

# Open-closed Iterative Learning Control Algorithm for Exoskeleton Rehabilitation Purposes

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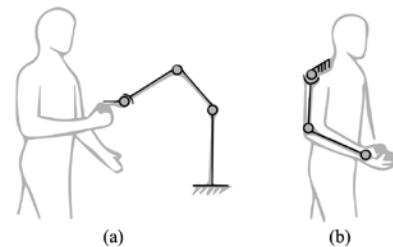
**Abstract.** The paper designs an appropriate iterative learning control(ILC) algorithm based on the trajectory characteristics of upper exoskeleton robotic system .The procedure of mathematical modelling of an exoskeleton system for rehabilitation is given and synthesis of a control law with two loops. First (inner) loop represents exact linearization of a given system, and the second (outer) loop is synthesis of an iterative learning control law which consists of two loops, open and closed loop. In open loop ILC sgnPDD2 is applied, while in feedback classical PD control law is used. Finally, a simulation example is presented to illustrate the feasibility and effectiveness of the proposed advanced open-closed iterative learning control scheme.

## 1 Introduction

Stroke is second cause of mortality and disability in the world, [1]. Patients who survive stroke are faced with some degree of limb impairment, depending on the place in brain structure and size of caused damage. It is widely accepted that brain structure can be reorganized after stroke and thus some functions can be fully or partially recovered. Rehabilitative training plays crucial role in recovery of lost functions, [2]. In order to enhance therapy delivered by therapists, use of robotics emerged as aid in rehabilitation process,[3]. It is noted in [4] and [5] that robot-aided sensorimotor training, especially in upper limbs, shows that more activity leads to better recovery and that recovered functions are sustained over long period.

Rehabilitative robotics of upper limbs started with end-effector robots research. End-effector robot supports patients arm in one point of contact, usually patients hand or forearm. End-effector robot joints movement doesn't coincide with movement of patients arm. These drawbacks with end-effector robots influenced research of exoskeleton rehabilitation robots. Exoskeletons mitigate important flaws of end-effector robots mentioned above [6]. Rehabilitation robots can be developed to assist rehabilitation in individuals with stroke.

Also, in the last three decades, iterative learning control (ILC) has been extensively studied, achieves significant progress in both theory and application, and becomes one of the most active fields in intelligent control and system control,[7-11].



**Fig. 1.** Schematics of end-effector robots (a) and exoskeleton robots (b)

ILC is an intelligent control method for systems which perform tasks repetitively over a finite time interval where ILC approach is an imitation of a human learning process. Intelligent beings tend to learn by performing a trial (i.e. selecting a control input) and observing what was the end result of this control input selection. Emulating human learning, ILC uses knowledge obtained from the previous trial to adjust the control input for the current trial so that a better performance can be achieved. The basic idea of ILC schemes is to refine the control input to make better operation performance of the system on the next trial by use of updated data of the previous trial.

On the other side, rehabilitation training is a kind of repetitive training. Body state of patients will improve with an increase in the number of training while the auxiliary level of robot and electrical stimulation will be reduced. In ILC the control input is directly updated between trials and it is this feature that makes it suitable for exoskeleton robots (i.e robotic assisted stroke rehabilitation),[12].

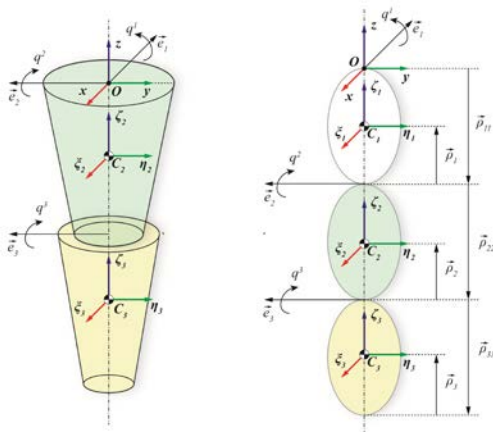
In this paper, a advanced robust open-closed iterative learning control for exoskeleton rehabilitation robots is

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suggested and introduced. First, the procedure of mathematical modeling of an exoskeleton robotic system for rehabilitation is presented using the Rodrigues approach, [13,14]. Then, we propose a joint space trajectory tracking control system consisting of two loops. First (inner) loop represents exact linearization of a given system, and the second (outer) loop is synthesis of a linear control law which consists of two branches, feedforward and feedback branch of ILC control. In feedforward ILC algorithm is applied, while in feedback classical PD control law is applied. Finally, a simulation example is presented to illustrate the feasibility and effectiveness of the proposed advanced open-closed ILC scheme.

## 2 Nonlinear mathematical model of exoskeleton robot

Control object is modelled using Rodrigues approach. This approach is more viable, than Denavit-Hartenberg method, for setting up kinematics and dynamics of biomechanical systems [7]. Validation of control laws is carried out using 3DOF biomechanical system, (Fig 2b). The system consists of two bodies - links. Links are simplified model of human arm attached to exoskeleton. Links are modeled as two truncated cones (Fig.2a).



**Fig. 2.** a) Biomechanical system of upper limb as open chain of the rigid bodies system b) Biomechanical system presented as 3DOFs system –Rodrigues approach

The mechanical structure of a proposed system consists of a sequence of rigid bodies (or links) interconnected by means of one-degree-of-freedom joints forming kinematical pairs of the fifth class, [13]. The open chain system of rigid bodies (V1), (V2),(V3) is shown in Fig. 2b. Two neighboring bodies  $(V_{i-1})$  and  $(V_i)$  are connected with a joint  $i=1,2,3$ , which allows rotation of body  $(V_i)$  in respect to the body  $(V_{i-1})$ . The values  $q^i, i=1,2,3$  represent generalized coordinates and define a configuration of the mechanical model, where  $n=3$  is a number of bodies in the system. The reference frame  $Oxyz$  is the inertial Cartesian frame, and the reference frame  $O_{\xi\eta\zeta}$  is a local body-frame which is associated with the body  $(V_i)$ . At an initial time, the corresponding

axes of reference frames were parallel, i.e. all the variables  $q^i=0, i=1,2,3$  and the robotic system is in reference position. Parameters  $\xi_i, \bar{\xi}_i=1-\xi_i$ , denote parameters in general case for recognizing joints between bodies  $(V_{i-1})$  and  $(V_i)$ , ( $\xi_i=1$ , -prismatic, 0-cylindrical joint). The geometry of the system is defined by the unit vectors  $\bar{e}_i$  and the position vectors  $\bar{\rho}_i$  and  $\bar{\rho}_{ii}$  expressed in local coordinate systems  $C_{i\xi\eta\zeta}$  are connected to mass centers of bodies in a multibody system [13,14]. Unit vectors  $\bar{e}_i, i=1,2,3$  are describing the axis of rotation of the  $i=1,2,3$ -th segment with respect to the previous segment, and  $\bar{\rho}_{ii}$  denotes a vector between two neighboring joints in a multi body system, while the position of the center of mass of  $i$ -th segment is expressed by vectors  $\bar{\rho}_i$ . For the entire determination of this system, it is necessary to specify masses  $m_i$  and tensors of inertia  $J_{Ci}$  expressed in local coordinate systems. Dynamic equations of motion for the robot system can be obtained by applying Lagrange equations of the second kind in the covariant form as follows:

$$\sum_{\alpha=1}^n a_{\gamma\alpha}(q)\ddot{q}_\alpha + \sum_{\alpha=1}^n \sum_{\beta=1}^n \Gamma_{\alpha\beta,\gamma}(q)\dot{q}_\alpha\dot{q}_\beta = Q_\gamma \quad \gamma=1,2,\dots,n \quad (1)$$

where the coefficients  $a_{\gamma\alpha}=a_{\alpha\gamma}$  are the covariant coordinates of the basic metric tensor  $[a_{\gamma\alpha}] \in R^{3 \times 3}$  and  $\Gamma_{\alpha\beta,\gamma}$  present Christoffel symbols of the first kind. Coefficients of the metric tensor are defined as, [13]:

$$a_{\alpha\beta} = \sum_{i=1}^n m_i (\bar{T}_{\alpha(i)}) \{ \bar{T}_{\beta(i)} \} + (\bar{\Omega}_{\alpha(i)}) [J_{Ci}] \{ \bar{\Omega}_{\beta(i)} \}, \quad (2)$$

where quasi-base vectors  $\bar{T}_{\alpha(i)}$  and  $\bar{\Omega}_{\alpha(i)}$  are

$$\bar{T}_{\alpha(i)} = \begin{cases} \bar{\xi}_\alpha \bar{e}_\alpha \times \left( \sum_{k=\alpha}^i (\bar{\rho}_{kk} + \xi_k \bar{e}_k q^k) + \bar{\rho}_i \right) + \xi_\alpha \bar{e}_\alpha, & \forall \alpha \leq i, \\ 0, & \forall \alpha > i, \end{cases} \quad (3)$$

$$\bar{\Omega}_{\alpha(i)} = \begin{cases} \bar{\xi}_\alpha \bar{e}_\alpha, & \forall \alpha \leq i, \\ 0, & \forall \alpha > i, \end{cases} \quad (4)$$

and Cristoffel symbols are

$$\Gamma_{\alpha\beta,\gamma} = \frac{1}{2} \left( \frac{\partial a_{\beta\gamma}}{\partial q^\alpha} + \frac{\partial a_{\gamma\alpha}}{\partial q^\beta} - \frac{\partial a_{\alpha\beta}}{\partial q^\gamma} \right), \quad \alpha, \beta, \gamma = 1, \dots, n. \quad (5)$$

The generalized forces  $Q_\gamma$  can be presented in the following expression (6), wherein  $Q_\gamma^a, Q_\gamma^g, Q_\gamma^v, Q_\gamma^c, Q_\gamma^f$ , denote the generalized control, gravitational, viscous, spring and friction forces, respectively.

$$Q_\gamma = Q_\gamma^a + Q_\gamma^g + Q_\gamma^v + Q_\gamma^c + Q_\gamma^f, \quad \gamma = 1, 2, \dots, n \quad (6)$$

The robot arm dynamics can be written in compact matrix form as (where in our case  $Q_\gamma^v, Q_\gamma^c, Q_\gamma^f = 0, Q^a = v$ ):

$$A(q)\ddot{q} + (C(q, \dot{q}) - Q^s) = A(q)\ddot{q} + n(q, \dot{q}) = v \quad (7)$$

where  $A(q)$  represents basic metric tensor (or inertia matrix),  $C(q, \dot{q})$  is a matrix that includes centrifugal and Coriolis effects, respectively, [15].

### 2.1. State-space representation

In our case, the state vector of the nonlinear robot arm system is introduced as:

$\tilde{x} = [\tilde{x}_1, \tilde{x}_2]^T = (q, \dot{q}) = (q_1, q_2, q_3, \dot{q}_1, \dot{q}_2, \dot{q}_3)^T \in R^{2 \times 3}$  so one can obtain (7) in state space form:

$$\dot{\tilde{x}}(t) = A(\tilde{x}) + B(\tilde{x})v(t) \quad (8a)$$

$$A(\tilde{x}) = \begin{bmatrix} \tilde{x}_2(t) \\ -A(\tilde{x}_1(t))^{-1}(C(\tilde{x}(t)) - Q^s) \end{bmatrix}, B(\tilde{x}) = \begin{bmatrix} 0 \\ A(\tilde{x}_1(t))^{-1} \end{bmatrix} \quad (8b)$$

$$y(t) = h(\tilde{x}(t)) = [1 \ 0] \tilde{x}(t) \quad (8c)$$

## 3 Control Design

Given biomechanical system is nonlinear MIMO time varying system, hence two levels of control laws are applied. First level is Computed torque (Inverse dynamics, Feedback linearization). Role of this control law is to linearize given nonlinear system, so linear control law can be applied afterwards.

### 3.1 Feedback Linearization (Computed Torque control)

The idea of Computed torque control is to provide exact linearization of all nonlinearities in biomechanical system via closed loop. In other words direct linear connection between input and output is achieved with application of Computed torque. It is a special application of feedback linearization technique used in nonlinear control systems, [16]. Computed torque controllers can be very effective, since they provide us independent joint control, which can then be used together with some classical and modern design techniques, such as iterative learning control. The nonlinear transformation (9) has converted a complicated nonlinear controls design problem into a simple design problem for a linear system consisting of  $n$  decoupled subsystems. A nonlinear controller will be realized as:

$$v = (L_g L_f^{r-1} h)^{-1} (L_f^r h + u) = \dots = A(q) \cdot u + n(q, \dot{q}) \quad (9)$$

where  $L_g, L_f$  denote corresponding Lie derivatives, [17]

A schematic diagram of a feedback linearization technique is illustrated in Fig. 3.

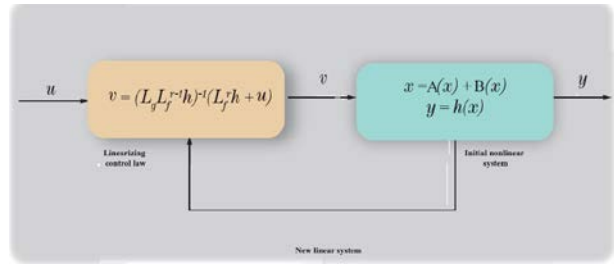


Fig. 3. Block diagram of exact feedback linearization

So, one can linearize the dynamics in ideal case, as follows:

$$\ddot{q}(t) = u(t) \quad (10)$$

where taking into account under the influence of model uncertainties  $\eta_k = \eta_k(t), i = 1, 2, 3$  we have:

$$\ddot{q}_k(t) = u_k(t) + \eta_k(t), i = 1, 2, 3 \quad (11)$$

### 3.2 Advanced open-closed loop iterative learning control

We investigate the problem where the exoskeleton robot must repeatedly follow the desired trajectory  $q_d(t) \in R^n, t \in J, J = [0, T], J \subset R$  in the joint space under the influence of model uncertainties,  $\eta_k(t) \in R^n$ , where  $T$  is the time duration,  $k$  denotes the iteration index. For the linearized dynamics of the robot arm (11), the open-closed ILC algorithm is suggested which comprises two types of control laws: a feed-forward  $\text{sgn} PDD^2$  control law and a PD feedback law, see Fig. 4.

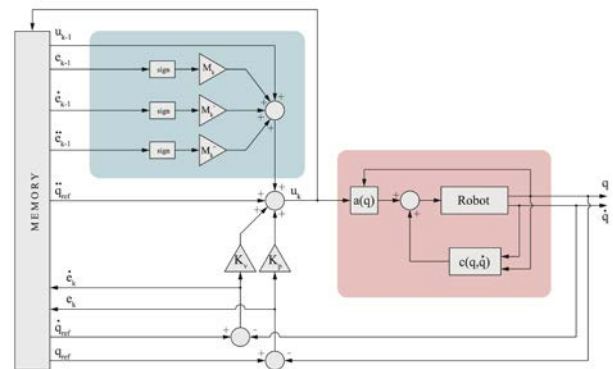


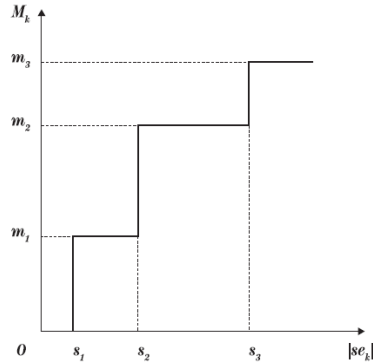
Fig. 4. Block diagram of the open-closed  $\text{sgn} PDD^2 / PD$  type of ILC for a robotic system

$$\begin{aligned} u_k(t) &= u_{ffk}(t) + u_{fbk}(t) = \\ &= u_{k-1} + \text{sgn}(e_{k-1})M_k + \text{sgn}(\dot{e}_{k-1})M'_k + \text{sgn}(\ddot{e}_{k-1})M''_k \\ &\quad + K_p e_k + K_v \dot{e}_k \end{aligned} \quad (12)$$

where  $e_k(t) = y_d(t) - y_k(t)$  is the trajectory tracking error in  $k$ -kth iteration,  $y_d(t)$  denotes desired output trajectory.  $K_p, K_v \in R^{3 \times 3}$  are closed-loop positive-

definite diagonal learning matrices and  $M_k, M'_k, M''_k$ , are functions of  $|se_k|, |s\dot{e}_k|, |s\ddot{e}_k|$ , (Fig.5) respectively where we define the error sums for the  $k$ -th iterative in the form, [18]:

$$se_k = \sum_0^T e_k(t), s\dot{e}_k = \sum_0^T \dot{e}_k(t), s\ddot{e}_k = \sum_0^T \ddot{e}_k(t) \quad (13)$$



**Fig. 5.** Scheme of function  $M_k = f(|se_k|)$

To reduce the computation and storage size for the proposed ILC control functions are introduced as step functions.

Also, the following assumptions on the system (11) are imposed.

A1. The desired trajectories  $y_d(t), x_d(t)$  are continuously differentiable on  $[0, T]$ .

A2. The system (11) is causal. Specifically, for a given desired output trajectory  $y_d(t)$ , there exists a unique bounded control input  $u_d(t)$  such that the system has a unique bounded state  $x_d(t)$  and  $y_d(t)$ , i.e:

$$\dot{x}_d(t) = Ax_d(t) + Bu_d(t), \quad (14)$$

$$y_d(t) = Cx_d(t), \quad (15)$$

A3. The initial resetting conditions hold for all iterations, i.e.

$$x_k(0) = x_d(0), k = 1, 2, 3, \dots \quad (16)$$

A4. The uncertainties  $\eta_k(t) \in R^3$ , are uniformly bounded.

Convergence analysis of the proposed method is omitted here, some more details, see [18].

## 4 Simulation results and discussion

Here, we used a main exoskeleton system due to biomechanical system with revolute joints, with three DOFs, Fig. 2, to solve the trajectory tracking problem in joint space. For the simulation, we use the next model parameters of robot arm  $m_1 = 0kg, m_2 = 1.4kg, m_3 = 1.1kg$  where first segment is fictive due to decomposition [14]. Numerical simulations were carried out to demonstrate the feasibility and effectiveness of the proposed advanced ILC PDD<sup>2</sup>/PD

type. The desired trajectories are given as polynomial of fifth order  $q_d(t) = a_0 + a_1t + a_2t^2 + a_3t^3 + a_4t^4 + a_5t^5$  with constraints

$$\begin{aligned} q_{dk}(0) &= 0, k = 1, 2, 3, \\ q_{d1}(T) &= \pi/2, q_{d2}(T) = \pi/4, q_{d3}(T) = \pi/6, \\ \dot{q}_{dk}(0) &= \dot{q}_{dk}(T) = 0, k = 1, 2, 3, \\ \ddot{q}_{dk}(0) &= \ddot{q}_{dk}(T) = 0, k = 1, 2, 3, \end{aligned} \quad (17)$$

Model of feedback linearized robotic system in state space is given as:

$$\dot{x}_k(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_k(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_k(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \eta_k(t), k = 1, 2, 3$$

$$y_k(t) = [1 \ 0] x_k(t)$$

where  $\eta_k(t), k = 1, 2, 3$  are model uncertainties:

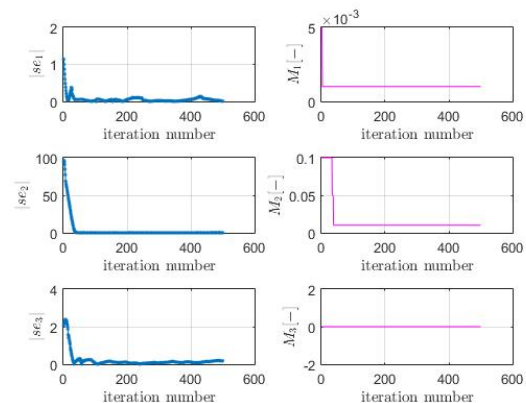
$$\begin{aligned} \eta_1(t) &= 0.1 \cdot \exp(1-t), \quad \eta_2(t) = (10 - 0.2 \cdot t^3), \\ \eta_3(t) &= 0.2 \cdot \sin(\pi \cdot t) \quad t \in [0, 5] \end{aligned} \quad (19)$$

For the elements of learning gain matrices,  $K_p, K_v$  the following values are adopted:

$$K_p = \text{diag}\{10, 25, 2\}, K_v = \text{diag}\{4.6, 15, 6\}, \quad (20)$$

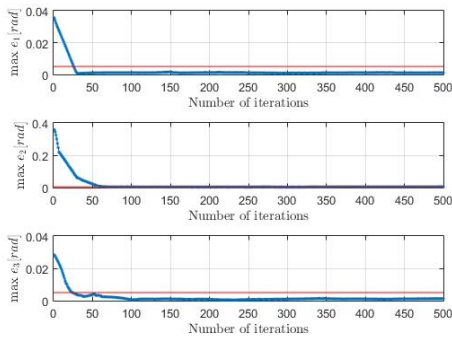
as well as:

$$\begin{aligned} M_{k1} = M_{k2} = M_{k3} &= \begin{cases} 0.09, & |se_{1,2,3}| \geq 0.5 \\ 0.005, & 0.5 \geq |se_{1,2,3}| \geq 0.1 \\ 0.001, & |se_{1,2,3}| \leq 0.1 \end{cases} \\ M'_{k1} = M'_{k2} = M'_{k3} &= \begin{cases} 0.09, & |se_{1,2,3}| \geq 0.5 \\ 0.05, & 0.2 \geq |se_{1,2,3}| \geq 0.1 \\ 0.01, & |se_{1,2,3}| \leq 0.1 \end{cases} \\ M''_{k1} = M''_{k2} = M''_{k3} &= \begin{cases} 0.01, & |se_{1,2,3}| \geq 1 \\ 0.005, & 1 \geq |se_{1,2,3}| \geq 0.5 \\ 0.001, & |se_{1,2,3}| \leq 0.5 \end{cases} \end{aligned} \quad (21)$$



**Fig. 6.**  $M_k, (|se_k|)$  in function of iteration number

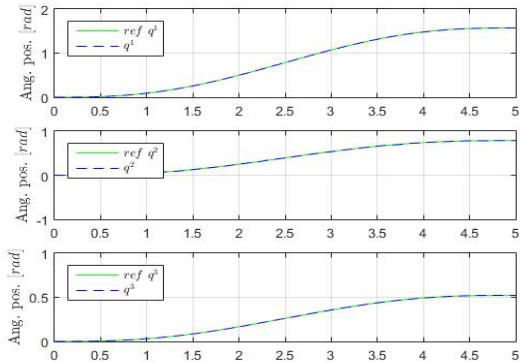
Figure 6 shows the maximum  $M_k$ ,  $|se_k|$  from iteration to iteration.



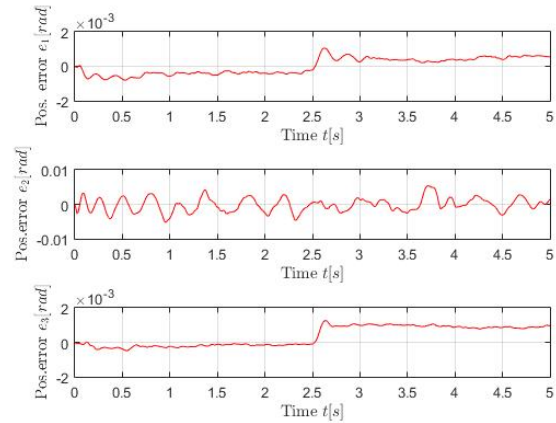
**Fig. 7** Maximum error bounds in each iteration

It is clear that the trajectory tracking error decreases through the iterations. Also, we can find (see Fig. 7), that proposed requirement of tracking performance is achieved at th 50th iteration.

Fig. 8 shows that the ILC control law drives the considered robotic system output on the interval  $t \in [0,5]$  through the desired trajectory as closely as possible after 100 iterations. Also on Fig. 9 presents the tracking errors  $e_i(t), i=1,2,3$  of the system output after 100 iterations.



**Fig.8** The tracking performance of the system output ( $q_{di}(t), i=1,2,3$ -solid line,  $q_i(t), i=1,2,3$ , (- line))



**Fig.9** The tracking errors  $e_i(t), i=1,2,3$  of the system output

## 5 Conclusion

In this paper, we studied the tracking problem of exoskeleton robotic system robot with revolute joints via intelligent control which includes advanced ILC control. First, a feedback linearization control technique is applied on a given robotic system. Then, the proposed intelligent control algorithm takes the advantages offered by closed-loop control PD type and open-loop control sgnPDD2 type of ILC. Suggested robust ILC algorithm is applied to the linearized system to further enhance tracking performance for repetitive tasks and deal with the model uncertainties. Finally, a simulation example is presented to illustrate the effectiveness of the proposed robust ILC scheme for a robot arm.

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