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MINIMIZATION OF THE CONTROL TIME OF MOTION IN MECHANICAL SYSTEMS HAVING A CYCLICAL INTEGRAL IN AN UNCONTROLLED MOTION

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The solution of the problem of minimization of the time interval of the controlled motion in mechanical systems, in the case when the control vector coordinates are generalized forces, makes sence only if the field of permissible controls is a closed set. In this sense, the field of permissible controls is represented in the present paper by a constant set of the polyhedral form. In addition to that, a holonomic, scleronomic mechanical systems having a cyclical integral in an uncontrolled motion have been considered. Upon the introduction of the controlled forces, this cyclical integral disappears. By using the Pontryagin's maximum principle, it has been demonstrated that in the case mentioned above, under definite conditions, there also exists a cyclical integral in the corresponding coupled system of differential equations. An important fact has been observed, i.e., the optimum control should be looked for at the boundary of permissible vector control fields. This indicates the possibility of the existence of singular controls only along a part of the control vector coordinates. These features, alongside with Kelley's conditions for singular controls, have been used for a more detailed analysis of the optimum control structure over the whole time interval.

The procedure described in the present paper has been illustrated by determining the optimum control for a two degree freedom manipulator, where one control vector coordinate is singular. Otherwise, exactly in most manipulators with the uncontrolled motion a cyclical integral is present.

Minimization of the Control Time of Motion in Mechanical Systems Having a Cyclical Integral in an Uncontrolled Motion

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1. Introduction

Let us consider a holonomic, scleronomic mechanical system whose equations of motion can be presented in the canonical form [7] :

$$(1) \quad \dot{q}^i = \frac{\partial H}{\partial p_i}$$

$$\dot{p}_i = -\frac{\partial H}{\partial q^i} + Q_i^N, \quad i = 1, \dots, n,$$

where $H = T + \Pi$ is the Hamilton's function, $T = \frac{1}{2} a^{ij} p_i p_j$ is the kinetic energy, $\Pi(q^i)$ is the potential energy, $a^{ij}(q^k)$ is the contravariant metric tensor of the configuration space R_n, q^i are the generalized coordinates, p_i are the impulses, Q_i^N are the non-potential generalized forces and n is the number of degrees of freedom (in this state, the tensor notation will be used in a usual way of analytic mechanics [7]). The initial and the final states are:

$$(2) \quad q^i(0) = q^{io}, \quad p_i(0) = p_{io}, \quad q^i(\tau) = q^{i\tau}, \quad p_i(\tau) = p_{i\tau}, \quad i = 1, \dots, n.$$

The minimum-time control problem lies in determing the non-potential generalized forces $Q_i^N = Q_i^N(t)$ (we shall call them controls u_i in the following text), they must bring the mechanical system, whose equation of motion are (1), from the given initial state to the final state in the minimum possible travelling time τ ($\tau \to$ inf). Along with all of these things, the constraints of the controls, $|u_i| \le C_i$, $i = 1, \dots, n$, must be satisfied, where C_i are the given positive constants. Let us assume that this problem has got a solution.

For solving the problem we shall use the necessary conditions of optimum, which are given in the maximum principle of Pontryagin [10] . In that sense, let us write the Pontryagin's function κ:

$$(3) \quad \kappa = \lambda_o + \lambda_i a^{ij} p_j + \nu^k \left(-\frac{1}{2} \frac{\partial a^{ij}}{\partial q^k} p_i p_j - \frac{\partial \Pi}{\partial q^k} + u_k \right), \quad i, j, k = 1, \dots, n,$$

and of the basis of it the adjoint system:

$$(4) \quad \dot{\lambda}_i = -\frac{\partial \kappa}{\partial q^i} = -\lambda_k \frac{\partial a^{jk}}{\partial q^i} p_j + \nu^k \left(\frac{1}{2} \frac{\partial^2 a^{jl}}{\partial q^k \partial q^i} p_j p_l + \frac{\partial^2 \Pi}{\partial q^k \partial q^i} \right)$$

$$\dot{\nu}^k = -\frac{\partial \kappa}{\partial p_k} = -\lambda_i a^{ik} + \nu^j \frac{\partial a^{ik}}{\partial q^j} p_i \,, \quad i,j,k,l = 1, \dots n \,,$$

where λ_i, ν^k are the coordinates of the adjoint vector.

The optimal controls are determined according to the condition of supremum of the Pontryagin's function κ ($(\kappa)_{opt} = \sup_{u_i} \kappa$, $|u_i| \leq C_i, i = 1, \dots, n$). Beside that, all the solutions that can satisfy the maximum principle of Pontryagin should be found, and the optimum one chosen among them. The basic purpose of this paper is to show the possibility of the existence of the singular controls [3,5] . That possibility was not considered in some papers, for example in [6] , which was quite wrong.

Now it will be shown that the singular controls in all the coordinates u_k ($\nu^k = 0$, $k = 1, \dots, n$) do not satisfy the maximum principle, because, in that case, (4) and det[a^{ij}] >0 make $\lambda_i = 0$, $i = 1, \dots, n$, which together with the condition $\kappa = 0$ gives $\lambda_o = 0$, which is unacceptable case according to the Pontryagin's principle. This was shown in [9] and [11] as well as [1] ; however, det[a_{ij}] was unnecessary calculated in [1] , with purpose of showing that det[a_{ij}] > 0, although fact, that [a_{ij}] is positive definite, is well-known in mechanics. An important fact has been observed, i.e, the optimum control should be looked for at the boundary of permissible vector control fields. This indicates the possiblility of the existance of singular controls only along a part of the control vector coordinates.

It is very difficult to say something more about this possibility in the general case. In [9] it was shown that it exists:

a) when (1) can be divided into many uncoupled subsystems;

b) when the control is $u_i = -\dfrac{\partial \Pi}{\partial q_i} + Q_i^N$ and the configuration space is Euclid's space (a_{ij} =const);

c) in some special cases of mechanical systems, when $a_{1j} = 0$ $(j \neq 1)$ and when the cyclical coordinate is q^1.

The purpose of this paper is to search for the possibility of the existance of the singular control in a more general case, when a cyclical integral exists during an uncontrolled motion. Let q^1 be the cyclical coordinate i.e. the Hamilton's function does not depend on it. Then the cyclical integral p_1 = const exists there. Upon the introduction of the controlled forces this cyclical integral disappears in the general case, but the corresponding adjoint system has got λ_1 = const integral. That integral gives us the opportunity to analyse the existance of the singular control in detail.

The possibility of the existance of the singular control in the non-cyclical (cyclical) coordinate will be examined in Chapter 3 (Chapter 4). It will be illustrated with the examples of mechanical systems with two degrees of freedom, whose mechanical models will be presented in Chapter 2. Since the problem accomplicated one as a result of the existance of the singular controls, we have considered only two degrees of freedom. That is what was done in [1] , [4] and [8] too.

Paper [4] is amongst the first ones that put the question of the existance of the singular controls for some manipulator types. Some of them will be considered in this paper. In [4] , the possibility of the existance of the singular control was stated. But, in [8] it was prooved that such a possibility does not exist (for the particular problem that was treated in [4]). As in [8] , in Chapter 4 of this state non-existance of the possibility of the singular control in the cyclical coordinate (specific type) will be shown.

In addition to this, exactly in most manipulators with the uncontrolled motion the cyclical integral is present. And, this work comes as a result of solving many practical tasks.

2. Mechanical Models of Systems with Two Degrees od Freedom

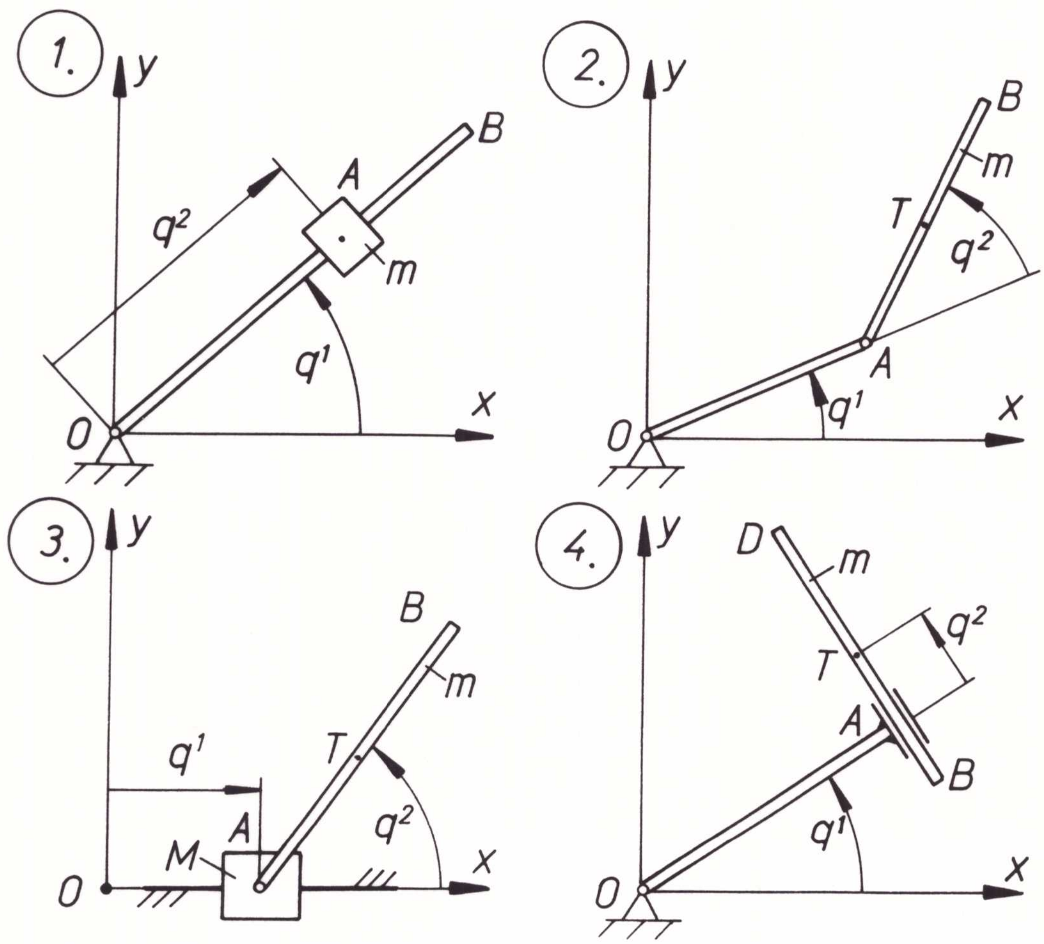

Figure 1: Examples of body-systems

Examples, which are necessary for further consideration, are shown in Figure 1. First two are the manipulators, analysed in [4] , and the third one is also the manipulator but from [2] .

What all of them have in common is that:

a) they have two degrees of freedom;

b) they move on horizontal plane (Π = const);

c) they have a cyclical integral during the uncontrolled motion.

The covariant coordinates of metric tensor of the configuration space differ, and in matrix notation they are:

(5) *1. example*

$$[a_{ij}]=\begin{bmatrix} J_{O_z}^{OB}+J_{A_z}^{A}+m(q^2)^2 & 0 \\ 0 & m \end{bmatrix}$$

2. example

$$[a_{ij}]=\begin{bmatrix} J_{O_z}^{OA}+J_{T_z}^{AB}+(\overline{OA}^2+\overline{AT}^2)m+2\,\overline{OA}\,\overline{AT}m\cos q^2 & J_{T_z}^{AB}+m\overline{AT}^2+m\,\overline{OA}\,\overline{AT}\cos q^2 \\ J_{T_z}^{AB}+m\overline{AT}^2+m\,\overline{OA}\,\overline{AT}\cos q^2 & J_{T_z}^{AB}+m\overline{AT}^2 \end{bmatrix}$$

3. example

$$[a_{ij}]=\begin{bmatrix} M+m & -m\overline{AT}\sin q^2 \\ -m\overline{AT}\sin q^2 & J_{T_z}^{AB}+m\overline{AT}^2 \end{bmatrix}$$

4.example

$$[a_{ij}]=\begin{bmatrix} J_{O_z}^{OA}+J_{T_z}^{BD}+m\overline{OA}^2+m(q^2)^2 & m\overline{OA} \\ m\overline{OA} & m \end{bmatrix}$$

On the basis of all the previous considerations equations of motion (1) have the form:

(6) $\dot{q}^1 = a^{11}p_1 + a^{12}p_2$

$$\dot{q}^2 = a^{21} p_1 + a^{22} p_2$$

$$\dot{p}_1 = u_1$$

$$\dot{p}_2 = u_2 - \frac{\partial T}{\partial q^2}$$

3. On the Possibility of the Existance of the Singular Control in the Non-cyclical Coordinate

Let us search the possibility of the existance of the singular control in the non-cyclical coordinate q^2 during some subinterval into time interval $[0, \tau]$. In that case, in that interval $\nu^2 = 0$ and the maximum principle is ineffective for the determining control u_2, where $u_1 = C_1 \operatorname{sign} \nu^1$. By differentiating ν^2 twice, in accordance with adjoint system (4), for n=2, we get the condition which must be satsified by phase and adjoint variables:

$$\text{(7)} \quad a^{22} p_1 \lambda_1 \frac{\partial}{\partial q^2}\left(\frac{1}{a_{11}}\right) = 0 .$$

Since $a^{22} \neq 0$ (in all cases) and $p_1 \neq 0$ (in this case), then $\lambda_1 = 0$ or $\frac{\partial}{\partial q^2}\left(\frac{1}{a_{11}}\right) = 0$. Adjoint system (4) can be expressed as:

$$\text{(8)} \quad \dot{\lambda}_1 = 0$$

$$\dot{\nu}^1 = -\frac{1}{a_{11}} \lambda_1$$

$$\lambda_2 = \frac{a_{12}}{a_{11}} \lambda_1$$

$$\nu^2 = 0$$

and the Pontryagin's function has this form:

$$\text{(9)} \quad \kappa = \lambda_o + \lambda_1 \frac{1}{a_{11}} p_1 + \nu^1 u_1$$

When $\lambda_1 = 0$, then $\lambda_2 = 0$, ν^1 = const, u_1 = const and u_2 is determined by satisfying the initial and the final conditions for phase variables. When, during all the time interval $[0, \tau]$, there is this type of the singular control, then the final time is:

(10) $\tau = \frac{p_{1\tau} - p_{1o}}{u_1}.$

As u_2 during the time interval $[0, \tau]$ must satisfy condition (2), this solution is possible only in a special case of the initial and the final conditions. Nothing specific, connected with optimality of this solution and his adjoint with the non-singular arcs, can be concluded.

The second case $\frac{\partial}{\partial q^2}\left(\frac{1}{a_{11}}\right) = 0$ $\left(\frac{\partial a_{11}}{\partial q^2} = 0\right)$ is more interesting. From this condition, the value of the coordinate q^2 is determined. During this subinterval, this must have a constant value ($\dot{q}^2 = 0$). In the examples from Chapter 2, $q^2 = 0$ for the first and the fourth example, while $q^2 = k\pi$, k=0,1,2,..., for the second example. All of this results have their own explanations, from the point of view of mechanics, because they indicate minimization of the mass moment of inertia of the all system about the first axis rotation.

However, if those singular arcs must be adjoint with singular ones, due to satisfying the initial and the final conditions, then it is necessary that some corresponding junction conditions [3,5] are fulfilled. On the basis of it, it was shown in [8] that during the time subinterval with $q^2 = 0$ (for the first example in Chapter 2) this type of control is not optimal, which is contrary to the results from [4] .

In the previous cases $\frac{\partial T}{\partial q^2} = 0$ is due to $\frac{\partial}{\partial q^2}\left(\frac{1}{a_{11}}\right) = 0$ so the optimal controls have this form:

(11) $u_1 = C_1 \operatorname{sign} \nu^1 , \quad u_2 = \frac{a_{12}}{a_{11}} u_1,$

while both of them can have interruption at the most, because ν^1 is a linear function.

All of these can be applied to the mechanical systems with a_{11} = const (third example), but in this case q^2 can have any constant value. It is also possible that q^2 is changeable during that subinterval, only in that case $\frac{\partial T}{\partial q^2} \neq 0$, and then (11) is disturbed.

4. On the Possibility of the Existance of the Singular Control in the Cyclical Coordinate

Like in the previous Chapter, we search for the possibility that control u_1 is singular (the singular control in cyclical coordinate q^1) into some subinterval. Then $\nu^1 = 0$ and $u_2 = C_2 \operatorname{sign} \nu^2$. The authors have not managed to analyse this possibility in the general case, but they have focused on the arcs of the optimal traectories with $q^1 = const$.

In that case adjoint system (4) becomes:

$$(12)\quad \lambda_1 = -\frac{d}{dt}(\nu^2 a_{12}) = \text{const}$$

$$\nu^1 = 0$$

$$\dot{\lambda}_2 = -\lambda_2 p_2 \frac{\partial}{\partial q^2}\left(\frac{1}{a_{22}}\right) + \frac{1}{2}\nu^2 p_2 p_2 \frac{\partial^2}{\partial q^2 \partial q^2}\left(\frac{1}{a_{22}}\right)$$

$$\dot{\nu}_2 = -\lambda_2 \frac{1}{a_{22}} + \nu^2 p_2 \frac{\partial}{\partial q^2}\left(\frac{1}{a_{22}}\right)$$

In all the examples considered in this paper $a_{22} = \text{const}$, so that ν^2 is a linear function i.e. control u_2 has one interruption at the most. The control u_1 depending u_2 is determined by differentiating the relation q^1=const according to the equations of motion (6).

Besides, on the basis of (12) we can draw a conclusion that a mechanical system must have a_{12} =const. The second and the third example do not satisfy that condition, and so the singular control of this type (with q^1 =const) cannot exist.

What is interesting here is the result coming from Kelley's conditions [3,5] for the singular control u_1:

$$(13)\quad \frac{\partial}{\partial u_1}\frac{d^2}{dt^2}\frac{\partial \kappa}{\partial u_1} \geq 0.$$

Relation (13) becomes:

$$(14)\quad \nu^2 \frac{\partial a^{11}}{\partial q^2} \geq 0.$$

For the first and the fourth example (14) can be expressed as:

$$(15)\quad -\nu^2 q^2 \geq 0.$$

In the first example, this result can be explained from the point of view of mechanics. Let us consider the motion with $q^2 > 0$. In that case ν^2 as well as u_2 must be negative. That means that these conditions indicate that u_2 must be negative when u_1 is singular. This can be explained by so-called "inertial force" whose existance is unwelcome when $u_2 < 0$ (in order to minimize the final time τ). This justifies, in some degree, examining the singular control with q^1=const.

Some more examples can be found in [9] , where Kelley's conditions can be explained from the point of view of mechanics although their applications is not completely searched in the tasks of this type.

Otherwise, in the already mentioned [9] , the numerical solution of the optimal control problem, for the first example (Chapter 2), has been given. In that example there are jointed singular and non-singular arcs.

Conclusion

This work has shown that upon solving the problems of this kind, all the solutions which satisfy the maximum principle should be examined and among them the optimum one chosen. The optimum traectories can consist of not only non-singular, but also singular arcs in some coordinates (but not in all).

The work has not dealt with the especially important and difficult problem of joining some parts of the optimum traectory. Beside that, the question of application of Kelley's conditions is still open.

Some of these results can be generalized in the case when the number of degrees of freedom is more than two. In mechanical systems with two degrees of freedom and the cyclical coordinate, the possibility of the existance of some other singular controls (beside those analysed here) is not out of question.

The existance of the cyclical integral in an uncontrolled motion has provided more detailed analyses of the structure of the optimal control. The singular controls have particularly been considered.

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The procedure described in the present paper has been illustrated by determining the optimum control for a two degree freedom manipulator, where one control vector coordinate is singular. Otherwise, exactly in most manipulators with the uncontrolled motion the cyclical integral is present.

Minimizacija vremena upravljanja kretanjem mehaničkih sistema koji na neupravljanom kretanju imaju ciklični integral

A.Obradović, J.Vuković

Rešavanje problema minimizacije vremenskog intervala kretanja upravljanog mehaničkog sistema, u slučaju kada su koordinate vektora upravljanja generalisane sile, ima smisla samo ukoliko je oblast upravljanja neki zatvoreni skup. U tom smislu, u ovom radu, oblast dopustivih upravljanja predstavljena je konstantnim skupom oblika pravilnog poliedra. Pored toga, razmatra se holonomni, skleronomni mehanički sistem koji na neupravljanom kretanju ima ciklični integral. Uvođenjem upravljačkih sila ovaj ciklični integral se gubi. Korišćenjem Pontrjaginovog principa maksimuma pokazuje se da ova osobina cikličnosti, uz određene uslove, uzrokuje pojavu cikličnog integrala kod odgovarajućeg spregnutog sistema diferencijalnih jednačina. Uočava se važna činjenica da optimalno upravljanje treba tražiti na granici oblasti dopustivih vektora upravljanja, što ukazuje na mogućnost postojanja singularnih upravljanja samo po delu koordinata vektora upravljanja. Ova svojstva, uz Kelijeve uslove za singularna upravljanja, koriste se za detaljniju analizu strukture optimalnog upravljanja na celom vremenskom intervalu.

Postupak izložen u ovom radu ilustrovan je određivanjem optimalnog upravljanja za manipulator sa dva stepena slobode, gde je jedna koordinata vektora upravljanja singularna. Inače, upravo kod većine tipova manipulatora na neupravljanom kretanju postoji ciklični integral.