Vlada Gašić
Teaching Assistant
University of Belgrade
Faculty of Mechanical Engineering

Nenad Zrnić
Assistant Proffesor
University of Belgrade
Faculty of Mechanical Engineering

Aleksandar Obradović
Assistant Professor
University of Belgrade
Faculty of Mechanical Engineering

Milorad Milovančević
Professor
University of Belgrade
Faculty of Mechanical Engineering

Revisiting the use of finite element packages for moving load problem at bridge cranes

This paper deals with moving load considerations in material handling engineering. It revisits the technique for using standard finite element packages for analysis of dynamic response of bridge crane structure due to moving hoist. Hoist was treated as moving load, ignoring inertia effects. Illustration and validation of tehnique is applied to a simply supported uniform beam subjected to a single load moving at constant speed along the beam. It is obtained frequencies and dynamic deflections of structure of bridge crane finite element model. Obtained results show that standard finite element packages, appart from modal analysis and time hystory analysis, can be use to describe response of structure to time-variant moving loads.

Keywords : *moving load, FEA, dynamic response, beam, cranes.*

1. INTRODUCTION

Moving load problem is a special topic in structural dynamics. In the past this topic was mainly related to transportation engineering problems, but nowdays is getting more importance in material handling problems. This is due to increasment of speeds and structural flexibility at structures with moving hoists, such as cranes.

There are proposed many analitical methods for solving the simple cases of moving load problems. Many of them refer to excellent monograph by Fryba [8]. For more complex problems numerical methods have to be used. Although varying positions of the present dynamic loads need some special considerations, the finite element method (FEM) is especially powerful due to its versatility in spatial discretization. Major steps for finite element analysis of moving load problems are done by Olsson [3], and later improved with moving mass problem by Tretheway [1], along with Wu [2] and others. To the authors knowledge FEM, with mentioned considerations, is now only used method for crane structural dynamics at reviews of well known researchers.

Standard finite element packages are not usually set up to apply moving loads. Although better FE softwares provide analysis due to dynamic loads, application of moving and time-variant loads demands certain considerations. This paper shows techniques for describing such loads in finite element software SAP 2000.

The algoritham is presented on a model of overhead bridge crane as typical material handling machine with moving element-hoist. There is shown comparation of results with analytical solution of uniform, simply supported Euler Bernoulli beam subjected to a constant vertical force moving at constant speed.

Correspondence to: Mr Vlada Gašić, teaching assistant
Faculty of Mechanical Engineering,
Kraljice Marije 16, 11120 Belgrade 35, Serbia
E-mail: vgasic@mas.bg.ac.rs

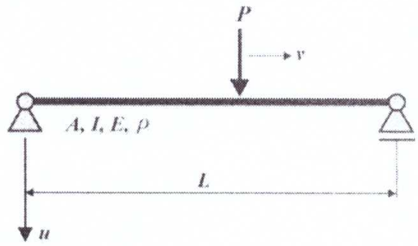

Figure 1. Uniform, simply supported beam subjected to a constant vertical force *P* moving at constant speed *v*

2. DEFINITION OF NODAL FORCES

Basic principle of simulation of moving load is to apply forces and moments to all the nodes of the finite element model, making these loads functions of time. As expected nodes near to the instantaneous force application point can then be given relatively large force values, whereas nodes away from the instantaneous force application point will have zero values. To develop techniques for deriving appropriate time-loads for all the nodes of the structure, a beam subjected to a single concentrated force will be studied first.

The equation of motion of multi-degree-of-freedom structural system is represented as follows:

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{F(t)\} \quad (1)$$

where $[M],[C],[K]$ are the mass, damping and stiffness matrices of structure, respectively; $\{\ddot{u}\},\{\dot{u}\},\{u\}$ are the acceleration, velocity and displacement vectors for whole structure, respectively; and $\{F(t)\}$ is the external force vector. With beam subjected to a concentrated force P, the forces on all the nodes of the beam are equal to zero except the nodes of element s that is subjected to concentrated force, fig. 2.

©FME Belgrade, 2009. All rights reserved

According to Clough and Penzien [6], the external force vector takes the following form:

$$\{F(t)\} = \{000....f_1^{(s)} f_2^{(s)} f_3^{(s)} f_4^{(s)}....000\} \qquad (2)$$

where $f_i^{(s)} = f_i^{(s)}(t), (i = 1,2,3,4)$
represent the equivalent nodal forces,

$$\{f^{(s)}(t)\} = P\{N\} \qquad (3)$$

and

$$\{N\} = [N_1 N_2 N_3 N_4]^T \qquad (4)$$

such that these represent shape functions, hermitian polynomials, given by

$$N_1 = 1 - 3\xi^2 + 2\xi^3 \qquad (5)$$

$$N_2 = l(\xi - 2\xi^2 + \xi^3) \qquad (6)$$

$$N_3 = 3\xi^2 - 2\xi^3 \qquad (7)$$

$$N_4 = l(-\xi^2 + \xi^3) \qquad (8)$$

$$\xi = x / l \qquad (9)$$

noting l as element length and x as distance along the element to the point of application of force P, as shown in fig.2.

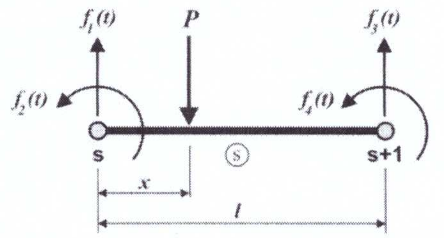

Figure 3. The equivalent forces of the element s subjected to concentrated force

In order to simulate the moving load there is considered beam composed of n-1 elements, with n nodes, with concentrated force moving with velocity v, fig... Considering m time steps and choosing a time interval Δt, the total time is then given by

$$\tau = m \cdot \Delta t \qquad (10)$$

The force and moment vectors contain information for all the nodes on the beam at all time steps:

$$[F]_{m+1}^i = [F_{t=0}^i F_{t=1\cdot\Delta t}^i F_{t=2\cdot\Delta t}^i ... F_{t=\tau}^i]_{m+1}^i \qquad (11)$$

$$[M]_{m+1}^i = [M_{t=0}^i M_{t=1\cdot\Delta t}^i M_{t=2\cdot\Delta t}^i ... M_{t=\tau}^i]_{m+1}^i \qquad (12)$$

for $i = 1$ to n.

At any time $t = r\,\Delta t$ ($r = 1$ to m), the position of the moving force, relative to the left end of the beam, is given by

$$x_p(t) = v \cdot r \cdot \Delta t \qquad (13)$$

We can find the element number s, that the moving concentrated force is applied to at any time t, as

$$s = \mathrm{Int}[\frac{x_p(t)}{l}] + 1. \qquad (14)$$

Element s has two nodes s and s+1. Therefore, the following equations for nodal forces and moments are formed when the moving concentrated force, P, is on the s[th] beam element at any time.

$$F_{t=r\Delta t}^s = PN_1 \qquad (15)$$

$$F_{t=r\Delta t}^{s+1} = PN_3 \qquad (16)$$

$$M_{t=r\Delta t}^s = PN_2 \qquad (17)$$

$$M_{t=r\Delta t}^{s+1} = PN_4 \qquad (18)$$

$$F_{t=r\Delta t}^i = 0,\ M_{t=r\Delta t}^i = 0 \qquad (19)$$

($i = 1$ to n except s and $s+1$).

Equation .. can be re-written in terms of the global $x_p(t)$ with

$$\xi = \frac{x_p(t) - (s-1)l}{l} \qquad (20)$$

Hence, the time-force and time-moment functions are determined for all the nodes of the beam when is subjected to a moving concentrated force. There is simplification referred as "no moment" method, setting N_2, N_4 to zero. However, in this paper is used "full" method concerning all of previously mentioned.

2.1 Illustration of equivalent nodal forces

As seen, previous algorithm can be performed for any simply supported beam. Here, given agorithm is written in software MathCad and illustrates the priciples considering the simply supported beam of lenght 40 m, with 11 nodes equally spaced along the beam. Force of 1 N travel with a constant velocity of 2 m/s from one end to the other. Fig. 3 and fig. 4 show the force/time and moment/time graphs for each node of the beam.

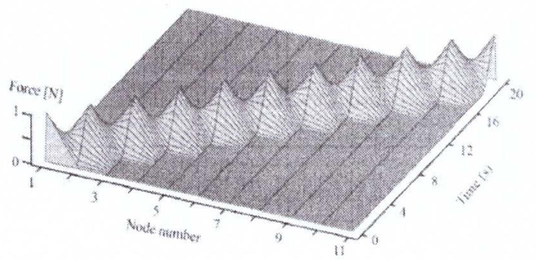

Figure 4. Force-time graph for each node of beam

All these figures illustrate a feature of technique that both the force and moment on each node are zero for all times other than while the force is travelling from the previous node to the next node.

©FME Belgrade 2009. All rights reserved

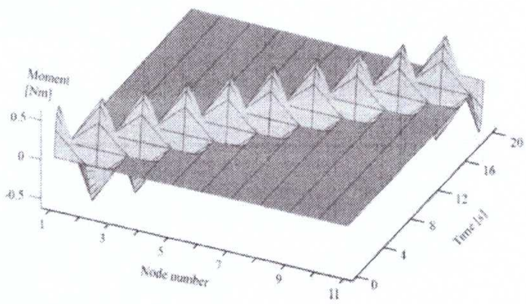

Figure 5. Moment-time graph for each node of beam

3. FEM MODELLING

Implementation of described principles is done on finite element model of overhead bridge crane with box section girders. Model is created in finite element package SAP 2000, as single-span beam model that consists of 10 elements, i.e. 11 nodes. All the elements are the same because of uniform girder. Analysis is done for push-to-limits span of bridge crane L = 40 m and velocity of hoist v = 2 m/s. Another parametar is capacity per girder of 10 t. Other data of crane are tabulated below. It should be mentioned here that are followed recomendatations for discretization of structure continuum in moving load finite element beam models.

Table 1. Material-section properties

No	Par.	Value
1.	ρ	$7850\, kgm^{-3}$
2.	E	$2.1 \cdot 10^{11}\ Pa$
3.	I	$0{,}00667\ m^{-4}$
4.	A	$0{,}04\ m^{-2}$

First, it is performed Modal analysis in SAP 2000. It is always first step in dynamic analysis to obtain frequencies. According to obtained participating mass ratios, it is given only first 3 frequencies of structure and tabulated below. Since model is uniform, simply supported Euler-Bernoulli beam, results can be compared with exact theoretical values, table 2. Mode shapes are not presented here because of well-known shapes of simply supported beam [4,5].

Table 2. Modal frequencies

Mode	SAP 2000 values			Exact
No	Period [s]	Freq., [Hz]	Circ. freq. [rad/s]	Circ. freq. [rad/s]
1.	0,480	2,080	13,074	13,025
2.	0,121	8,255	51,872	52,100
3.	0,054	18,307	115,02	117,225

For given data of moving hoist, it is used equivalent forces and moments for each node obtained with mathematical software, according to chapter 2. Each load is given as Time hystory function in FE software. Linear modal hystory case is created with all the functions. It is used step-by-step method in 100 steps to obtain response.

4. RESULTS AND VALIDATION

After creating the appropriate model of crane, defining load cases-time hystory functions-load combination one can find response of structure due to moving load-hoist. It is common in structural analysis of cranes to search for middle section diplacement and max. flexural moment. In following picture it is shown time-plot displacement of joint representing middle section.

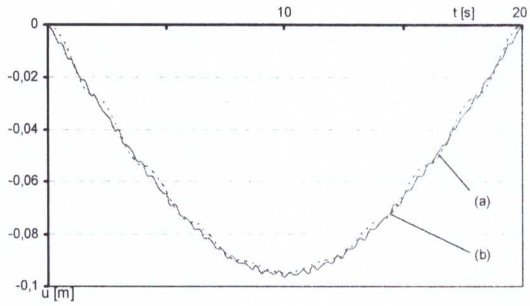

Figure 6. Time-plot of mid-span displacement; (a) SAP 2000 dynamic response; (b) analytical solution [3]

Following picture presents flexural moment in structure, in envelope representation (time step max. value).

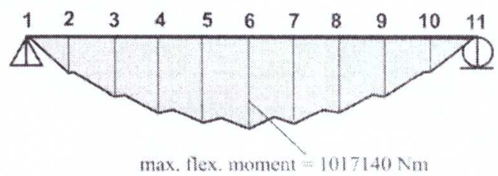

Figure 7. Moment envelope graph at beam

As known, given dynamic problem belongs to very few moving load problems that can be solved analytically. Exact solution is presented in [3], which is used for validation of given model in SAP 2000. Transverse beam displacement, with data from table 1, is given in fig. 2, with obvius validation.

It should be noted here that basic parameter for dynamic response of moving load at simple beam, as in crane, is non-dimensional parameter which is ratio of half of fundamental perod of beam and time for passing beam from left to right end, which is for this model

$$\alpha = \frac{T_1}{2 \cdot \tau} = \frac{0{,}48}{2 \cdot 20} = 0{,}012\,,$$

which can be used for mid-spin dynamic magnification factor of displacement and moment [3]. The ratio of hoist mass and beam is not condsidered because inertial effects are neglected.

©FME Belgrade 2009. All rights reserved

5. CONCLUSION

This paper deals with moving load problem in dynamic analysis of bridge cranes. There is presented accepted tehnique for using standard finite element package to analyse the dynamic response of structure to time-variant moving load. They are incorporated in FE package SAP 2000 with data from computer program created according to presented algorithm. Equivalent nodal forces and moments are gained with full method and illustrated for chosen parameters. The procedure is applied to a model of bridge crane with parameters with upper limits of standard ones, for span and velocity of hoist. Static analysis is first performed to approve design parameters of crane. There are followed recomendatations for discretization of structure continuum in moving load finite element beam models. There are obtained frequencies of structure with modal analysis. It is obtained mid-span displacement and moment due to time-variant moving load, as main design requests. Furthermore, whole procedure is validated with analytical solution where used crane model is assumed to be simply supported beam with one moving force traveling at constant speed.

The aim of this work is to emphasise the technique for describing moving load with standard FE packages. It is convinient to be applied at models of material handling machines with various structural types. But, it can be used only where inertia effects of moving hoist is neglected which is reffered as moving mass problem in structural dynamics. Also, important aim is to improve moving load considerations in serbian literature.

ACKNOWLEDGMENT

This paper is a part of the research project no 14052 supported by Serbian Ministry of Science and Technological Development.

REFERENCES

[1] Rieker, J.R., Lin, YH, Tretheway, M.W.: *Discretization considerations in moving load finite element beam models*, Finite Elements in Analysis and Design 21, pp. 129-144, 1996.

[2] Wu, J.J., Whittaker, A.R., Cartmell, M.P.: *Dynamic responses of structures to moving bodies using combined finite element and analytical methods*, International Journal of Mechanical Sciences 43, pp. 2555-2579, 2001.

[3] Olsson, M.: *Analysis of structures subjected to moving loads*, Lund Institute of technology, Sweden, 1986.

[4] Meirovich, L.: *Elements of vibration analysis*, McGraw-Hill, International edition, 1986.

[5] Karnovsky, I., Lebed, O.: *Formulas for structural dynamics*, McGraw-Hill, 2004.

[6] Clough, RW., Penzien, J.: *Dynamics of structures*, McGraw-Hill, New York, 1993.

[7] Paz, M., Leigh, W.: *Structural dynamics*, Theory and computation, Kluwer Academic Publishers, USA, 2004.

[8] Fryba, L.: *Vibration of solids and structures under moving loads*, Czech Republic, 1972.

[9] *SAP 2000 v10*, Computers and Structures, Inc., University of Berkeley, 2006.

NOMENCLATURE

$[M]$	structural mass matrix of beam
$[C]$	structural damping matrix of beam
$[K]$	structural stiffness matrix of beam
$\{u\}$	nodal displacement vector
$\{\dot{u}\}$	nodal velocity vector
$\{\ddot{u}\}$	nodal acceleration vector
$\{F(t)\}$	external force vector
N	shape functions
v	velocity of moving hoist
x	distance between the contact position of moving load and left end of beam element
T_l	fundamental period of beam
τ	time for moving hoist from left end to right end of beam
A	cross-sectional area of a beam
I	moment of inertia
E	Young's modulus
ρ	mass density of beam

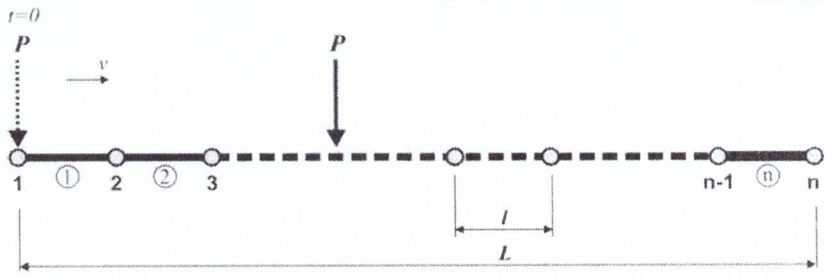

Figure 2. Modelling of beam subjected to concentrated force P moving with constant velocity

©FME Belgrade 2009. All rights reserved