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Robust PID Control for Robot Manipulators with Parametric Uncertainties

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***Abstract*—Most of the industrial robots use a classical PID-type controller for positioning tasks. A main reason for this is its effectiveness in regulation tasks, simple linear structure, and easy implementation. Moreover, it is well known that the robustness properties of a PID controller make it an excellent choice for set point control of robot manipulators. In this paper, a robust PID controller is designed in order to cope with modeling uncertainties and unmodelled dynamics. First, dynamic model of robot manipulator is derived using the Rodriquez approach. Then, a PID controller is applied to feedback linearized robotic system in order to suppress constant disturbance and achieve good set-point control. The proposed controller contains one adjustable parameter λ, which has a direct influence on the time constant of the closed loop system. By adjusting it, one can accomplish a compromise between the robustness and performance indices. Using this algorithm, simulation results of a NeuroArm robotic manipulator executing positioning tasks are shown to demonstrate the efficiency of the proposed controller.**

***Index Terms*— PID control; robust control; robot manipulator.**

I. Introduction

MODEL uncertainties are frequently encountered in robotics. Friction, backlash, joint or robot link flexibility are some examples of these model uncertainties for which only simplified dynamical models exist, and which may cause significant deviations between simulation and experimental results. That is why the development of robust control strategies has attracted considerable attention from robotics community during the past decades. By robust control, we mean strategies designed to cope with parametric uncertainties or unmodelled dynamics. Neglecting these effects significantly decreases system performances. In other words, the smaller the uncertainty on a robot model, the higher performance is attained.

There are a vast amount of different robot control strategies published in the literature. A good survey of early results up to 1999 is given by Sage et al. [1], and a more recent survey is presented in [2]. Classical control strategies in robotics can be found in textbooks like [3-5]. Based on a very large number of published papers, robust control strategies for robot manipulators can be roughly organized in following categories: linear schemes, H_∞ approach, Lyapunov based schemes, passivity based techniques, sliding mode schemes, robust adaptive control, and disturbance observer based control. The first category of control algorithms, which is of interest in this paper, follows linear approach. It is based mostly on feedback linearization technique, known in the literature also under the names computed torque control and inverse dynamics control. This technique is aimed at linearizing and decoupling robot manipulator dynamics. Nonlinearities in a dynamical model can be simply compensated by adding these forces to the control input. This way, the overall closed loop system behaves like a linear system, and a simple linear algorithm can be designed to control it.

Classical PID control is a usual tool for the stabilization of robot manipulators in real applications. It is proven that a simple PD controller can be a good robust solution for set point control of robot manipulators [6]. The main reason for this is because the nonlinear robot dynamics is equivalent to a linear model described by a set of double integrators for which a PD controller can be designed. Also, nonlinearities become less important if actuators with high gear ratio are used. This way, dynamic coupling effects from the motion of other joints decrease, and the robot control can be decoupled into independent joint control. These are one of the main reasons why linear approach has been accepted in the implementation of current industrial robots. Also in practice, integral term (PID control) is added when we want to reject constant perturbations.

One of the first attempts for adjusting the parameters of PID controllers was given by Ziegler and Nichols [7], who proposed to use sustained oscillations for process dynamics characterization and PID controller tuning. Since then, numerous methods for determining the controller parameters have been reported in the literature. A good overview of tuning formulae is given in [8], while some of the optimization methods are presented in [9-12]. The largest number of tuning formulae is derived for the ideal PID controller, taking the noise filter time constant T_f equal to zero. According to [13], the derivative (noise) filter must be an integral part of the PID optimization and tuning procedures. Also, most of the PID controllers operate as regulators [14], and rejection of the step load disturbance is one of their primary goals. In this paper, controller parameters

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are determined based on one adjustable parameter λ which has a direct influence on the time constant of the closed loop system. PID controller is designed under the constraints on robustness and sensitivity to measurement noise. By adjusting it, one accomplishes a compromise between performance and robustness indices, which is a key issue in control design. This is of most importance, since there always exists a difference between simulated and real robotic system, and well designed robust controller will guarantee the stability of closed loop system in all situations.

The rest of the paper is organized as follows. In Section 2 dynamic model of a robot manipulator is derived using the Rodriquez approach. After the well-known feedback linearization procedure is applied to robotic system, robust PID controller is designed for the nominally linear system in order to cope with modeling errors. This is covered in Section 3. The efficiency of the controller is demonstrated in Section 4 through the simulation of NeuroArm robotic manipulator. Section 5 concludes the paper.

II. Robot Dynamics

The mechanical structure of a robot manipulator consists of a sequence of rigid bodies (or links) interconnected by means of joints. In this paper Rodriquez approach [15] is used to obtain equations of motion. The open chain system of rigid bodies (V_1), (V_2), … , (V_n) is shown Figure 1. The rigid body (V_1) is connected to the fixed stand. Two neighboring bodies (V_{i-1}) and (V_i) are connected together with a joint (i), which allows translation or rotation of the body (V_i) in respect to body (V_{i-1}). The values q^i represent generalized coordinates. The reference frame $Oxyz$ is inertial Cartesian frame, and the reference frame $O\xi\eta\zeta$ is local body frame which is associated to the body (V_i). At initial time, the corresponding axes of reference frames were parallel. This configuration is called reference configuration and it is denoted by (0). The symbol $\overline{\xi_i}$ and ξ_i can be introduced, which are defined as

$$\xi_i = 1,\ \ \overline{\xi_i} = 0, \tag{1}$$

in the case when bodies (V_{i-1}) and (V_i) are connected with prismatic joint, and

$$\xi_i = 0,\ \ \overline{\xi_i} = 1, \tag{2}$$

in the case when bodies (V_i) and (V_{i-1}) are connected with cylindrical joint [16]. The geometry of the system is defined by the unit vectors $\vec{e}_i$ and position vectors $\vec{\rho}_i$ and $\vec{\rho}_{ii}$ expressed in local coordinate systems $C_i\xi_i\eta_i\zeta_i$ connected to mass centers of bodies in a multibody system [17]. Unit vectors $\vec{e}_i$, i=1,2, … , n is describing the axis of rotation (translation) of the i-th segment with respect to the previous segment, and $\vec{\rho}_{ii} = \overrightarrow{O_i^{'}O_{i+1}}$ denotes a vector between two neighboring joints in a multibody system, while position of the center of mass of i-th segment is expressed by vectors $\vec{\rho}_{ii} = \overrightarrow{O_{i+1}^{'}C_i}$. For the entire determination of this mechanical system, it is necessary to specify masses m_i and tensors of inertia J_{Ci} expressed in local coordinate systems.

If we have a kinetic energy of the system in terms of generalized coordinates and its derivatives, one can write dynamic equations of the system in terms of Lagrange equations of the second kind. After some transformations, equations of motion of a multibody system in a covariant form can be written as

$$\sum_{\alpha=1}^{n} a_{\gamma\alpha}(q)\ddot{q}^{\alpha} + \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\Gamma_{\alpha\beta,\gamma}(q)\dot{q}^{\alpha}\dot{q}^{\beta} = Q_{\gamma},\quad \gamma = 1,\ldots,n. \tag{3}$$

Here, q^{α} and q^{β} denote generalized coordinates, n is a number of bodies in the system, $a_{\gamma\alpha} = a_{\alpha\gamma}$ are elements of the basic metric tensor, and $\Gamma_{\alpha\beta,\gamma}$ are Cristoffel symbols of the first kind. Coefficients of the metric tensor are defined as

$$a_{\alpha\beta} = \sum_{i=1}^{n} m_i\left(\vec{T}_{\alpha(i)}\right)\left\{\vec{T}_{\beta(i)}\right\} + \left(\vec{\Omega}_{\alpha(i)}\right)\left[J_{Ci}\right]\left\{\vec{\Omega}_{\beta(i)}\right\}, \tag{4}$$

where quasi-base vectors $\vec{T}_{\alpha(i)}$ and $\vec{\Omega}_{\alpha(i)}$ are

$$\vec{T}_{\alpha(i)} = \begin{cases} \overline{\xi}_{\alpha}\vec{e}_{\alpha} \times \left(\sum_{k=\alpha}^{i}\left(\vec{\rho}_{kk} + \xi_k\vec{e}_k q^k\right) + \vec{\rho}_i\right) + \xi_{\alpha}\vec{e}_{\alpha}, & \forall\alpha \le i, \\ 0, & \forall\alpha > i, \end{cases} \tag{5}$$

$$\vec{\Omega}_{\alpha(i)} = \begin{cases} \overline{\xi}_{\alpha}\vec{e}_{\alpha}, & \forall\alpha \le i, \\ 0, & \forall\alpha > i, \end{cases} \tag{6}$$

and Cristoffel symbols are

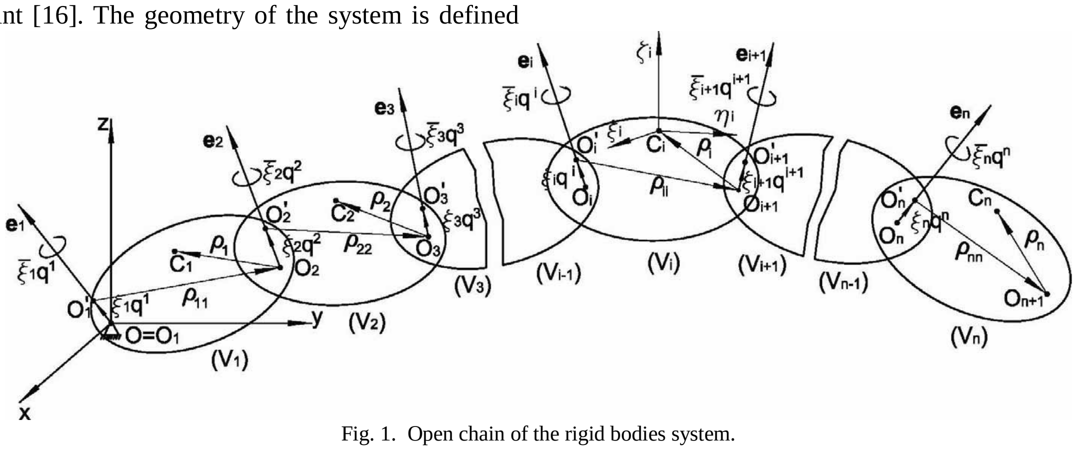

Fig. 1. Open chain of the rigid bodies system.

$$\Gamma_{\alpha\beta,\gamma}=\frac{1}{2}\left(\frac{\partial a_{\beta\gamma}}{\partial q^{\alpha}}+\frac{\partial a_{\gamma\alpha}}{\partial q^{\beta}}-\frac{\partial a_{\alpha\beta}}{\partial q^{\gamma}}\right),\quad \alpha,\beta,\gamma=1,\ldots,n. \tag{7}$$

Regardless of the chosen theoretical approach, in [15] it is shown that it could be started from different theoretical aspects (e.g. general theorems of dynamics, d'Alambert's principle, Lagrange's equation of second kind, Appell's equations, etc.) to get to the equations of motion of the robotic system expressed in the covariant form as (3). It is also shown that for the above system of differential equations it is convenient to use Rodriquez approach for matrices of coordinate transformations. On the right hand side of (3), the generalized forces Q_γ represent external forces Q_γ^g, Q_γ^m which denote the generalized gravitational forces and motor torques, respectively. For details of the calculation of the basic metric tensor and Cristoffel symbols for robot manipulators, the reader is referred to [15].

The robot arm dynamics given by (3) can be rewritten in compact matrix form as

$$A(\boldsymbol{q})\ddot{\boldsymbol{q}}+C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}-\boldsymbol{Q^g}=\boldsymbol{Q^m} \tag{8}$$

where $\boldsymbol{q}(t)\in\mathbb{R}^n$ is the vector of the generalized coordinates, $A(\boldsymbol{q})\in\mathbb{R}^{n\times n}$ represents basic metric tensor (or inertia matrix), $C(\boldsymbol{q},\dot{\boldsymbol{q}})\in\mathbb{R}^{n\times n}$ is a matrix that includes centrifugal and Coriolis effects, and $\boldsymbol{Q^g}\in\mathbb{R}^n$ and $\boldsymbol{Q^m}\in\mathbb{R}^n$ are gravity term and motor torque vectors, respectively. Equation (8) is more suitable for designing control system, which is covered in next section.

III. Control Design

A. Feedback Linearization

A widely used approach for the control of a nonlinear system is feedback linearization [18]. The idea is to perform a nonlinear feedback operation which cancels the nonlinearities of the system dynamics. In this manner, the closed loop system becomes nominally linear. This approach, as mentioned above, is also known in the literature as the computed torque control, and inverse dynamics control [4].

In order to incorporate modeling uncertainties into the model of the rigid robot (8), the matrices A and C, and the vector $\boldsymbol{Q^g}$ are split up into a nominal part and an uncertain part

$$\left(A_N(\boldsymbol{q})+\Delta A\right)\ddot{\boldsymbol{q}}+\left(C_N(\boldsymbol{q},\dot{\boldsymbol{q}})+\Delta C\right)\dot{\boldsymbol{q}}-\left(\boldsymbol{Q_N^g}+\Delta\boldsymbol{Q^g}\right)=\boldsymbol{Q^m} \tag{9}$$

Only the nominal part of the model can be linearized. Choosing

$$\boldsymbol{Q^m}=A_N(\boldsymbol{q})\boldsymbol{u}+C_N(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}-\boldsymbol{Q_N^g} \tag{10}$$

leads to

$$\ddot{\boldsymbol{q}}=\boldsymbol{u}+\boldsymbol{\eta} \tag{11}$$

where $\boldsymbol{u}$ represents the new control signal, and $\boldsymbol{\eta}$ is the model uncertainty

$$\boldsymbol{\eta}=-A^{-1}\left(\Delta A\boldsymbol{u}+\Delta C\dot{\boldsymbol{q}}-\Delta\boldsymbol{Q^g}\right) \tag{12}$$

which can be regarded as disturbance signal. The nonlinear transformation (10) has converted a complicated nonlinear controls design problem into a simpler one, consisting of n decoupled subsystems, each obeying Newton's laws. There are several ways for selecting $\boldsymbol{u(t)}$, including some robust control techniques. One way to select $\boldsymbol{u(t)}$ is as the proportional-integral-derivative feedback, i.e. as PID controller:

$$\boldsymbol{u}(t)=-K_p\boldsymbol{q}_f-K_d\dot{\boldsymbol{q}}_f+K_i\int\left(\boldsymbol{q}_d-\boldsymbol{q}_f\right)dt \tag{13}$$

wherein $\boldsymbol{q}_f$ is filtered signal $\boldsymbol{q}$, $\boldsymbol{q}_d$ is the reference signal, and K_p, K_d, and K_i are diagonal matrices containing proportional, derivative, and integral gains, respectively. The resulting control scheme appears in Figure 2. It consists of an inner nonlinear loop plus an outer control signal $\boldsymbol{u(t)}$.

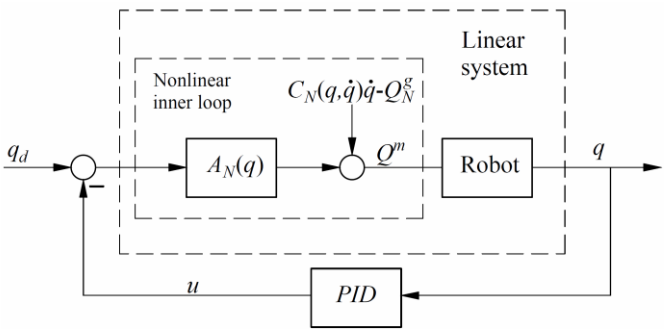

Fig. 2. Computed torque control scheme, showing inner and outer loop

PID controller must be robust enough in order to guarantee closed loop stability due to model uncertainties $\boldsymbol{\eta}(t)$. On the other hand, it also must satisfy desired performance specifications, at least for the nominal case.

B. Tuning of the PID Controller

For the purpose of tuning the PID controller, we observe the linear closed loop system given in Fig. 3. The plant dynamics is defined by the following transfer function

$$G(s)=\frac{1}{s^2} \tag{14}$$

which follows directly from (11). Parallel PID controller in series with the noise filter is given by

$$C(s)=\frac{k_d s^2+k_p s+k_i}{s\left(T_f s+1\right)} \tag{15}$$

wherein T_f is the filter time constant, and k_d, k_p, and k_i are derivative, proportional, and integral gain, respectively. Since original nonlinear system (8) resulted into n linear, decoupled subsystems, it is enough to tune PID controller in Fig. 3 for one arbitrary joint, and use those parameters for other joints.

Tuning method derived below is explained thoroughly in [19], and reader is referred to it for detailed explanations. In this paper, parameters of the controller are derived specifically for plant given by (14), since the nonlinear robot dynamics is equivalent to that linear model. To begin with, we start from the load disturbance response of the closed loop system in Fig. 3, defined by

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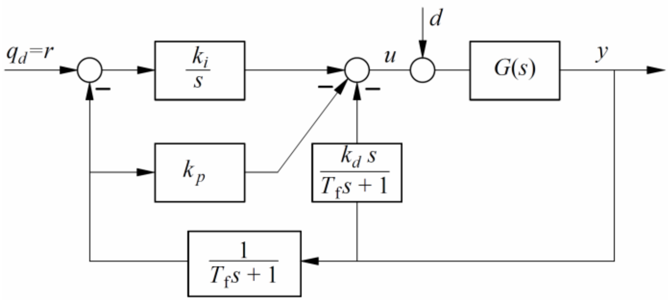

Fig. 3. Plant $G(s)$ with controller $C(s)$

$$G_d(s) = \frac{G(s)}{1 + C(s)G(s)} \quad (16)$$

$$R(s) = 0,\ D(s) = \frac{1}{s} \quad (17)$$

The complementary sensitivity function [20] of the closed loop system is given by $T(s) = C(s)G(s)/(1 + C(s)G(s))$. Let the complementary sensitivity function be defined by

$$T(s) = \frac{\eta_2 s^2 + \eta_1 s + 1}{(\lambda^2 s^2 + 2\zeta\lambda s + 1)^2} \quad (18)$$

wherein time constant $\lambda > 0$, η_1 and η_2 are free parameters which will be determined to obtain the desired dynamic characteristics of the closed loop system. The parameter $\zeta > 0$ is a damping ratio, and from here onwards we adopt $\zeta = 1$, since it corresponds to critically damped system. Then, after some calculations [19], one obtains the controller $C^*(s)$ in the form

$$C^*(s) = \frac{1}{k_u} \frac{(s^2 + k_u)(\eta_2 s^2 + \eta_1 s + 1)}{(\lambda s + 1)^4 - (\eta_2 s^2 + \eta_1 s + 1)} \quad (19)$$

where $C(s) = k_u(1 + C^*(s))$. Parameter $k_u > 0$ is the ultimate gain. Using (19), and for an oscillatory process $G^*(s) = k_u/(s^2 + k_u)$, one obtains that the load disturbance transfer function $G_d^*(s)$ is defined by

$$G_d^*(s) = \frac{G^*(s)}{1 + C^*(s)G^*(s)} = \frac{(\lambda s + 1)^4 - (\eta_2 s^2 + \eta_1 s + 1)}{s^2 + k_u} \frac{k_u}{(\lambda s + 1)^4} \quad (20)$$

To avoid oscillatory load disturbance response, free parameters η_1 and η_2 are determined to satisfy the condition

$$(\lambda s + 1)^4 - (\eta_2 s^2 + \eta_1 s + 1)\Big|_{s = \pm i\sqrt{k_u}} = 0 \quad (21)$$

in order to cancel the pair of complex conjugate poles $s_{1,2} = \pm i\sqrt{k_u}$ in (20). These values of η_1 and η_2 are given by

$$\eta_1 = 4\lambda(1 - k_u\lambda^2),\ \eta_2 = \lambda^2(6 - k_u\lambda^2) \quad (22)$$

Now, one obtains the desired load disturbance transfer function of the closed loop system in the form

$$G_d^*(s) = k_u\lambda^3 \frac{s(\lambda s + 4)}{(\lambda s + 1)^4} \quad (23)$$

Implementation of the PID controller given by (15) is obtained by combining (19), (22), and $C(s) = k_u(1 + C^*(s))$

$$k_d s^2 + k_p s + k_i = s(T_f s + 1)(1 + C^*(s))k_u \quad (24)$$

Finally, parameters of the PID controller are defined by

$$k_d = \frac{3}{2\lambda},\ k_p = \frac{1}{\lambda^2},\ k_i = \frac{1}{4\lambda^3},\ T_f = \frac{\lambda}{4} \quad (25)$$

Tuning formulae (25) contain one free parameter: closed loop time constant λ. One can easily verify from (23) that system response is faster as λ gets smaller. Hence, this parameter can be used to obtain a desired settling time of the closed loop system or, to specify the desired sensitivity to the high frequency measurement noise M_n, defined by

$$M_n = \lim_{\omega \to \infty} \left| \frac{-C(i\omega)}{1 + C(i\omega)G(i\omega)} \right| \approx \frac{k_d}{T_f} \quad (26)$$

This is very important property, since an inadequate sensitivity to measurement noise is the reason why derivative action is often excluded in industry control. Also, by decreasing time constant λ one can reject load disturbance very efficiently, as we will show in next section. With proposed method, one can accomplish very good robustness/performance trade-off, defined by the desired M_n and the desired maximum sensitivity M_s, given by

$$M_s = \max_{\omega} \left| \frac{1}{1 + C(i\omega)G(i\omega)} \right| \quad (27)$$

The maximum sensitivity M_s is good robustness measure. Smaller values of M_s are desirable, meaning the system is more stable and robust with respect to process variations. This is very important, since we want to ensure that even we have inaccuracies in our robot model, we can still guarantee stability and performance.

IV. Simulation Results

The effectiveness of the proposed control method was examined by computer simulation. The manipulator used for simulation is a NeuroArm robotic arm, shown in Fig. 4. It is an integral part of the Laboratory of Applied Mechanics, at Faculty of Mechanical Engineering in Belgrade. This arm has seven degrees of freedom. First three revolute joints are used for positioning of the end-effector, and the following three joints form the spherical wrist, used to accomplish orientation. The last joint is the gripper. Since the goal of this paper is designing a controller for positioning tasks, we will simplify our robotic arm to a three degree of freedom model, considering only first three joints. Position of the end-effector depends only on the configuration of joints 1, 2 and 3, and hence it follows $q^4 = q^5 = q^6 = q^7 = 0 = \text{const}$.

The considered problem in simulations is to move the robot end-effector from an initial to a distant final desired position.

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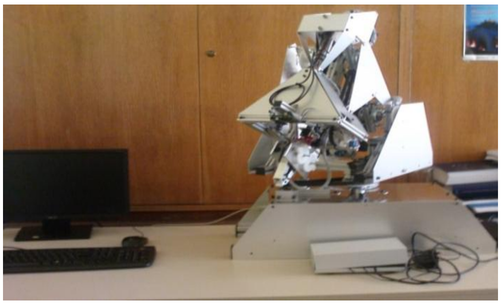

Fig. 4. Laboratory NeuroArm robotic arm

An initial configuration is given by $q_0^1 = q_0^2 = 0$, and $q_0^2 = \pi/4$ rad (initial value q_0^2 is different from zero in order to avoid singular position), while the final positions are $q_d^1 = q_d^2 = q_d^3 = 1$ rad. Simulation tests were conducted for different values of λ parameter and for different physical parameters of the system, to illustrate the features and performances of the PID control system.

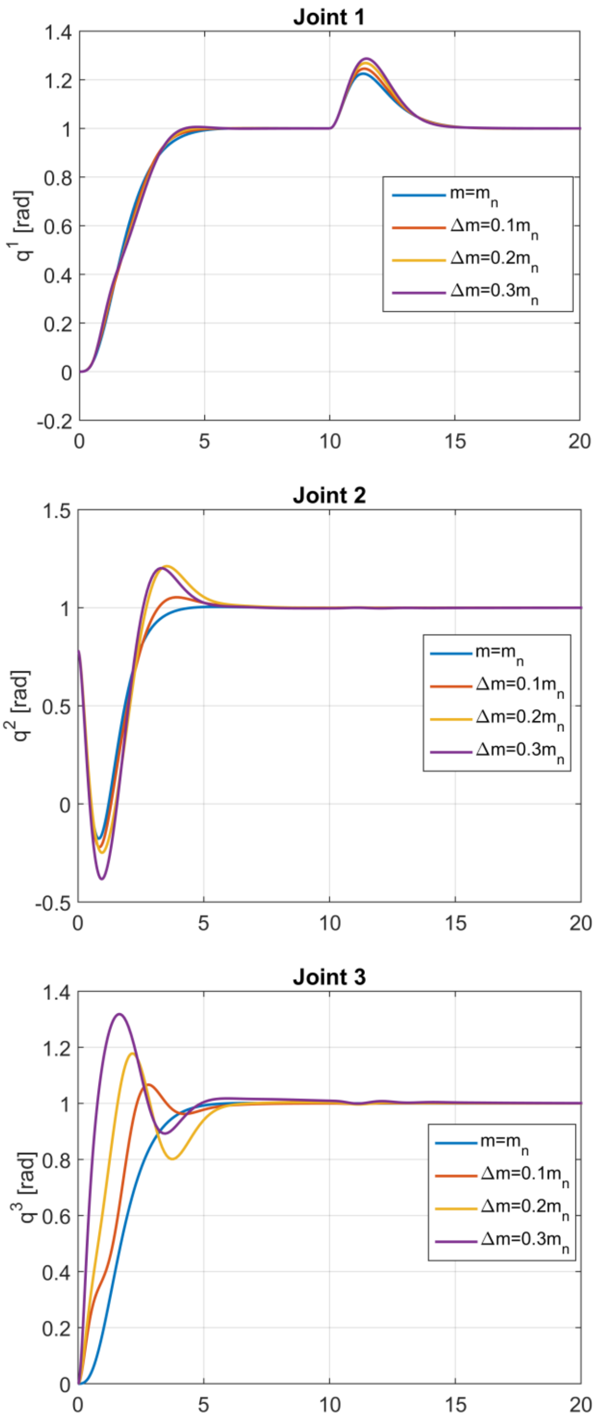

Fig. 5. Reference step responses for $\lambda = 0.5$ and mass variations.

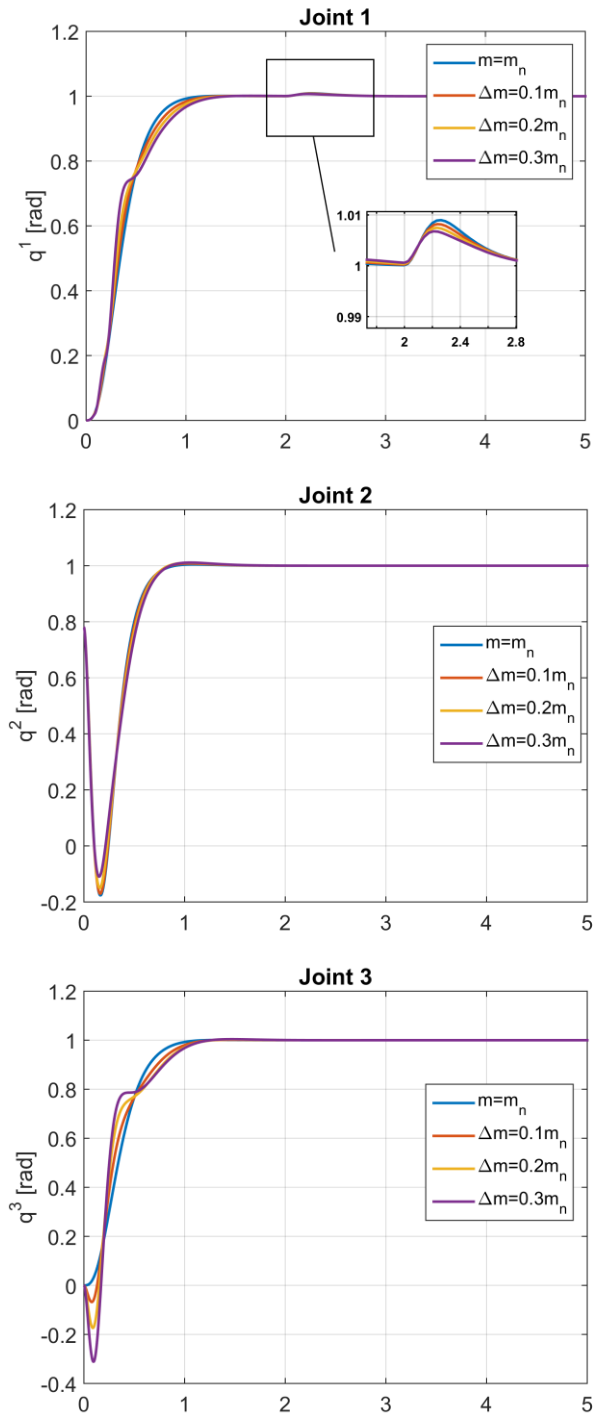

Fig. 6. Reference step responses for $\lambda = 0.1$ and mass variations.

In order to incorporate modeling uncertainties in simulations, masses of robot links are changing 10%, 20% and 30% with respect to nominal mass values. This directly affects on the dynamical parameters of the system, which in turn results in different closed loop responses. On the other side, changing parameter λ also greatly influence on performance characteristics of the system.

Figure 5 shows the reference step responses of first three joints for $\lambda = 0.5$. Using (25), one obtains the following parameters of PID controller: $k_d = 3$, $k_p = 4$, $k_i = 2$, $T_f = 0.125$. We can see that the joint positions reach asymptotically the desired ones. Step responses are without overshoot for nominal case, as predicted. As masses of robot links more deviate from their nominal values, closed loop responses for second and third joint deteriorate. Overshoot increases, and settling time becomes sligthly larger. However, the stability of the system is preserved, which is a proof of robustness of the designed controller. It is interesting to note that step response of the first joint remains practically the same with mass variations. Also, load disturbance of unit amplitude acting on the first joint and starting at $t = 10$s, is successfully compensated by PID controller.

Now, in Fig. 6 simulations are repeated for different value of parameter λ. For $\lambda = 0.1$, parameters of the PID controller are as follows: $k_d = 15$, $k_p = 100$, $k_i = 250$, $T_f = 0.025$. By decreasing λ we obtain more efficient controller. Settling time for each joint is shorter, and mass variations do not affect on overshoot which practically can be neglected. Also, step load disturbance acting on the first joint at $t = 2$s is so efficiently compensated by the controller that it can barely be seen without zooming on it. Comparing Fig. 5 and 6, one could conclude that performances of the closed loop system are improving as λ decreases. However, there is one drawback that should be pointed out here, and which cannot be seen from these figures. It regards to system's sensitivity to measurement noise. It can be easily proven from (25) and (26) that sensitivity to measurement noise M_n is increasing with λ getting smaller. If there exist a high frequency noise in the system, one should be careful and choose not too small value of λ.

V. Conclusion

In this paper, robust control system is designed in order to cope with parametric uncertainties of robot manipulator. Dynamic equations of NeuroArm robotic system are derived, and then feedback linearization technique is utilized in order to obtain a linear system, which is more suitable for control design. PID controller is introduced, and a procedure for tuning of its parameters is explained. Tuning formulae contain one adjustable parameter, closed loop time constant λ. Simulation results show performance improvements as λ decreases: settling time is shorter, zero overshoot is preserved with regard to mass variations, and load disturbance response is greatly improved. However, sensitivity to measurement noise is increasing with smaller values of λ, which should be paid attention on when adjusting the controller.

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