6th International Congress of Serbian Society of Mechanics
Mountain Tara, Serbia, June 19-21, 2017

DYNAMIC MODELLING AND CONTROL DESIGN OF SEVEN DEGREES OF FREEDOM ROBOTIC ARM

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Abstract:

In this paper kinematic and dynamic model or robot manipulator with 7 degrees of freedom is given. Lagrange's equations of second kind in the covariant form are obtained by applying Rodriguez method, instead of regular Newton-Euler or Lagrange method. By implementing inverse dynamics control, linear and decoupled system is obtained, after which classical *PID* controller is introduced. In order to verify mathematical model is well derived, along with its control system design, numerical simulation of given robot is presented, wherein trajectory tracking problem is investigated.

Key words: robotic arm, modeling, control design, computed torque control.

1. Introduction

Robots today are making a considerable impact on many aspects of modern life, from manufacturing to health care [1]. Unlike the industrial robotics domain where the workspace of machines and humans can be segmented, applications of intelligent machines that work in contact with humans are increasing, which involve e.g. haptic interfaces and teleoperators, cooperative material handling, power extenders and such high volume markets as rehabilitation, physical training, entertainment [2]. In that way, robotic systems are more and more ubiquitous in the field of direct interactions with humans, in a so called friendly home environment [3]. As one of these robotic systems capable of operating in such environments is NeuroArm robotic system. It is an integral part of the Laboratory of Applied Mechanics, at Faculty of Mechanical Engineering in Belgrade (Figure 1).

From the mechanical point of view NeuroArm robotic arm has seven degrees of freedom. Given manipulator is useful for studying kinematics, dynamics, as well as for research of control systems design [4]. Schematic view of NeuroArm robot is given in Figure 2. With powerful DC motors and high resolution encoders, a smooth control of all seven cylindrical joints is provided. Accurate robot control and realistic robot simulation require an accurate dynamic robot model. Hence, by applying the Rodriques approach detailed mathematical model of NeuroArm robot will be given first. Using the information about dynamic model, nonlinear technique known as computed torque control is implemented, with aim to simplify equations of motion. Then, classical *PID* controllers for each joint can be used, which in return allow robot to follow prescribed trajectory. Finally, numerical simulation of robot model is given, wherein tracking problem is considered so as to validate the control design task is well performed.

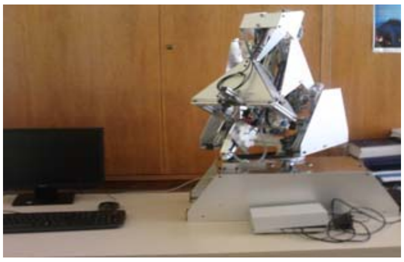

Fig. 1. Laboratory NeuroArm robotic arm.

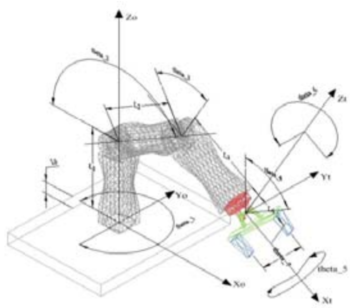

Fig. 2. NeuroArm model with 7 degrees of freedom

2. Mathematical model

The mechanical structure of a robot manipulator consists of a sequence of rigid bodies (or links) interconnected by means of joints [5]. The open chain system of rigid bodies (V_1), (V_2), … , (V_n) is shown Figure 3. The rigid body (V_1) is connected to the fixed stand. Two neighboring bodies (V_{i-1}) and (V_i) are connected together with joint (i), which allows translation or rotation of body (V_i) in respect to body (V_{i-1}). The values q^i represent generalized coordinates.

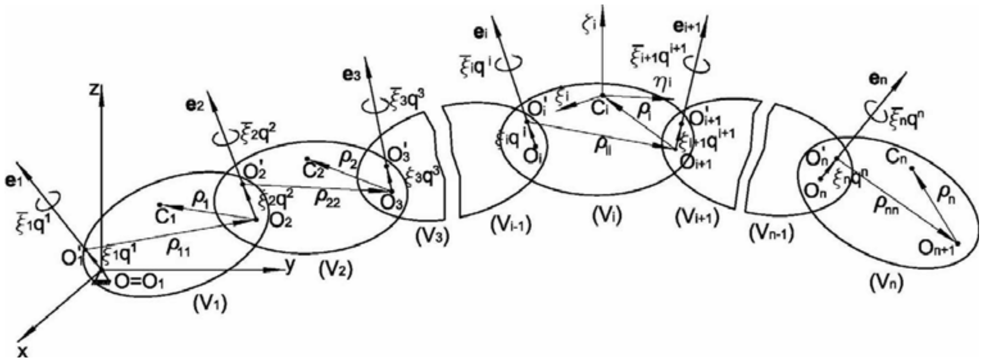

Fig. 3. Open chain of the rigid bodies system.

The reference frame $Oxyz$ is inertial Cartesian frame, and the reference frame $O\xi\eta\zeta$ is local body –frame which is associated to the body (V_i). At initial time, corresponding axis of reference frames were parallel. This configuration is called reference configuration and it is denoted by (0). The symbol $\overline{\xi_i}$ and ξ_i can be introduced, which are defined as:

$$\xi_i = 1, \;\; \overline{\xi_i} = 0, \tag{1}$$

in the case when bodies (V_{i-1}) and (V_i) are connected with prismatic joint, and

$$\xi_i = 0, \;\; \overline{\xi_i} = 1, \tag{2}$$

in the case when bodies (V_i) and (V_{i-1}) are connected with cylindrical joint [6]. The geometry of the system is defined by the unit vectors $\vec{e}_i$ and position vectors $\vec{\rho}_i$ and $\vec{\rho}_{ii}$ expressed in local coordinate systems $C_i\xi_i\eta_i\zeta_i$ connected to mass centers of bodies in a multibody system [7]. Unit vectors $\vec{e}_i$, i=1,2, … , n is describing the axis of rotation (translation) of the i-th segment with respect to the previous segment, and $\vec{\rho}_{ii} = \overrightarrow{O_i^{'}O_{i+1}}$ denotes a vector between two neighboring joints in a multi body system, while position of the center of mass of ***i***-th segment is expressed by vectors $\vec{\rho}_{ii} = \overrightarrow{O'_{i+1}C_i}$. For the entire determination of this mechanical system, it is necessary to specify masses m_i and tensors of inertia J_{Ci} expressed in local coordinate systems.

If we have a kinetic energy of the system in terms of generalized coordinates and its derivatives, one can write dynamic equations of the system in terms of Lagrange equations of the second kind. After some transformations, equations of motion of a multibody system in a covariant form can be written as [5,8]

$$\sum_{\alpha=1}^{n} a_{\gamma\alpha}(q)\ddot{q}^{\alpha} + \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} \Gamma_{\alpha\beta,\gamma}(q)\dot{q}^{\alpha}\dot{q}^{\beta} = Q_{\gamma}, \quad \gamma = 1,\ldots,n. \tag{3}$$

Here, q^{α} and q^{β} denote generalized coordinates, n is a number of bodies in the system, $a_{\gamma\alpha} = a_{\alpha\gamma}$ are elements of the basic metric tensor, and $\Gamma_{\alpha\beta,\gamma}$ are Cristoffel symbols of the first kind. Coefficients of the metric tensor are defined as

$$a_{\alpha\beta} = \sum_{i=1}^{n} m_i \left(\vec{T}_{\alpha(i)}\right)\left\{\vec{T}_{\beta(i)}\right\} + \left(\vec{\Omega}_{\alpha(i)}\right)\left[J_{Ci}\right]\left\{\vec{\Omega}_{\beta(i)}\right\}, \tag{4}$$

where quasi-base vectors $\vec{T}_{\alpha(i)}$ and $\vec{\Omega}_{\alpha(i)}$ are

$$\vec{T}_{\alpha(i)} = \begin{cases} \bar{\xi}_{\alpha}\vec{e}_{\alpha} \times \left(\sum_{k=\alpha}^{i}\left(\vec{\rho}_{kk} + \xi_k \vec{e}_k q^k\right) + \vec{\rho}_i\right) + \xi_{\alpha}\vec{e}_{\alpha}, & \forall\alpha \le i, \\ 0, & \forall\alpha > i, \end{cases} \tag{5}$$

$$\vec{\Omega}_{\alpha(i)} = \begin{cases} \bar{\xi}_{\alpha}\vec{e}_{\alpha}, & \forall\alpha \le i, \\ 0, & \forall\alpha > i, \end{cases} \tag{6}$$

and Cristoffel symbols are

$$\Gamma_{\alpha\beta,\gamma} = \frac{1}{2}\left(\frac{\partial a_{\beta\gamma}}{\partial q^{\alpha}} + \frac{\partial a_{\gamma\alpha}}{\partial q^{\beta}} - \frac{\partial a_{\alpha\beta}}{\partial q^{\gamma}}\right), \quad \alpha,\beta,\gamma = 1,\ldots,n. \tag{7}$$

Eq. (3) has a suitable form of motion equation for automated setting of analytical expressions dedicated to the proposed mechanical system. Regardless of the chosen theoretical approach, in [5] it is shown that it could be started from different theoretical aspects (e.g. general theorems of dynamics, d'Alambert's principle, Lagrange's equation of second kind, Appell's equations, etc.) to get to the equations of motion of the robotic system expressed in the covariant form as Eq. (3). It is also shown that for the above system of differential equations it is convenient to use Rodriquez approach for matrices of coordinate transformations.

On the right hand side of Eq. (3), the generalized forces Q_{γ} represent external forces Q_{γ}^{g}, Q_{γ}^{m} which denote the generalized gravitational forces and motor torques, respectively. For details of the calculation of the basic metric tensor and Cristoffel symbols of the NeuroArm robot, the reader is referred to Appendix A.

3. Controller design

There have been proposed many different schemes of robot control. However, many of them can be regarded as special cases of the class known as computed torque control [9]. It is a special application of feedback linearization technique used in nonlinear control systems [10]. Computed torque controllers can be very effective, since they provide us independent joint control, which can then be used together with some classical and modern design techniques, as we will see in rest of this chapter.

The robot arm dynamics can be written in compact matrix form as:

$$A(q)\ddot{q}+C(q,\dot{q})\dot{q}-Q^g=Q^m \tag{8}$$

where $A(q)$ represents basic metric tensor (or inertia matrix), $C(q,\dot{q})$ is matrix that includes centrifugal and Coriolis effects, and Q^g and Q^m are gravity term and motor torque vectors, respectively [3]. Now, suppose that a desired trajectory $q_d(t)$ has been selected for the arm motion. To ensure trajectory tracking, define an output or tracking error as:

$$e(t)=q_d(t)-q(t). \tag{9}$$

To demonstrate the influence of the input $Q^m(t)$ on the tracking error, differentiate twice to obtain:

$$\dot{e}(t)=\dot{q}_d(t)-\dot{q}(t), \tag{10}$$

$$\ddot{e}(t)=\ddot{q}_d(t)-\ddot{q}(t). \tag{11}$$

Solving now for $\ddot{q}(t)$ in Eq.(8) and substituting into the last equation yields:

$$\ddot{e}(t)=\ddot{q}_d(t)+A^{-1}\left(C(q,\dot{q})\dot{q}-Q^g-Q^m\right). \tag{12}$$

Defining the control input function as:

$$u(t)=\ddot{q}_d(t)+A^{-1}\left(C(q,\dot{q})\dot{q}-Q^g-Q^m\right), \tag{13}$$

the feedback linearizing transformation may be inverted to yield:

$$Q^m(t)=A\left(\ddot{q}_d(t)-u(t)\right)+C(q,\dot{q})\dot{q}-Q^g. \tag{14}$$

Equation above is known as computed torque control law [3,9]. If we select a control *u(t)* that stabilizes *e(t)* so that it goes to zero, than the nonlinear control input given by Eq.(14) will cause the robot arm to follow the desired trajectory $q_d(t)$. In fact, substituting Eq.(14) into Eq.(8) yields:

$$A(q)\ddot{q}+C(q,\dot{q})\dot{q}-Q^g=A(q)\left(\ddot{q}_d-u\right)+C(q,\dot{q})\dot{q}-Q^g. \tag{15}$$

or

$$\ddot{e}=u. \tag{16}$$

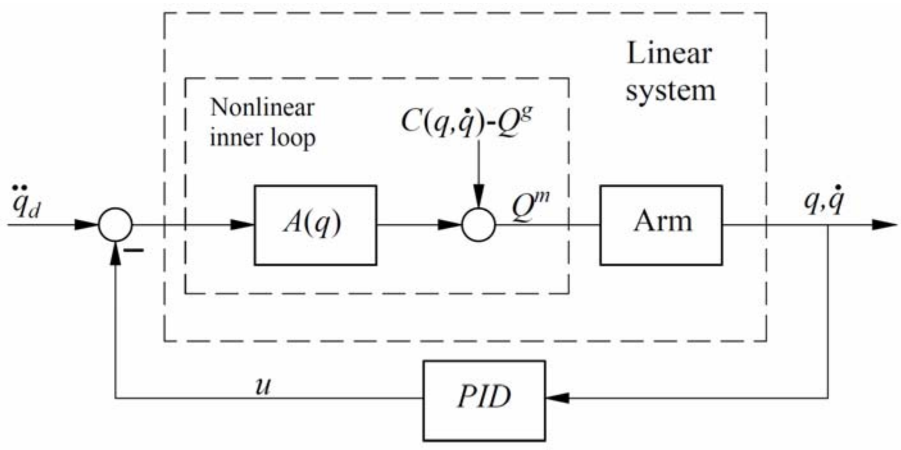

Fig. 4. Computed torque control scheme, showing inner and outer loop.

The nonlinear transformation (13) has converted a complicated nonlinear controls design problem into a simple design problem for a linear system consisting of *n* decoupled subsystems, each obeying Newton's laws. The resulting control scheme appears in Figure 4. It consists of an inner nonlinear loop plus an outer control signal *u(t)*. Since *u(t)* will depend on *q(t)* and $q_d(t)$, the outer loop will be a feedback loop. There are several ways for selecting *u(t)*, including robust and adaptive control techniques. One way to select *u(t)* is as the proportional plus derivative feedback, i.e. as *PD* controller:

$$u(t) = -K_d \dot{e}(t) - K_p e(t). \tag{17}$$

Then the overall robot arm input becomes:

$$Q^m(t) = A\left(\ddot{q}_d(t) + K_d \dot{e}(t) + K_p e(t)\right) + C(q, \dot{q})\dot{q} - Q^g. \tag{18}$$

The closed loop error dynamics are:

$$\ddot{e}(t) + K_d \dot{e}(t) + K_p e(t) = 0, \tag{19}$$

We can see the error system asymptotically stable as long as the K_p and K_d are all positive. However, in the presence of constant disturbances, *PD* control gives a nonzero steady-state error. In that case, proportional-integral-derivative (*PID*) controllers of the following form:

$$u(t) = -K_d \dot{e}(t) - K_p e(t) - K_i \int e(t), \tag{20}$$

will cause the error ***e(t)*** to go to zero. Selection of controller and its parameters depends on the performance objectives, e.g. the type of the desired trajectory, whether the closed loop response should be with or without overshoot, are there unknown disturbances or not, etc.

4. Simulation results

In this section we showed numerical simulation of NeuroArm robotic arm controlled with a PD computed torque controller. The desired trajectories for all seven joints are:

$$q_{d,i}(t) = \sin(3\pi t), \quad i = 1, 2, \ldots, 7. \tag{21}$$

PD controller gains are chosen to achieve zero overshoot of closed loop response:

$$K_{p,i} = 25, \quad K_{d,i} = 10, \quad i = 1, 2, \ldots, 7. \tag{22}$$

By applying computed torque and *PD* control, closed loop dynamics for every joint q_i, $i = 1, 2, \ldots, 7$, is described with Eq.(19). For the same desired trajectory $q_{d,i}$ given by (21), and same controller parameters (22), closed loop response will also be the same for all seven joints. Hence, it is enough to show the closed loop response for one joint, since all other time responses will be equal. The result of the simulation is shown in Figure 5. The initial conditions results in a large initial error that vanishes within approximately one second. Disturbance signal $d(t)$ of amplitude 20 applied to the system at time interval $t \in [2,\ 2.1]$ was successfully canceled by the *PD* control action.

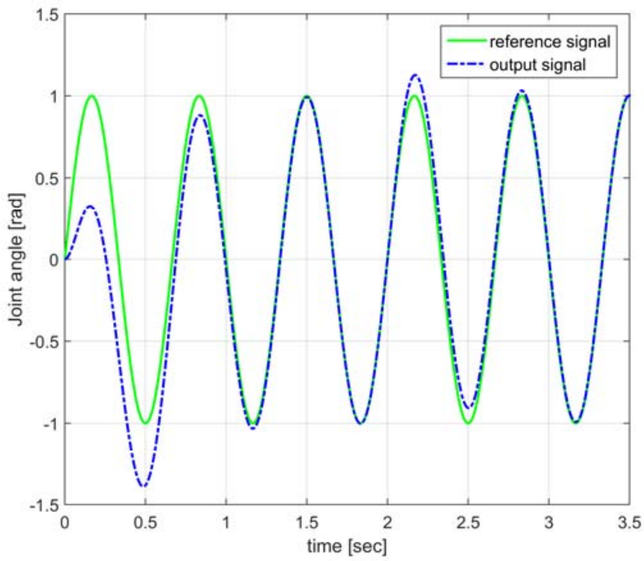

Fig. 5. Joint angles $q_i(t)$, $i = 1, 2, \ldots, 7$.

Figures 6, 7 and 8 show control torques for all joints. The larger torque corresponds to the inner motors, which must move more links than motors which are positioned higher in the kinematic chain. It is important to note that although inverse dynamics control results in decoupled subsystems at the outer-loop level, it does not result in a decoupled joint-control level. This is because the inner loop scrambles the signal $u(t)$ among all the joints. Thus, information on all joint position $q(t)$ and velocities $\dot{q}(t)$ is generally needed to compute the control $Q^m(t)$ for any given joint.

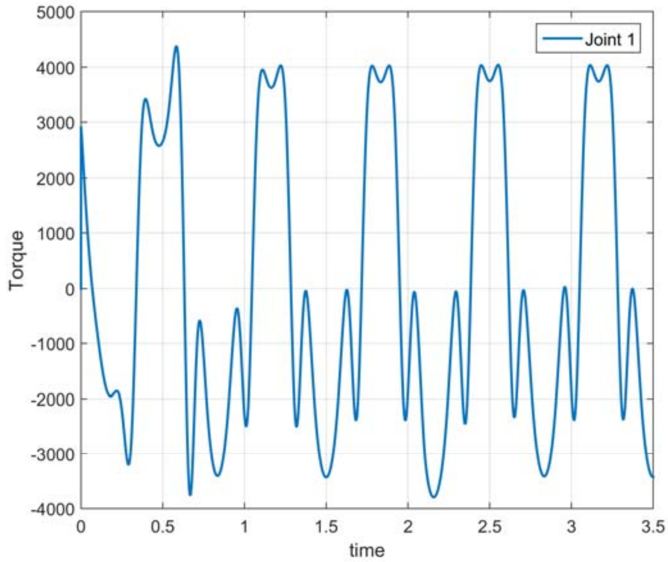

Fig. 6. Torque input $Q_1^m(t)$.

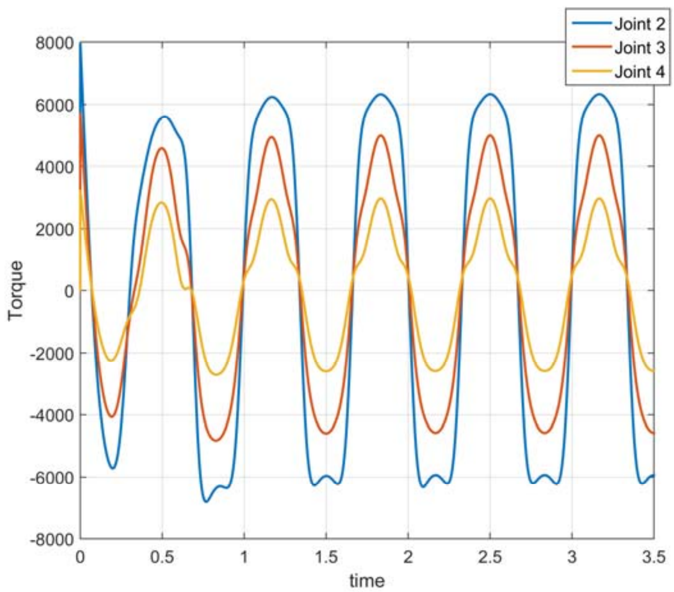

Fig. 7. Torque inputs $Q_i^m(t)$, $i = 2,3,4$.

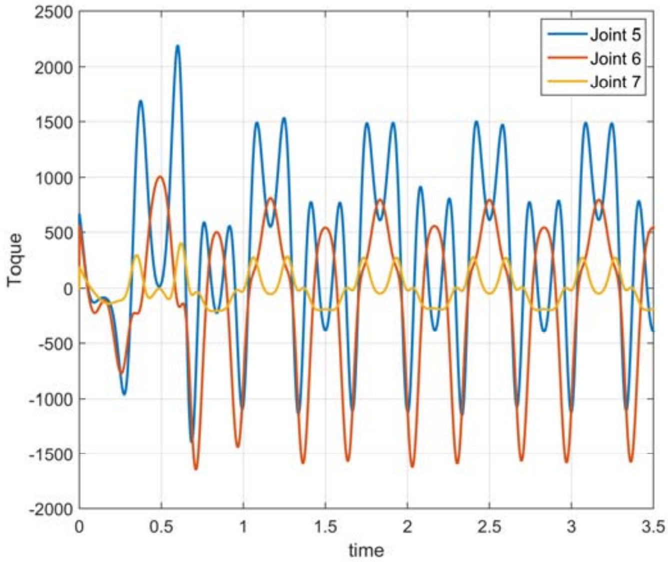

Fig. 8. Torque inputs $Q_i^m(t),\ i=5,6,7.$

5. Conclusions

In this paper kinematic and dynamic model of NeuroArm robotic arm with 7 degrees of freedom is presented. By applying Rodriguez method, Lagrange's equations of second kind in the covariant form are obtained. Computed torque control, as a special application of feedback linearization technique, is used to obtain linear and decoupled dynamic model, after which classical *PD* controller is introduced. Proposed manipulator control scheme is numerically simulated and tested. Trajectory tracking problem is considered in order to verify that robot dynamics model is well derived, along with its control system.

Acknowledgments

Authors gratefully acknowledge the support of Ministry of Education, Science and Technological Development of the Republic of Serbia under the project TR 33047 (P.D.M.), as well as supported by project TR 35006 (M.P.L.) and TR 33020 (T.B.Š).

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Appendix A.

Kinetic energy of the system can be expressed in the form

$$E_k = \frac{1}{2}\sum_{\alpha=1}^{7}\sum_{\beta=1}^{7} a_{\alpha\beta}\dot{q}^{\alpha}\dot{q}^{\beta} = \frac{1}{2}(\dot{\boldsymbol{q}})[a]\{\dot{\boldsymbol{q}}\}, \tag{A.1}$$

where formula for the coefficients of the metric tensor is of the form

$$a_{\alpha\beta} = \frac{\partial^2 E_k}{\partial\dot{q}^{\beta}\partial\dot{q}^{\alpha}} \Rightarrow a_{\alpha\beta} = a_{\beta\alpha}. \tag{A.2}$$

The coefficients of the basic metric tensor can be calculated by using the metrics of a multibody system

$$a_{\alpha\beta} = \sum_{i=1}^{n} m_i \left(\vec{T}_{\alpha(i)}\right)\left\{\vec{T}_{\beta(i)}\right\} + \left(\vec{\Omega}_{\alpha(i)}\right)\left[J_{Ci}\right]\left\{\vec{\Omega}_{\beta(i)}\right\}. \tag{A.3}$$

For example, coefficient a_{34} can be obtained by using (A.3)

$$\begin{aligned} a_{34} &= \sum_{i=\sup(3,4)}^{7} m_i \left(\vec{T}_{3(i)}\right)\left\{\vec{T}_{4(i)}\right\} + \left(\vec{\Omega}_{3(i)}\right)\left[J_{Ci}\right]\left\{\vec{\Omega}_{4(i)}\right\} \\ &= m_4\left(\vec{T}_{3(4)}\right)\left\{\vec{T}_{4(4)}\right\} + \left(\vec{\Omega}_{3(4)}\right)\left[J_{C4}\right]\left\{\vec{\Omega}_{4(4)}\right\} + m_5\left(\vec{T}_{3(5)}\right)\left\{\vec{T}_{4(5)}\right\} + \left(\vec{\Omega}_{3(5)}\right)\left[J_{C5}\right]\left\{\vec{\Omega}_{4(5)}\right\} \\ &+ m_6\left(\vec{T}_{3(6)}\right)\left\{\vec{T}_{4(6)}\right\} + \left(\vec{\Omega}_{3(6)}\right)\left[J_{C6}\right]\left\{\vec{\Omega}_{4(6)}\right\} + m_7\left(\vec{T}_{3(7)}\right)\left\{\vec{T}_{4(7)}\right\} + \left(\vec{\Omega}_{3(7)}\right)\left[J_{C7}\right]\left\{\vec{\Omega}_{4(7)}\right\}. \end{aligned} \tag{A.4}$$

In the similar manner, other coefficients $a_{11}, a_{12}, \ldots, a_{17}, a_{21}, a_{22}, \ldots, a_{27}, \ldots, a_{71}, \ldots, a_{77}$, can be calculated.

By definition, expression for Christoffel symbols of the first kind is of the form

$$\Gamma_{\alpha\beta,\gamma} = \sum_{i=\sup(\beta,\gamma)}^{7} m_i\overline{\xi}_{\alpha}\left(\vec{e}_{\alpha}\times\vec{T}_{\beta(i)}\right)\left\{\vec{T}_{\gamma(i)}\right\} + \xi_{\alpha}\xi_{\beta}\xi_{\gamma}\left(\vec{e}^{(i)}_{\beta\gamma}\right)\left[\Pi^{(i)}_{Ci}\right]\left\{\vec{e}^{(i)}_{\alpha}\right\},\ \alpha\le\beta,\ \vec{e}^{(i)}_{\beta\gamma} = \vec{e}_{\beta}\times\vec{e}_{\gamma}, \tag{A.5}$$

where $\Pi^{(i)}_{Ci}$ is the planar tensor of inertia given in the form

$$\Pi^{(i)}_{Ci} = \begin{bmatrix} J_{\eta_i C_i \zeta_i} & J_{\eta_i\xi_i} & J_{\zeta_i\xi_i} \\ J_{\eta_i\xi_i} & J_{\xi_i C_i \zeta_i} & J_{\zeta_i\eta_i} \\ J_{\zeta_i\xi_i} & J_{\zeta_i\eta_i} & J_{\eta_i C_i \xi_i} \end{bmatrix}. \tag{A.6}$$

By taking into account that $\alpha \le \beta$, and properties of symmetry and antisymmetry, we have

$$\Gamma_{\alpha\beta,\gamma} = \Gamma_{\beta\alpha,\gamma},\ \Gamma_{\alpha\beta,\gamma} = -\Gamma_{\alpha\gamma,\beta},\ \alpha\le\beta. \tag{A.7}$$

For example, Christoffel symbol $\Gamma_{23,5}$ is then determined as

$$\begin{aligned} \Gamma_{23,5} &= \sum_{i=\sup(3,5)}^{7} m_i\overline{\xi}_2\left(\vec{e}_2\times\vec{T}_{3(i)}\right)\left\{\vec{T}_{5(i)}\right\} + \xi_2\xi_3\xi_5\left(\vec{e}^{(i)}_{35}\right)\left[\Pi^{(i)}_{Ci}\right]\left\{\vec{e}^{(i)}_{2}\right\} \\ &= m_5\overline{\xi}_2\left(\vec{e}_2\times\vec{T}_{3(5)}\right)\left\{\vec{T}_{5(5)}\right\} + \overline{\xi}_2\overline{\xi}_3\overline{\xi}_5\left(\vec{e}^{(5)}_{35}\right)\left[\Pi^{(5)}_{C5}\right]\left\{\vec{e}^{(5)}_{2}\right\} \\ &+ m_6\overline{\xi}_2\left(\vec{e}_2\times\vec{T}_{3(6)}\right)\left\{\vec{T}_{5(6)}\right\} + \overline{\xi}_2\overline{\xi}_3\overline{\xi}_5\left(\vec{e}^{(6)}_{35}\right)\left[\Pi^{(6)}_{C6}\right]\left\{\vec{e}^{(6)}_{2}\right\} \\ &+ m_7\overline{\xi}_2\left(\vec{e}_2\times\vec{T}_{3(7)}\right)\left\{\vec{T}_{5(7)}\right\} + \overline{\xi}_2\overline{\xi}_3\overline{\xi}_5\left(\vec{e}^{(7)}_{35}\right)\left[\Pi^{(7)}_{C7}\right]\left\{\vec{e}^{(7)}_{2}\right\}. \end{aligned} \tag{A.8}$$

In the similar manner, other Christoffel symbols can be calculated.