

For later use in proving the convergence of the proposed learning control, the following norms for n-dimensional Euclidean space R^n are introduced [1]:

the sup-norm $\|x\|_\infty = \sup_{1\le i\le n} |x_i| \quad x = [x_1, x_2, \ldots x_n]^T$,

and maximum norm $\|x\|_s = \max_{0\le t\le T} |x(t)| \quad x = [x_1, x_2, \ldots x_n]^T$;

the matrix norm as $\|A\|_\infty = \max_{1\le i\le m}\left(\sum_{j=1}^{n} |g_{i,j}|\right), \quad A = [a_{i,j}]_{mxn}$;

and the λ -norm for a real function:

$$h(t), \ \left(t \in [0,T]\right), \ h:[0,T] \to \Re^n \quad \|h(t)\|_\lambda = \sup_{t\in[0,T]} e^{-\lambda t}\|h(t)\|, \ \lambda > 0. \tag{1}$$

Induced norm of a matrix A is defined as:

$$\|A\| = \sup\left\{\frac{\|Ax\|}{\|x\|} : x \in X \ \ with \ \ \|x\| \neq 0\right\}, \tag{2}$$

where $\|(.)\|$ denotes an arbitrary vector norm. In case $\|(.)\|_\infty$ we have

$$\|Ax\|_\infty \leq \|A\|_\infty \|x\|_\infty \ , \tag{3}$$

where $\|A\|_\infty$ denotes the maximum value of matrix A. For the previous norms, note that

$$\|h(t)\|_\lambda \leq \|h(t)\|_\infty \leq e^{\lambda T} \|h(t)\|_\lambda \,. \tag{4}$$

The $\lambda-$norm is thus equivalent to the $\infty-$norm. For simplicity, in applying the norm $\|(.)\|_\infty$ index ∞ will be omitted.

Before giving the main result, we will first give the following Lemma 1 [3].

Lemma 1. Suppose that a real positive series $\{a_n\}_1^\infty$ satisfies

$$a_n \leq \rho_1 a_{n-1} + \rho_2 a_{n-2} + \ldots + \rho_N a_{n-N} + \varepsilon \qquad (n = N+1, N+2, \ldots), \tag{5}$$

where $\rho_i \geq 0 \quad (i = 1,2,\ldots,N), \ \ \varepsilon > 0$ and $\rho = \sum_{i=1}^{N} \rho_i < 1$. Then the following holds:

$$\lim_{n\to\infty} a_n \leq \varepsilon / (1-\rho). \tag{6}$$

2.2 Fractional integro-differential operators

In this paper, Caputo fractional-order operator is used, where definition of the left Caputo fractional-order derivatives is given [14,15] as follows:

$${}_{t_0}^{C}D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^{t} (t-\tau)^{n-\alpha-1} f^{(n)}(\tau) d\tau, \tag{7}$$

where $f^{(n)}(\tau) = d^n f(\tau) / d\tau^n$, $n-1 \leq \alpha < n \in \mathbb{Z}^+$, and $\Gamma(.)$ is the well-known Euler's gamma function. In the case $n=1$ we have $0 \leq \alpha < 1$ as well as

$${}_{t_0}^{C}D_t^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} (t-\tau)^{-\alpha} \frac{df(\tau)}{d\tau} d\tau. \tag{8}$$

2.3 Fractional-order autonomous linear singular system

Consider the following autonomous, singular, fractional-order system (SFOS) described by the state equations

$$ED^\alpha \boldsymbol{x}(t) = Ax(t), \quad n-1 < \alpha < n \ , \tag{9}$$

$$\boldsymbol{y}(t) = C\boldsymbol{x}(t), \tag{10}$$

where admissible initial conditions for (9) are given by

$$x^{(k)}(0) = x_{0,k} \qquad k = 0,1,2,\ldots n-1. \tag{11}$$

Here, ${}_0^C D_t^\alpha = D^\alpha$ denotes the αth-order Caputo fractional derivative with respect to *t*, while E, A, and C are matrices with appropriate dimensions [24]. In solving a singular problem, assuming regularity of the system, it is necessary to ensure the existence and uniqueness of the solution.

Definition 1. a) The SFOS system (9) is said to be regular if $\det\left(s^\alpha E - A\right) \neq 0$,

b) The SFOS system (10) is said to be impulse free if (10) applies and

$$\deg\left(\det\left(s^{\alpha}E-A\right)\right)=rankE. \tag{12}$$

Lemma 2. The triplet (E,A,α) is called regular if and only if $\det\left(s^{\alpha}E-A\right)\neq 0$ for some $s\in\mathbb{C}$ [24]. Also, if triplet (E,A,α) is regular, we call SFOS system (9) regular, and consequently SFOS system is solvable.

3. Open-closed-loop fractional-order iterative learning control

3.1 The fractional-order non-autonomous singular linear system

A non-integer (fractional) linear, singular system described in the form of pseudo state space and output equations is considered. The considered class of fractional-order $\alpha\in(0,1)$ non-autonomous singular linear system can be written as the state space equation and output equation

$$E\,D^{\alpha}\boldsymbol{x}(t)=Ax(t)+Bu(t),\quad 0<\alpha<1 \tag{13}$$

$$\boldsymbol{y}(t)=C\boldsymbol{x}(t). \tag{14}$$

Here, t is the time within the operation interval $J=\left[t_o,t_o+T\right], J\subset R$, while A,B, and C are matrices having appropriate dimensions. It is assumed that $\det E=0$ and that SFOS system is regular.

Also, the initial conditions of fractional differential equations which were compared to the given fractional derivatives were considered by different authors [24,25] assuming that there was no difficulty as regards the questions of existence, uniqueness, and continuity of solutions with respect to initial data. The following assumptions on the system (13), (14) are imposed.

A1. The desired trajectories $y_d(t), x_d(t)$ are continuously differentiable in $[0,T]$.

A2. For the given desired output trajectory $y_d(t)$, there exists a control input $u_d(t)$ such that

$$ED^{\alpha}\boldsymbol{x}_d(t)=Ax_d(t)+Bu_d(t),\ 0<\alpha<1 \tag{15}$$

$$\boldsymbol{y}_d(t)=C\boldsymbol{x}_d(t). \tag{16}$$

A3. SFOS system is controllable and observable.

A4. Resetting the initial conditions holds for all iterations, i.e $x_k(0)=x_d(0),\quad k=0,1,2...,$

3.2 Convergence Analysis

Here, for the singular system defined by (10), open-closed-loop P/PD-type iterative learning algorithm is proposed as follows:

$$u_{i+1}(t)=u_i(t)+\Gamma_1 e_i(t)+\Gamma_2\left({}_{C}D^{\alpha}_{0,t}e_{i+1}(t)+\Pi_2 e_{i+1}(t)\right), \tag{17}$$

where $u_i(t)$ and $y_i(t)$ are, respectively, the system input and output in the i_{th} iteration, $e_i(t)=y_d(t)-y_i(t)$ is the trajectory tracking error at $i-$th iteration, $u_{i+1}(t)$ is the system input of the $(i+1)_{th}$ trial, $y_d(t)=Cx_d(t)$ denotes desired output trajectory, and Γ_1,Γ_2,Π_2 are open-closed-loop learning matrices. In the closed loop, the PD^{α} controller $\Gamma_2\left({}_{C}D^{\alpha}_{0,t}e_{i+1}(t)+\Pi_2 e_{i+1}(t)\right)$ provides stability of the system and keeps its state errors within uniform bounds. A sufficient condition for convergence of the proposed open-closed-loop ILC is given by Theorem 1. The proof as follows:

Theorem 1: Suppose that the update law defined by (17) is applied to the non-autonomous singular linear system (13), (14) and assumptions $A_i, i = 1,2,3,4$ are satisfied. If matrix Γ_2, exist such that

$$\left\| \left[I - \Gamma_2 C\bar{B} \right] \right\| \le \rho < 1, \tag{18}$$

where is $\bar{B} = \left(E + B\Gamma_2 C\right)^{-1} B$ and matrix Γ_2 is such that $\left(E + B\Gamma_2 C\right)$ is invertible, then, when $i \to \infty$ the bounds of the tracking errors $\|x_d(t) - x_i(t)\|$ $\|y_d(t) - y_i(t)\|$, $\|u_d(t) - u_i(t)\|$ converge asymptotically to a residual ball centered at the origin.

Proof. Let $\delta h_i = h_d(t) - h_i(t), \quad h = x, x_d, u, u_d, f \quad D^{\alpha}\delta h_i\left(t\right) = \delta h_i^{(\alpha)} = h_d^{(\alpha)}(t) - h_i^{(\alpha)}(t)$. (19)

Tracking error can be obtained as follows:

$$e_i^{(\alpha)}(t) = \frac{d^{(\alpha)}}{dt^{(\alpha)}}\left(y_d(t) - y_i(t)\right) = C\delta x_i^{(\alpha)}(t). \tag{20}$$

Taking the proposed control law gives:

$$\delta u_{i+1} = u_d - u_{i+1} = \delta u_i - \Gamma_1 e_i - \Gamma_2\left(e_{i+1}^{(\alpha)} + \Pi_2 e_{i+1}\right), \tag{21}$$

or, taking into account (20) it yields:

$$\delta u_{i+1} = u_d - u_{i+1} = \delta u_i - \Gamma_1 C\delta x_{i+1} - \Gamma_2 C\delta x_{i+1}^{(\alpha)} - \Gamma_2 \Pi_2 C\delta x_{i+1}. \tag{22}$$

Also, from (13), (15) one can find that

$$E\delta x_{i+1}^{(\alpha)} = A\delta x_{i+1} + B\delta u_{i+1}. \tag{23}$$

Substitution of (22) into (23) results in

$$\left(E + B\Gamma_2 C\right)\delta x_{i+1}^{(\alpha)} = \left(A - B\Gamma_2\Pi_2 C\right)\delta x_{i+1} + B\delta u_i - B\Gamma_1 C\delta x_i. \tag{24}$$

By using suitable gain matrix Γ_2, as well as by taking into account the previously introduced assumptions, matrix $\left(E + B\Gamma_2 C\right)$ is invertible, i.e there exists $\left(E + B\Gamma_2 C\right)^{-1}$. By multiplying expression (24) by $\left(E + B\Gamma_2 C\right)^{-1}$, we obtain (25)

$$\delta x_{i+1}^{(\alpha)} = \bar{A}\delta x_{i+1} + \bar{A}_1\delta x_i + \bar{B}\delta u_i, \tag{25}$$

where are

$$\bar{A} = \left(E + B\Gamma_2 C\right)^{-1}\left(A - B\Gamma_2\Pi_2 C\right), \bar{B} = \left(E + B\Gamma_2 C\right)^{-1} B, \ \bar{A}_1 = \left(E + B\Gamma_2 C\right)^{-1} B\Gamma_1 C. \tag{26}$$

By replacing (23) into (20), we obtain

$$\delta u_{i+1} = \left[I - \Gamma_2 C\bar{B}\right]\delta u_i - \left[\Gamma_2 C\bar{A} + \Gamma_2\Pi_2 C\right]\delta x_{i+1} - \left[\Gamma_1 C + \Gamma_2 C\bar{A}_1\right]\delta x_i. \tag{27}$$

By estimating the norms of (25) with $\|(.)\|$ and using the condition of Theorem 1 one obtains

$$\|\delta u_{i+1}\| \le \rho\|\delta u_i\| + \left\|\left[\Gamma_2 C\bar{A} + \Gamma_2\Pi_2 C\right]\right\|\|\delta x_{i+1}\| + \left\|\left[\Gamma_1 C + \Gamma_2 C\bar{A}_1\right]\right\|\|\delta x_i\| = \rho\|\delta u_i\| + \beta_0\|\delta x_{i+1}\| + \beta_1\|\delta x_i\|. \tag{28}$$

Also, one can write the solutions of (25) in the form of the equivalent Volterra integral equations using assumption A4, as:

$$\delta x_{i+1}(t)=\frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\left(\bar{A}\delta x_{i+1}(s)+\bar{A}_1\delta x_i(s)+\bar{B}\delta u_i(s)\right)ds \tag{29}$$

By applying norm $\|(.)\|$ to equation (29), if the solution is unique [24, 25] one obtains:

$$\begin{aligned}\|\delta x_{i+1}(t)\| &\le \frac{1}{\Gamma(\alpha)}\int_0^t |(t-s)^{\alpha-1}|\|\bar{A}\|\|\delta x_{i+1}(s)\|ds+\frac{1}{\Gamma(\alpha)}\int_0^t |(t-s)^{\alpha-1}|\|\bar{A}_1\|\|\delta x_i(s)\|ds+ \\ &+\frac{1}{\Gamma(\alpha)}\int_0^t |(t-s)^{\alpha-1}|\|\bar{B}\|\|\delta u_i(s)\|ds \\ &\le \frac{a}{\Gamma(\alpha)}\int_0^t |(t-s)^{\alpha-1}|\|\delta x_{i+1}(s)\|ds+\frac{a_1}{\Gamma(\alpha)}\int_0^t |(t-s)^{\alpha-1}|\|\delta x_i(s)\|ds+\frac{b}{\Gamma(\alpha)}\int_0^t |(t-s)^{\alpha-1}|\|\delta u_i(s)\|ds+\end{aligned} \tag{30}$$

where are $a_{(..)}=\|\bar{A}_{(..)}\|$, $b=\|\bar{B}\|$. Furthermore, the next relation is fulfilled:

$$t\in[0,T],\quad \|\delta x_i(t)\|=\|\delta x_{i+1}(t)+x_{i+1}-x_i(t)\|\le\|\delta x_{i+1}(t)\|+\|x_{i+1}-x_i(t)\|. \tag{31}$$

Here, we may introduce $\eta_{i+1}=\sup_{t\in[0,T]}\|x_{i+1}-x_i(t)\|$, then $\|\delta x_i(t)\|\le\|\delta x_{i+1}(t)\|+\eta_{i+1}$ and

$$\begin{aligned}\|\delta x_{i+1}(t)\| &\le \frac{(a+a_1)}{\Gamma(\alpha)}\int_0^t |(t-s)^{\alpha-1}|\|\delta x_{i+1}(s)\|ds+\frac{a_1}{\Gamma(\alpha)}\int_0^t |(t-s)^{\alpha-1}|\eta_{i+1}ds+\frac{b}{\Gamma(\alpha)}\int_0^t |(t-s)^{\alpha-1}|\|\delta u_i(s)\|ds \\ &\le \frac{(a+a_1)}{\Gamma(\alpha)}\int_0^t |(t-s)^{\alpha-1}|\|\delta x_{i+1}(s)\|ds+\frac{b}{\Gamma(\alpha)}\int_0^t |(t-s)^{\alpha-1}|\|\delta u_i(s)\|ds+\frac{a_1\eta_{i+1}t^\alpha}{\Gamma(\alpha+1)}.\end{aligned} \tag{32}$$

Moreover, by applying λ norm to both sides of expression (32), it follows

$$\begin{aligned}\|\delta x_{i+1}(t)\|_\lambda &\le \sup_{0\le t\le T}\left\{\int_0^t e^{-\lambda t}\frac{|(t-s)|^{\alpha-1}}{\Gamma(\alpha)}\left[(a+a_1)\|\delta x_{i+1}(s)\|+b\|\delta u_i(s)\|\right]ds\right\}+\sup_{0\le t\le T}\left(e^{-\lambda t}\frac{a_1\eta_{i+1}t^\alpha}{\Gamma(\alpha+1)}\right)\le \\ &\sup_{0\le t\le T}\int_0^t e^{-\lambda(t-s)}\frac{|(t-s)|^{\alpha-1}}{\Gamma(\alpha)}\sup_{0\le t\le T}e^{-\lambda s}\left[(a+a_1)\|\delta x_{i+1}(s)\|+b\|\delta u_i(s)\|\right]ds+\frac{a_1\eta_{i+1}}{\Gamma(\alpha+1)},\end{aligned} \tag{33}$$

or

$$\le\left((a+a_1)\|\delta x_{i+1}(t)\|_\lambda+b\|\delta u_i(t)\|_\lambda\right)\cdot\sup_{0\le t\le T}\int_0^t e^{-\lambda(t-s)}ds\sup_{0\le t\le T}\int_0^t\frac{|(t-s)|^{\alpha-1}}{\Gamma(\alpha)}ds+\xi. \tag{34}$$

where $\xi=a_1\eta_{i+1}/\Gamma(\alpha+1)$. Further, we have

$$\|\delta x_{i+1}(t)\|_\lambda\le\left((a+a_1)\|\delta x_{i+1}(t)\|_\lambda+b\|\delta u_i(t)\|_\lambda\right)\cdot\frac{\left(1-e^{-\lambda T}\right)}{\lambda}\frac{T^\alpha}{\Gamma(\alpha+1)}+\xi. \tag{35}$$

Introducing $O(\lambda^{-1})$, as

$$O(\lambda^{-1})=\frac{\left(1-e^{-\lambda T}\right)}{\lambda}\frac{T^\alpha}{\Gamma(\alpha+1)}, \tag{36}$$

where (35) is simplified to

$$\|\delta x_{i+1}(t)\|_\lambda \leq \left((a+a_1)\|\delta x_{i+1}(t)\|_\lambda + b\|\delta u_i(t)\|_\lambda\right)\cdot O\left(\lambda^{-1}\right)+\xi, \tag{37}$$

one may conclude

$$\|\delta x_{i+1}(t)\|_\lambda \leq \frac{bO\left(\lambda^{-1}\right)\|\delta u_i(t)\|_\lambda+\xi}{\left(1-(a+a_1)O\left(\lambda^{-1}\right)\right)} \leq O_\gamma\left(\lambda^{-1}\right)\|\delta u_i(t)\|_\lambda+\xi'\left(\lambda^{-1}\right). \tag{38}$$

Then, if a sufficiently large λ is used**,** one can obtain that:

$$\lambda\Gamma(\alpha+1)-(a+a_1)\left(1-e^{-\lambda T}\right)T^\alpha>0. \tag{39}$$

After substitution of (31) into (28) it follows

$$\begin{aligned}\|\delta u_{i+1}\| &\leq \rho\|\delta u_i\|+\beta_0\|\delta x_{i+1}\|+\beta_1\|\delta x_i\| \leq \rho\|\delta u_i\|+\beta_0\|\delta x_{i+1}\|+\beta_1\|\delta x_{i+1}(t)\|+\beta_1\eta_{i+1}\\ &=\rho\|\delta u_i\|+(\beta_0+\beta_1)\|\delta x_{i+1}\|+\beta_1\eta_{i+1}.\end{aligned} \tag{40}$$

Taking the λ -norm of expression (40) leads to:

$$\|\delta u_{i+1}\|_\lambda \leq \rho\|\delta u_i\|_\lambda+\beta'\|\delta x_{i+1}\|_\lambda+\beta''. \tag{41}$$

Finally, taking into account (38) we have

$$\|\delta u_{i+1}\|_\lambda \leq \left(\rho+\beta' O_\gamma\left(\lambda^{-1}\right)\right)\|\delta u_i\|_\lambda+\beta'\xi'\left(\lambda^{-1}\right)+\beta''=\rho'\|\delta u_i\|_\lambda+\varepsilon. \tag{42}$$

Therefore, there exists a sufficiently large λ satisfying

$$\rho'=\left(\rho+\beta' O_\gamma\left(\lambda^{-1}\right)\right)<1. \tag{43}$$

According to Lemma 1[3], it can be concluded that:

$$\lim_{i\to\infty}\|\delta u_i\|_\lambda \leq \frac{1}{1-\rho'}\varepsilon. \tag{44}$$

This completes the proof of Theorem 1.

4. A numerical example

In this section, a numerical example is presented to demonstrate the validity of the design method based on open-closed-loop fractional-order ILC control. Consider the following fractional-order singular systems in pseudo state space form described by

$$\begin{bmatrix}1&0\\0&0\end{bmatrix}\begin{bmatrix}D^{0.5}x_1(t)\\D^{0.5}x_2(t)\end{bmatrix}=\begin{bmatrix}1&2\\1&-1\end{bmatrix}\begin{bmatrix}x_1(t)\\x_2(t)\end{bmatrix}+\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}u_1(t)\\u_2(t)\end{bmatrix} \tag{45}$$

$$\begin{bmatrix}y_1(t)\\y_2(t)\end{bmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}x_1(t)\\x_2(t)\end{bmatrix}, \tag{46}$$

where $t\in[0,1]$, $\alpha=0.5$. The desired trajectories are given by $y_{d1}(t)=1.5t\cdot(1-t)$, $y_{d2}(t)=0.5t^2$, $y_{d1,2}(0)=y_{i1,2}(0)=0$**.** In the simulation, we select the following gain matrices:

$$\Gamma_1 = \begin{bmatrix} 0.5 & 0.5 \\ 0 & 0.5 \end{bmatrix},\ \Gamma_2 = \begin{bmatrix} 0.95 & 0.4 \\ 0 & 0.95 \end{bmatrix},\ \Pi_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}. \quad (47)$$

To determine values of the gain matrices, it is necessary to satisfy the convergence condition of Theorem 1 and make a comprehensive consideration of the convergence speed. It is easy to show that the pair (E;A) is regular and $\left\| \left[I - \Gamma_2 C\bar{B} \right] \right\| = 0.7287 < 1$.

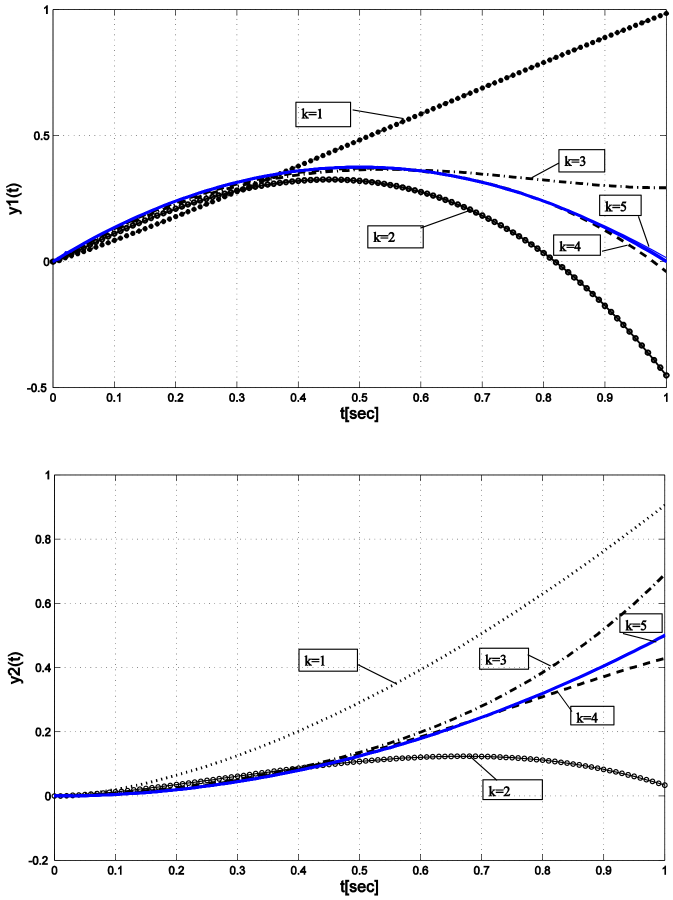

Figure 1. The tracking performance of the system output:
(a)-upper figure: $y_1(t)$ line $-(k=1,2,3,4,5..)$ iteration , $y_{d1}(t)$-bold blue line;
(b)-lower figure: $y_2(t)$ line$-(k=1,2,3,4,5..)$ iteration , $y_{d2}(t)$-bold blue line

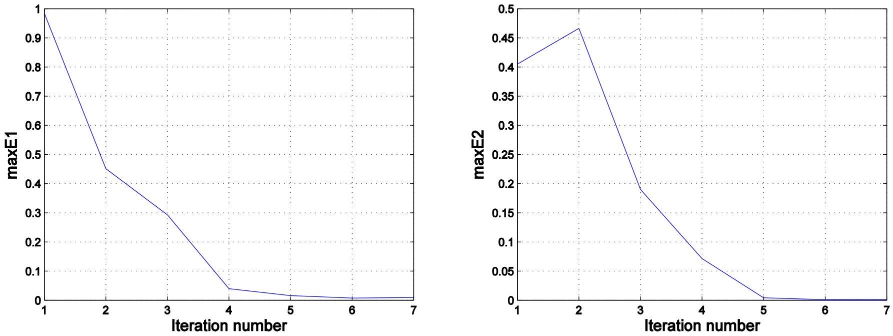

Figure 2. The sup-norms of tracking errors $e_1(t)$ and $e_2(t)$ at each iteration

The simulation results presented in Figs.1-2 demonstrate the effectiveness of the developed ILC control scheme for the system defined by (45),(46). The ILC rule (17) is used, where Figures 1,a),b) show the tracking performance of the ILC system outputs over interval $t\in[0,1]$. Also, we can find (see Figure 2) that proposed requirement for the tracking performance is achieved at the seventh iteration.

5. The Conclusions

For the first time the open-closed-loop fractional-order iterative learning control is proposed for a given class of fractional-order singular systems. In particular, the sufficient conditions for the convergence in time domain of the proposed ILC were defined, by the corresponding theorem, and proved. Finally, a numerical example is presented to illustrate the effectiveness of the proposed open-closed ILC scheme of fractional-order for a class of fractional-order, linear singular system.

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References

[1] Moore K.L., *Iterative learning control for deterministic systems, Advances* in Industrial Control, Springer-Verlag,London, 1993.

[2] Bien Z., Xu J., *Iterative Learning Control: Analysis, Design, Integration and Applications*, Kluwer Academic Publishers, 1989.

[3] Ahn H.-S., Moore K., Chen Y., *Iterative learning control robustness and monotonic convergence for interval systems*, Springer-Verlag London Limited,London, 2007.

[4] Uchiyama M., *Formulation of high-speed motion pattern of a mechanical arm by trial* (in Japanese), Trans. SICE (Soc. Instrum. Control Eng.) 14,706–712,1978.

[5] Arimoto S., Kawamura S., Miyazaki F., *Bettering operation of robots by learning*, J. Rob. Syst., 2(1),123-140, 1984.

[6] Kuc T. Y., Nam K., Lee J.-S., *An iterative learning control of robot manipulators,* IEEE Trans. on Robot. Auton., 7(6), 835 – 842, 1991.

[7] Rosenbrock H H., *Structural properties of linear dynamical systems*, Int. J. Control, 20(2), 191-202,1974.

[8] Müller P. C., *Stability of Linear Mechanical Systems with Holonomic Constraints*, Appl. Mech. Rev., 46(11),160–164,1993.

[9] I. Buzurovic, *Dynamic model of medical robot represented as descriptor system*, Int. J. Inf. Syst. Sci., 2(2), 316-333,2007.

[10] Fang C. H., Chang F.R., *Analysis of stability robustness for generalized state-space systems with structured perturbations*, Syst. Control Lett., 21(2), 109−114,1993.

[11] Dai L., *Singular Control Systems*,118 Lecture Notes in Control and Information Sciences. New York: Springer-Verlag,1989.

[12] Xie S.L,. Xie Z.D, Liu Y. Q., *Learning Control Algorithm for State Tracking of Singular Systems with Delay,* Syst. Eng. Electron., 21(5), 10-16,1999.

[13] Piao F. X., Zhang Q. L., Wang Z. F., *Iterative Learning Control for a Class of Singular Systems*, Acta Autom. Sin., 33(6), 658-659, 2007.

[14] Podlubny I., *Fractional Differential Equations*. Academic Press, San Diego,1999.

[15] Monje C.A, Chen Y. Q, Vinagre B. M, Xue D. Y, Feliu-Batlle V. *Fractional-order Systems and Controls:Fundamentals and Applications*.London:Springer-Verlag, 2010.

[16] Skruch,MitkowskiW., *Fractional-Order Models of the Ultracapacitors.In Mitkowski W, Kacprzyk J and Baranowsk J (eds*) Advances in the Theory and Applications of Non-integer Order Systems. Switzerland: Springer, 281-294,2013.

[17] Kaczorek T., *Singular fractional continuous-time and discrete-time linear systems*. Acta Mech. Autom. (7)1,26-33,2013.

[18] Cajić M., M.P.Lazarević, *Fractional-order spring/spring-pot/actuator element in a multibody system:Application of an expansion formula,* Mech. Res. Commun., 62, 44–56, 2014.

[19] Chen Y.Q, Moore K.L., *On D^{α} type iterative learning control,* In Proc. 40th IEEE Conference on Decision and Control, Orlando, FL USA,pp.4451-4456,2001.

[20] M.P. Lazarević, *PD^{α} -type iterative learning control for fractional LTI system*-16th Int.Cong. CHISA 2004,G2.9, Praha,Czech Republic, 22-26, August, 2004.

[21] M.Lazarević, P.Mandić, *Feedback-feedforward iterative learning control for fractional-order uncertain time delay system–PD alpha type,* ICFDA'14 Catania, 23-25 June 2014, IE86, 2014.

[22] Li Y., Chen Y.Q., Ahn H.S., *Fractional-order iterative learning control for fractional-order linear systems*,Asian J. Control,13 (1),54- 63,2011.

[23] Lazarević M.P., Tzekis P., *Robust second-order PD type iterative learning control for a class of uncertain fractional-order singular systems,* J. Vib. Control, Dec.,doi:10.1177/1077546314562241,2014.

[24] Yao Y, Zhuang J., Yin S. C., *Suffcient and Necessary Condition of Admissibility for Fractional-order Singular System*, Acta Autom. Sin.,(39),2160-2164,2013.

[25] Ortigueira M.,Coito F., *The initial conditions of Riemann-Liouville and Caputo derivatives,*in Proc. 6th EUROMACH Nonlinear Dynamics Conference, (ENOC 2008),2008.