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COMBINED SUB-HARMONIC RESONANCES OF NANOBEAM ON FRACTIONAL VISCO-PASTERNAK TYPE FOUNDATION

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Abstract:

In this work, we observe combined parametric and external sub-harmonic resonances of order one-third of a geometrically nonlinear nonlocal nanobeam model resting on a fractional visco-Pasternak type foundation. Euler-Bernoulli beam theory, nonlinear strain-displacement relation and nonlocal elasticity constitutive equation are employed to obtained fractional order governing equation for the transverse vibration of a system. Under the assumption of small fractional damping, we used the perturbation multiple-scales method to obtain an approximated analytical solution for the frequency-amplitude response for variable axial and transverse external loads. Several numerical examples are given to show the effects of different parameters on frequency-amplitude response.

Key words: combined resonance, nonlinear oscillations, fractional derivatives, visco-Pasternak, multiple scales, nonlocal elasticity, nanobeam.

1. Introduction

Nanostructures having similar shapes as macro engineering structures such as beams, plates and shells are developed using various nano-technological processes. Mechanical behavior of nano-scale structures is also similar to the behavior of macro structures with the main difference that influences of various size-effects cannot be neglected. Studying the dynamic behavior of nanostructures is especially important for the development of new types of nanoactuators, nanosensors or resonator devices. Geometrically nonlinear oscillations with different resonance conditions together with dissipation effects from different sources are important effects for the investigation and performance of nano-scale resonators. Very important is consideration of dissipation effects from external medium. Application of experimental and atomistic methods is limited to the analysis of nanostructures and it can be applied only for specific and less complex systems. Continuum based theoretical models have attracted a great attention of researchers in recent years. Eringen's nonlocal elasticity theory was one of the first that consider size effects and atomic forces through a single material parameter [1]. This theory shown to be very useful in describing the mechanical behavior of various nanostructures and nanocomposites compared to other methods [2]. Dissipation models based on fractional derivative viscoelasticity become widely used due to proven advantages compared to classical viscoelasticity models [3]. Recently, such models are successfully used to describe linear and nonlinear dynamic behavior of nanostructures considering size and damping effects [4, 5]. When solving nonlinear dynamic problems together with the fractional derivative damping, one can use numerical [5] or analytical approximation methods [6].

Here, we will investigate a combined parametric and external resonance of a nanobeam system subjected to time dependent axial and transverse loads. Effects of fractional derivative visco-Pasternak type foundation, representing some medium around nanobeam, and size effects on frequency-amplitude response will be examined for changes of model parameters. Our intention is to fill the gap in the literature by studying the combined resonance condition of a nanobeam system using the multiple scales perturbation method to solve nonlinear fractional order differential equations to obtain approximate expressions for amplitude-frequency response of the system.

2. Problem formulation and multiple-scales solution

Let as assume that a nanobeam is homogenous with length *L*, cross section area *A*, density *ρ*, and Young's modulus *E*, Fig. 1. Assuming the nonlocal elasticity stress-strain equation for one-dimensional case, von Karman strain-displacement relation and Euler-Bernoulli beam theory [6], we derive governing equation for nonlinear transverse vibration of a nanobeam resting on fractional visco-Pasternak type foundation [7] as

$$\rho A\frac{\partial^2 w}{\partial t^2}+\xi(x,t)-P\frac{\partial^2 w}{\partial x^2}-\mu\left[\rho A\frac{\partial^4 w}{\partial t^2\partial x^2}+\frac{\partial^4\xi(x,t)}{\partial x^2}-P\frac{\partial^4 w}{\partial x^4}\right]+EI\frac{\partial^4 w}{\partial x^4}=f(t) \tag{1}$$

where $f(t)=F_T\cos\Omega_T t$ is the assumed transverse harmonic load and the force-displacement relation from fractional visco-Pasternak type foundation [4] is given as $\xi(x,t)=\eta D_t^\alpha w-G_p D_t^\alpha \partial^2 w/\partial x^2$, where D_t^α is the Riemann-Liouville's fractional derivative operator. Axial force with static and dynamics part $F=-(F_0+F_1\cos\Omega_1 t)$ is given as

$$P=-(F_0+F_1\cos\Omega_1 t)+\frac{EA}{2L}\int_0^L\left(\frac{\partial w}{\partial x}\right)^2 dx \tag{2}$$

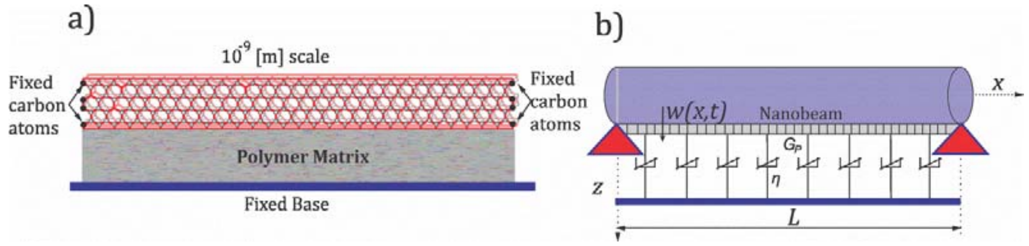

Fig. 1. Nanobeam on fractional visco-Pasternak foundation a) Physical model b) Mechanical model

To solve Eq. (1), first we derive dimensionless form of equation with assumed solution of the form

$$\bar{w}(\bar{x},\tau)=\phi_n(\bar{x})q_n(\tau),\; n=1,2,...,\infty \tag{3}$$

where $\bar{w}$ is dimensionless displacement, ϕ_n is the amplitude function and $q_n(\tau)$ is the time function in terms of dimensionless time τ.

Using the Galerkin discretization, we obtain the following forced Mathieu equation

$$\ddot{q} + \varepsilon \beta_{12} D_\tau^\alpha q + \left(\omega_0^2 - \varepsilon \beta_3 \bar{F}_1 \cos \bar{\Omega}_1 \tau\right) q + \varepsilon \beta_4 q^3 = \tilde{f} \cos \bar{\Omega}_2 \tau \tag{4}$$

where ε is the small parameter, β_{12}, β_3 and β_4 are corresponding dimensionless collecting parameters obtained from Eq. (1). Equation (4) is the fractional order nonlinear differential equation that will can be solved using perturbation multiple scales method. According to [3], we can adopt fast $\mathrm{T}_0 = \tau$ and slow $\mathrm{T}_1 = \varepsilon\tau$ time scales, where for the second perturbation we write fractional derivatives as $D_\tau^\alpha = D_{0+}^\alpha + \varepsilon\alpha D_{0+}^{\alpha-1} D_1$ and integer order derivatives as $D_n = \partial/\partial \mathrm{T}_n$ ($n = 1,2,\ldots$) with D_{0+}^α, $D_{0+}^{\alpha-1}$ denoting the Riemann-Liouville's fractional time derivatives. The following solvability condition is obtained for the sub-harmonic resonance of order one-third

$$-2i\omega_0 \mathrm{A}' - 6\beta_4 \mathrm{A}\Lambda^2 - 3\beta_4 \mathrm{A}^2 \tilde{\mathrm{A}} - \mathrm{A}\left(i\omega_0\right)^\alpha \beta_{12} - 3\beta_4 \tilde{\mathrm{A}}^2 \Lambda \exp\left[i\sigma \mathrm{T}_1\right] = 0 \tag{5}$$

where

$$\Lambda = \frac{\tilde{f}}{2\left(\omega_{on}^2 - \tilde{\Omega}^2\right)}. \tag{6}$$

Using polar coordinates $A(\mathrm{T}_1) = 1/2\, a e^{i\varphi}$ in Eq. (5), corresponding derivatives A' with respect to T_1 and separating real and imaginary parts yields

$$-\omega_0 a\varphi' - 6\beta_4 a\Lambda^2 - 3\beta_4 a^3 - a\omega_0^\alpha \beta_{12} \cos\left(\frac{\alpha\pi}{2}\right) - 3\beta_4 a^2 \Lambda \cos\left(\theta\right) = 0 \tag{7}$$

$$-\omega_0 a' - a\omega_0^\alpha \beta_{12} \sin\left(\frac{\alpha\pi}{2}\right) - 3\beta_4 a^2 \Lambda \sin\left(\theta\right) = 0 \tag{8}$$

where $\theta = \sigma \mathrm{T}_1 - 3\varphi$ is a new phase angle. To analyze the steady-state motion we assume that $a' = \theta' = 0$, which gives the following quadratic equation in terms of detuning parameter

$$\sigma^2 - 2K\sigma + M = 0 \tag{9}$$

where K and M are given as

$$K = 18\frac{\beta_4 \Lambda^2}{\omega_0} + 9\frac{\beta_4 a^2}{\omega_0} - 3\omega_0^{\alpha-1}\beta_{12}\cos\left(\frac{\alpha\pi}{2}\right), \tag{10}$$

$$M = K^2 + \left(3\omega_0^{\alpha-1}\beta_{12}\sin\left(\frac{\alpha\pi}{2}\right)\right)^2 - \left(9\frac{\beta_4 a\Lambda}{\omega_0}\right)^2. \tag{11}$$

By solving Eq. (9), we obtain the following expression for the amplitude-frequency response

$$\sigma_{1/2} = K \pm \sqrt{K^2 - M} \tag{12}$$

Based on Eq. (12), corresponding steady state frequency-amplitude curves will be plotted for simply supported and clamped-clamped nanobeam resting on fractional visco-Pasternak foundation. In addition, several numerical examples will be given to show the effects of nonlocal parameter, order of fractional derivative and parameters of visco-Pasternak type foundation on the amplitude-frequency response. Interaction of parametric resonance from variable axial load and external resonance from transverse harmonic load will be also discussed.

3. Conclusions

This paper studies the combined parametric and external subharmonic resonance of a nano-scale system composed of a nanobeam resting on fractional Pasternak-type viscoelastic foundation. Geometric nonlinearity is considered together with the Euler-Bernoulli beam theory and nonlocal elastic constitutive equation to derive motion equations of the observed system. The system is subjected to two different types of loads, transverse harmonic and axial loads with same frequencies. Solution of the governing equation is proposed via modified multiple-scales perturbation method for fractional derivative equations. Effects of different parameters on frequency-amplitude response and regions of instability are examined through several numerical examples.

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