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EFFECT OF PIEZOELECTRIC FIBER-REINFORCED COMPOSITE (PFRC) ACTUATOR ORIENTATION ON CONTROLLABILITY OF ANTISYMMETRIC COMPOSITE PLATES FOR ACTIVE VIBRATION CONTROL

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Abstract:

Piezoelectric materials have a wide range of application on the active vibration control of flexible structures as actuators and sensors due to its inverse and direct piezoelectric effect. In order to increase performance of active vibration control, piezoelectric fibers are stacked into single layer composite, making piezoelectric fiber-reinforced composite (PFRC) actuator and sensor. These actuators and sensors are used for active vibration control of a thin-walled structures, placing them at the surface of the structure. Since that, control performances depend on sizes, positions and orientations of PFRC actuators and sensors. The aim of this paper is to investigate the effect of PFRC actuator orientation and position (top or bottom) on controllability of cross-ply and angle-ply antisymmetric composite plates for active vibration control. Depending on layers' orientation, composite laminates possess coupling behavior (bending-stretching, bending-shear coupling). Since antisymmetric laminates possess bending-stretching coupling behavior, the effect of this behavior on controllability will be also discussed.

Key words: Piezoelectric actuator, composite plates, active vibration control, controllability

1. Introduction

Due to its strength-to-weight and stiffness-to weight ratios, composite materials are widely applied in industry such as aerospace and military industry, robotics, sports equipment, biomedical industry, etc. These industries require high performances of structures composed of composite materials. These performances can be easily disrupted by appearance of unwanted vibrations [1]. In order to reduce unwanted vibrations, piezoelectric materials are integrated into these structures as actuators and sensors making so-called smart structures. In order to increase performances, piezoelectric fibers are stacked into single layer composite, making piezoelectric fiber-reinforced composite (PFRC) actuator and sensor. Active vibration control performances of these structures directly depend on sizes, locations and orientations of piezoelectric elements [2, 3].

The aim of this paper is to investigate the effect of PFRC actuator orientation and position (top or bottom) on controllability of cross-ply and angle-ply antisymmetric composite plates for active vibration control. Depending on layers' orientation, composite laminates possess coupling behavior (bending-stretching, bending-shear coupling) [4]. In some cases this behavior can cause instability in active vibration control [5]. Since antisymmetric laminates possess bending-stretching coupling behavior, the effect of this behavior on controllability will be also discussed.

2. Mathematical model for active vibration control

Figure 1 presents the composite plate composed of a finite number of layers of uniform thickness. This plate is covered by piezoelectric fiber reinforced composite (PFRC) layers at the top and bottom. Both elastic and piezoelectric layers are supposed to be thin, such that a plane stress state can be assumed.

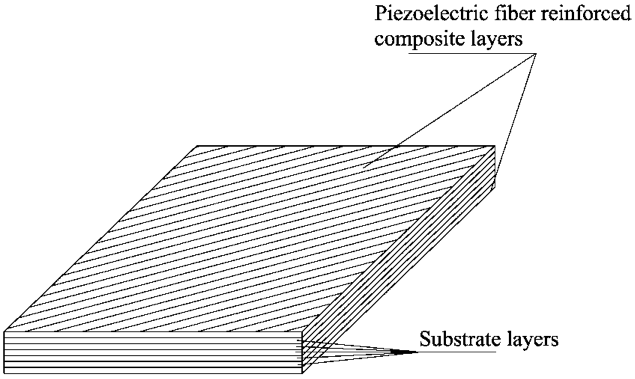


Fig. 1. Laminated composite plate with PFRC layers at the top and the bottom.

After discretization by using the finite element method based on the third-order shear deformation theory and modal analysis [6,7,8], the following equation is obtained

$$\{\ddot{\eta}\}+[\Lambda]\{\dot{\eta}\}+\lfloor\omega^2\rfloor\{\eta\}=[\Psi]^{\mathrm{T}}\{F_{\mathrm{m}}\}-[\Psi]^{\mathrm{T}}[K_{\mathrm{me}}]_{\mathrm{A}}\{\phi\}_{\mathrm{AA}}, \tag{1}$$

where $\{\eta\}$ represents the vector of modal coordinates, $\lfloor\omega^2\rfloor$ is the diagonal matrix of the squares of the natural frequencies, $[\Psi]$ is the modal matrix, $[K_{\mathrm{me}}]_{\mathrm{A}}$ is the piezoelectric stiffness matrix of actuator, $\{F_{\mathrm{m}}\}$ is the vector of external forces, $\{\phi\}_{\mathrm{AA}}$ is the vector of external applied voltage on actuators, and

$$[\Lambda]=\underset{i=1,r}{\mathrm{diag}}(2\zeta_i\omega_i), \tag{2}$$

presents the modal damping matrix in which ζ_i is natural modal damping ratio of the i-th mode. Equation (1) can be expressed in a state-space form on following way

$$\{\dot{X}\}=[A]\{X\}+[B]\{\phi\}_{\mathrm{AA}}+\{d\}, \tag{3}$$

where

$$\{X\}=\begin{Bmatrix}\eta\\ \dot{\eta}\end{Bmatrix},[A]=\begin{bmatrix}[0] & [\mathrm{I}]\\ -\left[\omega^2\right] & -[\Lambda]\end{bmatrix},[B]=\begin{bmatrix}[0]\\ -[\Psi]^{\mathrm{T}}[K_{\mathrm{me}}]_{\mathrm{A}}\end{bmatrix},\{d\}=\begin{bmatrix}[0]\\ [\Psi]^{\mathrm{T}}\{F_{\mathrm{m}}\}\end{bmatrix}, \tag{4}$$

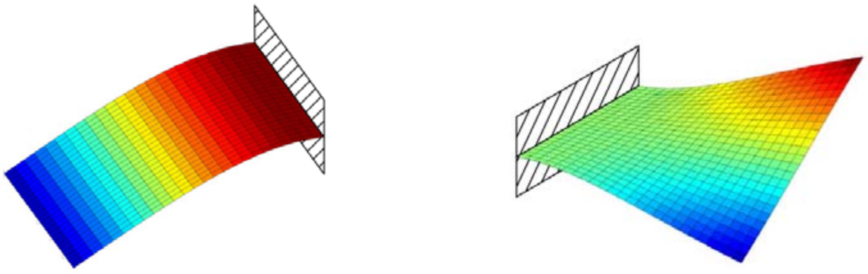
present the state vector, the system matrix, the control matrix and disturbance vector respectively, in which $[\mathrm{I}]$ and $[0]$ are the appropriately dimensioned identity and zero matrix.

2.1 Vibrational modes of antisymmetric cross and angle-ply composite plates

In this subsection, the quadratic cantilevers antisymmetric cross and angle-ply laminated plates are considered. Dimensions of plates are 0.5m x 0.5m and they are consist of eight graphite-epoxy layers with following orientations:

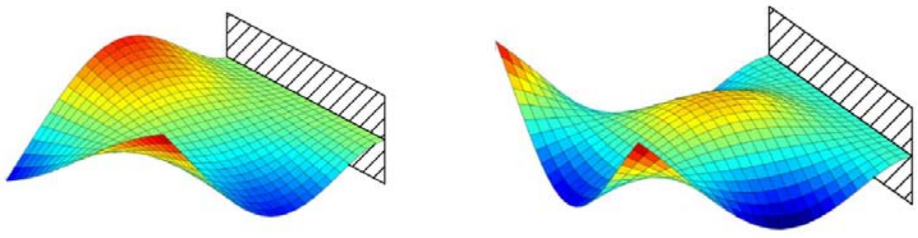
- cross-ply composite plate: (90°/0°/90°/0°/90°/0°/90°/0°),
- angle-ply composite plate: (45°/-45°/45°/-45°/45°/-45°/45°/-45°).

Material properties of the graphite-epoxy layer are presented in Table 1. The first six vibrational modes and correponding frequencies are presented in Figure 2 for antisymmetric cross-ply composite plate, and in Figure 3 for antisymmetric angle-ply composite plate. For this purpose, plates are discretized into 50x50 finite elements.



1st mode, 10.369 Hz   2nd mode, 15.747 Hz

3rd mode, 64.949 Hz   4th mode, 70.531 Hz

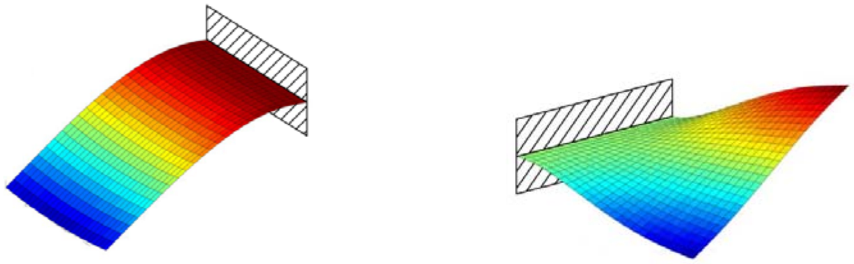


5th mode, 72.474 Hz   6th mode, 113.226 Hz

Fig. 2. Vibrational modes of cantilever antisymmetric cross-ply composite plate

Material properties	Graphite-Epoxy
E_1(GPa)	174
E_2(GPa)	10.3
G_{13}(GPa)	7.17
G_{23}(GPa)	6.21
ν_{12}	0.25
ρ(kg/m^3)	1389.23

Table 1. Material properties of graphite-epoxy

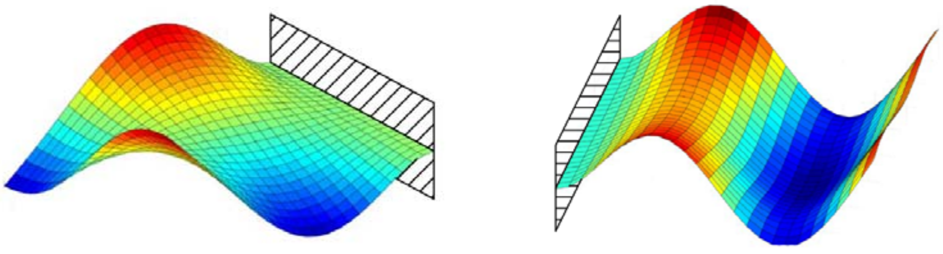


1st mode, 7.012 Hz

2nd mode, 25.814 Hz

3rd mode, 41.186 Hz

4th mode, 69.696Hz



5th mode, 84.667 Hz

6th mode, 125.914 Hz

Fig. 3. Vibrational modes of cantilever antisymmetric angle-ply composite plate

3. Controllability

Controllability is a structural properties and it can be defined as a system ability to control of all states of given system. It depends of system dynamics and the location, size, orientation and number of actuators. For active vibration control, the controllability of the whole system is a combination of controllability of individual modes and it can be expressed quantitatively by using controllability Grammian matrix [9]:

$$[W_{\mathrm{C}}(t)] = \int_0^t e^{[A]\tau}[B][B]^{\mathrm{T}} e^{[A]^{\mathrm{T}}\tau} \mathrm{d}\tau . \tag{5}$$

When structural damping is small, controllability Grammian expressed in modal coordinates is diagonally dominant [10]:

$$[W_{\mathrm{C}}] = \begin{bmatrix} W_{\mathrm{C}11} & 0 & \cdots & 0 \\ 0 & W_{\mathrm{C}22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & W_{\mathrm{C}nn} \end{bmatrix}, \tag{6}$$

and each diagonal term of controllability Grammian matrix can be expressed in a closed form eliminating time dependence of the solution [10]:

$$W_{\mathrm{C}ii} = \frac{1}{4\zeta_i \omega_i} \left(\overline{B}\right)_i \left(\overline{B}\right)_i^{\mathrm{T}} , \tag{7}$$

where $\left(\overline{B}\right)_i$ is *i*-th row of matrix $\left[\overline{B}\right]$. The value of this diagonal term gives information about the energy transmitted from the actuators to the structure for active control of corresponding mode. In order to control several modes simultaneously, Hac and Liu [10] presented the performance index:

$$J_{\mathrm{e}} = \mathrm{trace}([W_{\mathrm{C}}])(\det([W_{\mathrm{C}}]))^{1/(2N_{\mathrm{c}})}, \tag{8}$$

where N_{C} presents the number of controlled modes.

4. Effect of PFRC actuator layer orientation on controllability

In this section the effect of PFRC actuator layer's orientation on controllability of presented composite plates will be examined (Figure 4). Θ_{A} presents the orientation of the layer. Piezoelectric fibers are made of PZT5A, and properties of the actuator are given in Table 2. Orientation of PFRC layer is varied from -90° to 90°, on both, top and bottom sides of plates.

Figure 5 presents controllabilities of controlled modes versus orientation angle of the actuator layer, while Figure 6 presents the performance index versus orientation angle of actuator layer for cantilever antisymmetric cross-ply composite plate. Comparing Figures 5 and 2, it can be concluded that maximum controllability achieves when fibers of PFRC actuators are oriented in the direction of deformation for corresponding mode. From Figures 5 and 6 it can be noticed that diagonal terms of controllability Grammian matrix and the performance index are symmetric in respect to orientation angle Θ_{A} .

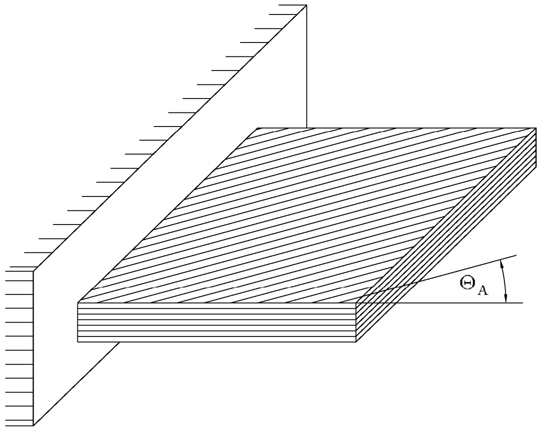


Fig. 4. Composite plate covered with PFRC layer

Material properties	PFRC layer
$E_1(\mathrm{GPa})$	30.2
$E_2(\mathrm{GPa})$	14.9
$G_{13}(\mathrm{GPa})$	5.13
$G_{23}(\mathrm{GPa})$	5.13
ν_{12}	0.45
$\rho(\mathrm{kg/m^3})$	4600
$e_{31}(\mathrm{C/m^2})$	9.41
$e_{32}(\mathrm{C/m^2})$	0.166
$k_{33}(\mathrm{F/m})$	6.1x10^{-9}

Table 2. Material properties of PFRC layer

Due to antisymmetry, there is a significant difference in controllability depending on which side the actuator is placed (top or bottom side of the plate). Controllabilities of 1st and 3rd mode are significantly larger if the actuator is placed on the bottom side (maximum value is reached for 0°). On the other side, controllabilities of 4th, 5th and 6th mode mode are significantly larger if the actuator is placed at the top side. For 4th mode maximum value is reached for orientation ±70°, while for 5th and 6th mode maximum value is reached for orientation ±90°. For 2nd mode controllability is almost equal for the top and the bottom position, while maximum controllability is reached for orientations ±45°. According to this analysis, larger controllability is achieved when the actuator is placed on the side where the angle between actuator's fibers and fibers of layer in contact has larger value. The reason of that is bending-stretching behavior of antisymmetric composites, which leads that strain on the one side of the plate is larger than on opposite side resulting in non-symmetry of the controllability.

Value of the performance index is 0 for angles 0° and 90° (Figure 6). This is obvious because the diagonal term of controllability Grammian matrix for some modes is 0 for these angles. Performance index is larger when the actuator is placed at the bottom side for angles between -35° and 35° and small interval around -60° and 60°. For other angles this index is larger when the actuator is placed at the top side. Its maximum value is reached for angles ±40° when the actuator is placed at the top of the plate.

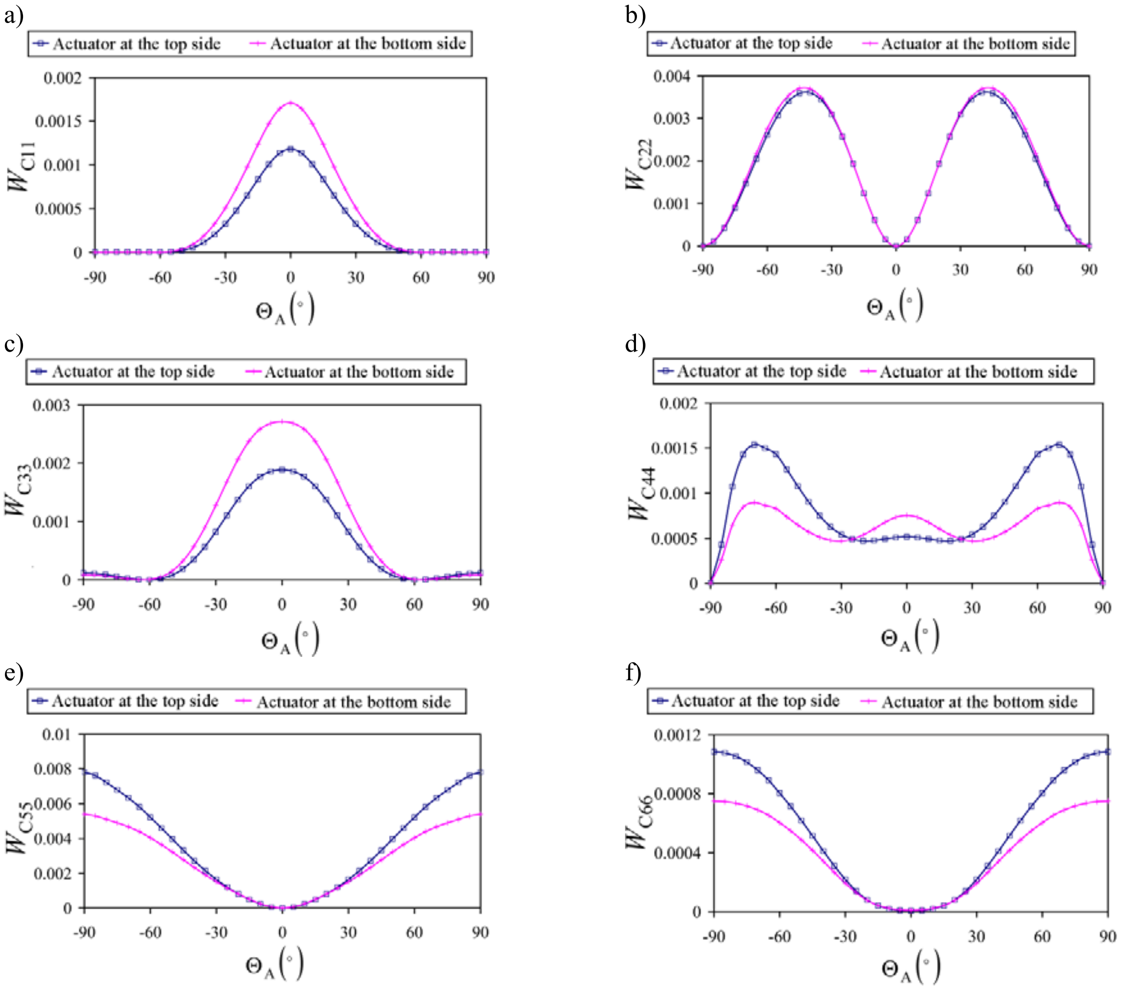


Fig. 5. Controllability of controlled mode versus orientation angle of actuator layer for cantilever antisymmetric cross-ply composite plate: a) 1st mode, b) 2nd mode, c) 3rd mode, d) 4th mode, e) 5th mode, f) 6th mode

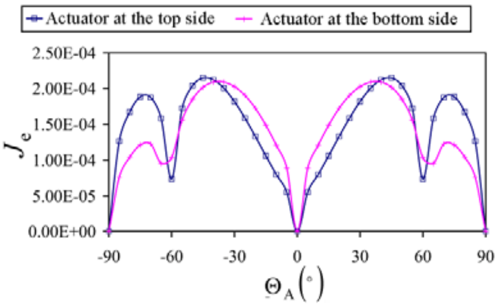


Fig. 6. Performance index versus orientation angle of actuator layer for cantilever antisymmetric cross-ply composite plate

Next analysis is performed for cantilever antisymmetric angle-ply composite plate. Figure 7 presents controllabilities of controlled modes versus orientation angle of the actuator layer, while Figure 8 presents the performance index versus orientation angle of actuator layer.

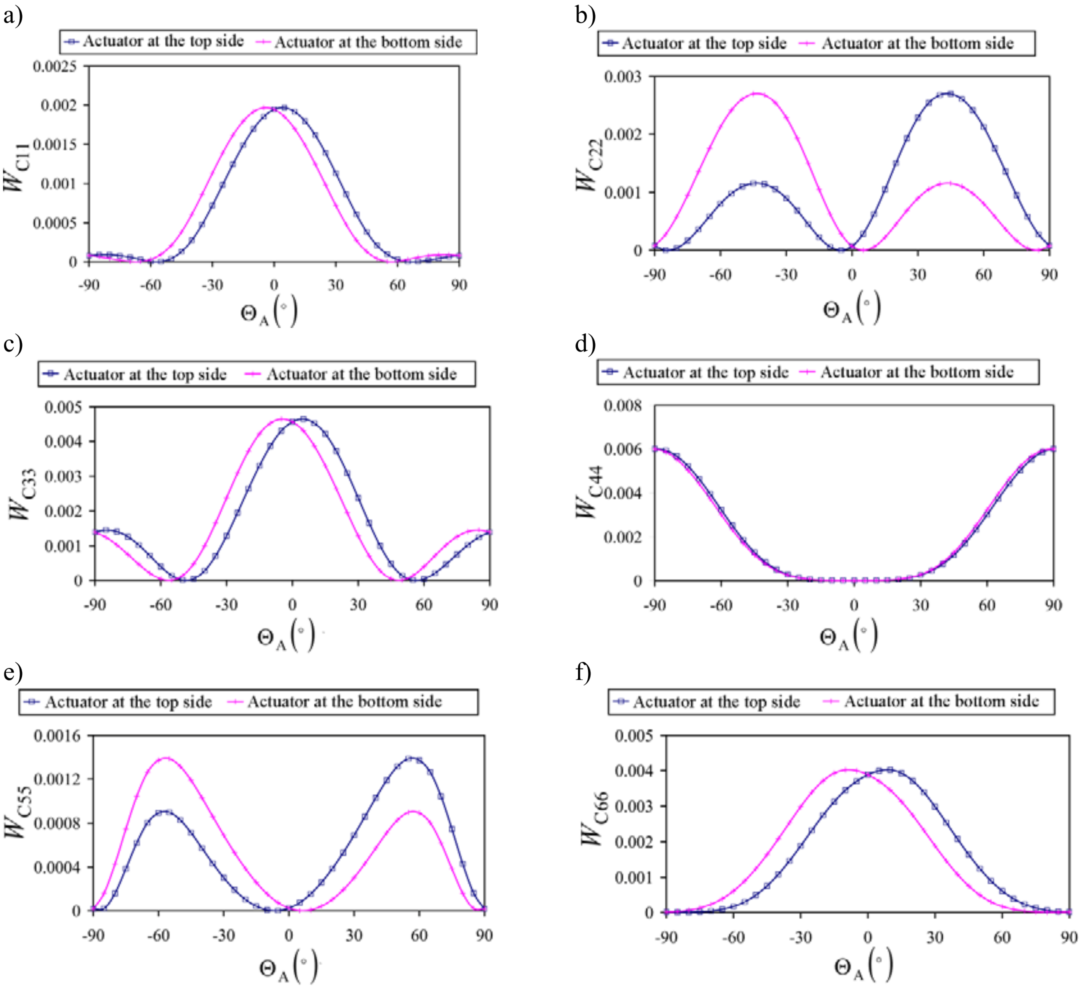


Fig. 7. Controllability of controlled mode versus orientation angle of actuator layer for cantilever antisymmetric angle-ply composite plate: a) 1st mode, b) 2nd mode, c) 3rd mode, d) 4th mode, e) 5th mode, f) 6th mode.

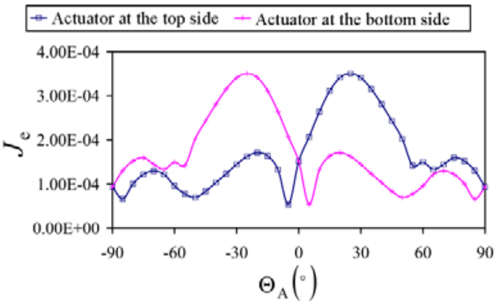


Fig. 8. Performance index versus orientation angle of actuator layer for cantilever antisymmetric angle-ply composite plate

Comparing Figures 7 and 3, it can be drawn the same conclusion as in previous case: that maximum controllability achieves when fibers of PFRC actuators are oriented in the direction of deformation for corresponding mode. From Figures 7 and 8 it can be noticed that unlike in the previous case, diagonal terms of controllability Grammian matrix are not symmetric with respect to orientation angle. This is the most visible for torsional modes (2nd and 5th mode). Better controllability is achieved when fibers' orientation of the actuator layer is opposite of the orientation of layer in contact. The reason for that is bending-stretching behavior of antisymmetric composites described in previous case.

Analyzing the placement of the actuator (top or bottom), it can be noticed that diagonal terms of controllability Grammian matrix are equal for opposite angles (controllability when actuator is set at angle of $+\Theta_A$ at the top of the plate is equal to controllability when actuator is set at angle of $-\Theta_A$ at the bottom of the plate and vice versa).

From Figure 8 it can be concluded that he performance index has the greatest value for angle of 25° in case when the actuator is placed at the top side and for angle of -25° in case when the actuator is placed at the bottom side of the plate.

5. Conclusions

This paper presents investigation of the effect of PFRC actuator orientation and position (top or bottom) on controllability of cross-ply and angle-ply antisymmetric composite plates for active vibration control. Numerical examples are provided for cantilever composite plates made of Graphite-Epoxy layers for following orientation: (90°/0°/90°/0°/90°/0°/90°/0°) (cross-ply) and (45°/-45°/45°/-45°/45°/-45°/45°/-45°) (angle-ply). It is found that there is a significant difference in controllability depending on which side the actuator is placed (top or bottom side of the plate). According to this analysis, larger controllability is achieved when the actuator is placed on the side where the angle between actuator's fibers and fibers of layer in contact has larger value. The reason for that is bending-stretching behavior of antisymmetric composites, which leads to larger strains on one side of the plate than on the opposite side, resulting in non-symmetry of the controllability.

Acknowledgements

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