TEORIJSKA I PRIMENJENA MEHANIKA 17, pp 121–132, 1991, UDK 531

OPTIMAL CONTROL OF A RIGID BODY SYSTEM IN A COMPLEX CASE

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(Received 25.09.1991; in revised form 24.02.1992)

1. Introduction

This paper is concerned with the time minimization problem in controlled motion of rigid body system in a complex case, when the optimal trajectory has portions with singular arcs [6]. Description of the problem of this kind of the optimal control with particular attention on (im)possibility of the singular arc existence is given in the second section. The third section is concerned with joining the singular and non-singular portions of the optimal trajectory.

This joining appears usually in practical tasks, for example in control of manipulators. Authors of the paper [4] tried to find solution with a singular part. However, they did not find a solution as it is shown in [9], because their solution does not satisfy junction conditions [3, 6]. The solution, for the same type of manipulator as in [4], with a singular control of the first order [7] (in [4] was considered singular control of the second order) on a part of the optimal trajectory, will be obtained in this paper. This solution satisfies the above mentioned junction conditions [3, 6] as well as all conditions of the Pontryagin's maximum principle [5] on the entire time interval of the controlled motion. Based on an analyses given in the fourth section, numerical solution is given in the fifth section.

Present difficulties related to numerical aspects of the optimal control theory have caused that certain number of papers cover only calculations of optimal controls in concrete cases (case studies [4], [9], [10]). A problem becomes more complex when possibility of singular controls arises, which is considered in this paper. In optimal control in manipulators, as the most frequent objects of implementation, an optimal control with singular parts has not been calculated till today, which is obvious from the overview of recent papers given in [11].

It is also known that numerical aspects limit application of the optimal control theory in cases of typical engineering tasks, and having it in mind, the numerical calculation of the optimal controls in the fifth chapter figures out as the basic contribution of this paper. In previous chapters it was necessary to examine all conditions derived from the theory for singular optimal controls, because its improper application, as it is shown in [4, 9], leads to wrong results.

2. Optimal Control with a Minimal Time (Problem Description)

Consider a holonomic, scleronomic mechanical system the motion of which is described by canonical equations:

$$\dot{q}^i = a^{ij} p_j$$
$$\dot{p}_k = -\frac{1}{2}\frac{\partial a^{ij}}{\partial q^k} p_i p_j - \frac{\partial \Pi}{\partial q^k} + Q_k^N \qquad i,j,k = 1,\dots,n \tag{1}$$

where q^i are generalized coordinates, p_i impulses, $\Pi(q^i)$ potential energy, $a^{ij}(q^k)$ contravariant coordinates of metric tensor of configuration space and n is number of degrees of freedom (DOF). Quantities Q_k^N are controls u_k with given limitations:

$$|u_k| \leq C_k \qquad k = 1,\dots,n \tag{2}$$

where C_k are given positive constants. The task is to establish how to change quantities $u_k = u_k(t)$ which satisfy (2), to bring system from initial to final state in a minimal time τ. These states are settled as:

$$q^i(0) = q^{i0}, \quad q^i(\tau) = q^{i\tau}, \quad p_i(0) = p_{i0}, \quad p_i(\tau) = p_{i\tau}, \qquad i = 1,\dots,n \tag{3}$$

Suppose in advance that problem formulated in this way has a solution.

To find a solution, Pontryagin's maximum principle [5] will be applied. In order to use it, we form Pontryagin's function H [12]:

$$H = \lambda_0 + \lambda_i a^{ij} p_j + \nu^k \left(-\frac{1}{2}\frac{\partial a^{ij}}{\partial q^k} p_i p_j - \frac{\partial \Pi}{\partial q^k} + u_k \right) \qquad i,j,k = 1,\dots,n \tag{4}$$

and after that co-state system:

$$\dot{\lambda}_i = -\frac{\partial H}{\partial q^i} = -\lambda_k \frac{\partial a^{kj}}{\partial q^i} p_j + \nu^k \left(\frac{1}{2}\frac{\partial^2 a^{jl}}{\partial q^k \partial q^i} p_j p_l + \frac{\partial^2 \Pi}{\partial q^k \partial q^i} \right)$$
$$\dot{\nu}^k = -\frac{\partial H}{\partial p_k} = -\lambda_i a^{ik} + \nu^j \frac{\partial a^{ik}}{\partial q^j} p_i, \qquad i,j,k,l = 1,\dots,n. \tag{5}$$

Before we go to deriving optimal controls from condition:

$$(H)_{\text{opt}} = \sup_{u_i} H, \qquad |u_i| \leq C_i, \qquad i = 1,\dots,n \tag{6}$$

it is necessary to check out whether in some subinterval of the time interval $[0,\tau]$, for some k, is $\nu^k = 0$. In that case maximum principle is ineffective for determining u_k and such a control is called singular [6].

We will show that controls, singular in all coordinates ($\nu^k = 0$, $k = 1,\dots,n$), do not fulfil necessary conditions of extremality. In this case, by (5) and $|a^{ij}| \neq 0$, we obtain $\lambda_i = 0$, $i = 1,\dots,n$. Uncertainty of τ causes $H = 0$, consequently

$\lambda_0 = 0$, which is contrary to Pontryagin's theorem. This means that during some subinterval in the time interval $[0, \tau]$ at least one control u_k is on the limit ($u_k = \pm C_k$).

When $\nu^k = 0$, $k = l+1, \dots, n$ the maximum principle gives:

$$u_M = c_M \operatorname{sign} \nu^M, \qquad M = 1, \dots, j \tag{7}$$

wherefrom it is obvious that l controls are on the limit ("bang-bang"), while the others are singular. A number of possible variants is $2^n - 1$, and among them extremals are to be found. One of the variants is the one with all controls of the "bang-bang" type. In the case of one DOF, the control is on the limit, while for a larger number all possibilities have to be checked out, and among all solutions (if there exists more than one) which satisfy maximum principle, the optimal is to be chosen. Application of the Pontryagin's maximum principle is limited by the difficulties due to numerical solution of the two-point boundary value problem of the system of equations (1) and (5). In [10], a very efficient numerical method, directly derived from the Pontryagin's principle, is implemented, where for the manipulator with two DOF the optimal control of the "bang-bang" type is obtained.

3. Extremal Trajectories With Singular Arcs

Consider a case of singular control in one coordinate u_k, when $\nu^k = 0$. All conditions of the Pontryagin's theorem are fulfilled, but by the theorem we cannot calculate u_k from the condition:

$$\frac{\partial H}{\partial u_k} = \nu^k = 0. \tag{8}$$

This case is often met in practical tasks and is not caused by the special task parameters but by the complexity of the case itself. To find u_k it is necessary to further differentiate (8) in accordance with (1) and (5). It is shown in [6] that u_k appears only in even derivatives:

$$\frac{d^{2q}}{dt^{2q}} \frac{\partial H}{\partial u_k} = 0. \tag{9}$$

First natural number q, for which u_k in (9) appears, establishes order of singular control.

In [1] possibility of singular control is shown in particular cases:

a) when (1) and (5) can be disassembled in several independent subsystems; minimal time is the largest one of the minimal times of the subsystems, while the other controls are singular, and determined in order to satisfy (3);

b) when $u_i = Q_i$ and configuration space is Euclid space ($a^{ij} =$ const); this case can be reduced to the previous one;

c) when q^1 is cyclical coordinate and $a^{1j} = 0$ for $j \neq 1$; at the same time, physical meanings of particulars singular controls are given.

Assuming that the cases when the control with singular arcs during the entire time interval $[0, \tau]$ can satisfy (3) are seldom, a problem of joining singular and non-singular portions of the optimal trajectory appears. Then, we first determine regions in $4n$-dimensional space of state and co-state variables, in the section of which singular trajectory lies. Then we intend to connect non-singular portions to it, which satisfy initial and final conditions.

For the optimal control, not only conditions of the maximum principle on the entire time interval $[0, \tau]$, but the junction conditions [3, 6] are to be satisfied. Observe such an optimal control which in $t = \theta$ has a discontinuity, so in the interval on the left of the point it is singular, and in the interval on the right of the point, the control is on a limit. Constraint conditions are reduced to the requirement that the quantity L given by:

$$L = (-1)^q \frac{\partial}{\partial u_k} \left(\frac{d^{2q}}{dt^{2q}} \frac{\partial H}{\partial u_k} \right) \tag{10}$$

achieves the first value less than zero at the point $t = \theta$, when q is an odd number.

These conditions are related to the trajectories with piecewise-continuous controls and admit the joining only for singular controls of odd order. Joining can be made even when they are not fulfilled, while the singular portions cannot be coupled with piecewise continuous controls, but with those which in a finite time interval have infinite number of discontinuities [7, 8].

4. Analysis of Solution for Case of Manipulator With Three DOF

Results of previous sections will be implemented in determination of the optimal control of manipulator with 3 DOF, a simplified scheme of which is shown in Fig. 1.

After transformations in nondimensional variables [1] equations (1) and constraints (2) become:

$$\begin{aligned}
\dot{q}^1 &= p_1 & \dot{p}_1 &= \frac{q^1}{(1+(q^1)^2)^2}(p_2)^2 + u_1 \\
\dot{q}^2 &= \frac{1}{1+(q^1)^2} p_2 & \dot{p}_2 &= u_2 \\
\dot{q}^3 &= p_3 & \dot{p}_3 &= u_3 - B_3
\end{aligned}$$

$$|u_1| \leq 1, \qquad |u_2| \leq c_2, \qquad |u_3| \leq c_3, \tag{11}$$

where c_2, c_3 and B_3 are non-dimensional test parameters.

The Pontryagin's function becomes:

$$H = \lambda_0 + \lambda_1 p_1 + \lambda_2 \frac{1}{1+(q^1)^2} p_2 + \lambda_3 p_3 + \nu^1 \left(\frac{q^1 (p_2)^2}{(1+(q^1)^2)^2} + u_1 \right) + \\ + \nu^2 u_2 + \nu^3 (u_3 - B_3) \tag{12}$$

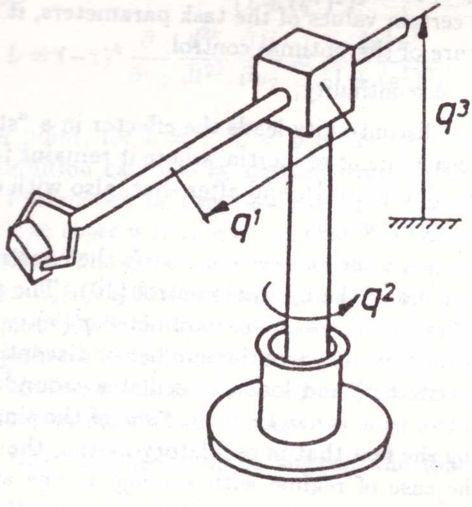

Fig. 1. Manipulator with 3 DOF

and over it, the co-state system:

$$\dot{\lambda}_1 = \frac{2p_2 q^1}{(1+(q^1)^2)^2}\lambda_2 + \frac{(3(q^1)^2-1)(p_2)^2}{(1+(q^1)^2)^3}\nu^1 \tag{13a}$$

$$\dot{\lambda}_2 = 0 \tag{13b}$$

$$\dot{\lambda}_3 = 0 \tag{13c}$$

$$\dot{\nu}^1 = -\lambda_1 \tag{13d}$$

$$\dot{\nu}^2 = \frac{-1}{1+(q^1)^2}\lambda_2 + \frac{-2q^1 p_2}{(1+(q^1)^2)^2}\nu^1 \tag{13e}$$

$$\dot{\nu}^3 = -\lambda_3 \,. \tag{13f}$$

By the analysis of equations (11), (12), (13) as well as initial and final conditions (3) it is clear that this system can be separated into two independent subsystems (the third DOF is independent of the other two). The task is resolved separately for each of them, and optimal time is the largest one of the minimal times [2]. In the paper the problem for the first two DOF will be solved, because the optimal solution for the third is basic and can be found in almost any textbook on the optimal control.

The question of existence of the singular arcs on the optimal trajectory for manipulators is open in [4] and the same type as in this paper is considered, among the other manipulators, with initial and final conditions:

$$q^1(0) = q^1(\tau) = q^{10} \qquad q^2(\tau) = q^{2\tau} > 0$$
$$q^2(0) = p_1(0) = p_2(0) = p_1(\tau) = p_2(\tau) = 0 \tag{14}$$

By the maximum principle, possibility of existence of the singular control in one coordinate is shown. For certain values of the task parameters, it is suggested that we follow the next structure of the optimal control:

a) control u_2 has one discontinuity;

b) control u_1 with one discontinuity leads the effector in a "static" state in the position with minimal mass moment of inertia, where it remains for maximum possible time ($u_1 = 0$ is a singular control), and after that, also with one discontinuity, moves back to the state $q^1(\tau) = q^1(0)$.

It is shown in [9] that this solution does not satisfy the constraint conditions of singular and non-singular part of the optimal control (10). The problem has been solved numerically for different values of the parameter $q^1(\tau)$, where it is obvious that decreasing of the parameter increases the number of discontinuity points. The mass center of the hand (effector) and loading oscillates around rotation axes. In that way the optimal control time is less than the time of the singular part regime, which can be explained by the fact that in oscillatory motion the slowdown of hand appears later than in the case of regime with staying on the axis. This reduces some average value of the mass moment of inertia for the rotation axis.

Controls with singular arcs of this type ($q = 2$) can exist only at some sufficiently low values of $q^1(\tau)$, but in combination with controls which on finite time interval have limitless number of discontinuity points, as it is shown in [8]. Such controls are not important in engineering.

The possibility of existence of a portion of the singular control in coordinate u_2 is investigated here. Such portions can be joined (as singular control of the first order, $q = 1$) with non-singular "bang-bang" portions. For the values of the non-dimensional task parameters, we take:

$$
\begin{array}{llll}
q^1(0) = 1, & q^2(0) = 0, & q^1(\tau) = 2, & q^2(\tau) = 0.363 \\
p_1(0) = 2, & p_2(0) = 0, & p_1(\tau) = 0, & p_2(\tau) = 0.
\end{array}
$$

$$
c_2 = 6 \tag{15}
$$

Based on previous considerations, the following possibilities of the optimal controls on the particular time subintervals inside time interval $[0, \tau]$ exist:

$$
u_1 = \operatorname{sign} \nu^1 \qquad u_2 = c_2 \operatorname{sign} \nu^2 \tag{16a}
$$

$$
u_1 = \operatorname{sign} \nu^1 \qquad u_2 = ? \quad (\nu^2 = 0) \tag{16b}
$$

We find the appropriate derivatives of Pontryagin's function which we use in further analysis:

$$
\frac{\partial H}{\partial u_2} = \nu^2 \tag{17a}
$$

$$
\frac{d}{dt}\frac{\partial H}{\partial u_2} = \frac{-(1 + (q^1)^2)\lambda_2 - 2q^1 p_2 \nu^1}{(1 + (q^1)^2)^2} \tag{17b}
$$

$$\frac{d^2}{dt^2}\frac{\partial H}{\partial u_2} = \frac{-2q^1p_1\lambda_2 - 2p_1p_2\nu^1 - 2q^1u_2\nu^1 + 2q^1p_2\lambda_1}{(1+(q^1)^2)^2} \tag{17c}$$

$$L = (-1)^k \frac{\partial}{\partial u_2}\frac{d^{2k}}{dt^{2k}}\frac{\partial H}{\partial u_2} = \frac{2q^1\nu^1}{(1+(q^1)^2)^2}, \qquad k=1 \tag{17d}$$

In the junction point, for $k=1$, $L<0$ is necessary, in other words $\nu^1<0$ ($u_1=-1$). This condition can also be derived from Kelley's conditions [**3**, **6**] on the singular part. Parameters of this task are such that initial velocity is to be decreased ($u_1=-1$), so there is reason to suppose that singular control starts from the beginning of motion. The right-hand sides of the expressions (17a,b,c) are equal to zero, $p_2(0)=0$, so $\lambda_2(0)=0$, and taken into account in (13b) $\lambda_2(t)=0$ on the full length of time interval $[0,\tau]$. It is then over (17b) on singular part $p_2(t)=0$, so we can finally say that $u_2=0$ (results from (17c)). This can be explained as, in initial interval, the hand does not rotate because if it rotated, the so-called "inertial force" would obstruct control while breaking.

All above, for this task, leads us to suggest the following structure of the optimal controls:

$$\begin{aligned} u_1 &= -1, & u_2 &= 0, & t &\in [0,t_1] \\ u_1 &= -1, & u_2 &= 6, & t &\in (t_1,t_2] \\ u_1 &= 1, & u_2 &= 6, & t &\in (t_2,t_3] \\ u_1 &= 1, & u_2 &= -6, & t &\in (t_3,\tau]. \end{aligned} \tag{18}$$

Now, we have to calculate all state and co-state variables on the entire time interval $[0,\tau]$, to confirm all conditions of the maximum principle.

5. Numerical Solution

At the initial, singular part we have analytical solution:

$$\begin{aligned} q^1 &= 1+2t-\tfrac{1}{2}t^2 & q^2 &= 0 \\ p_1 &= 2-t & p_2 &= 0 \\ \lambda_1 &= \lambda_1(0) & \lambda_2 &= 0 \\ \nu^1 &= \nu^1(0)-\lambda_1(0)t & \nu^2 &= 0. \end{aligned} \tag{19}$$

In further motion it is necessary to make numerical integration of the system of equations (11) and (13). Results are given in Table 1, diagrams of the state variables on entire interval $[0,\tau]$ are given in Fig. 2 and diagrams of the co-state variables in Fig. 3.

The task is solved in the same way as in [9]. First, the four unknown time instants t_1, t_2, t_3 and τ were determined from final conditions for state variables:

$$t_1 = 2.695 \qquad t_2 = 3.115 \qquad t_3 = 3.298 \qquad \tau = 3.900 \tag{20}$$

t	q^1	q^2	p_1	p_2	λ_1	ν^1	ν^2
2.695	2.758	0.000	-0.695	0.000	-9.959	-4.189	0.000
2.737	2.728	0.001	-0.737	0.252	-9.959	-3.771	0.002
2.779	2.697	0.003	-0.779	0.504	-9.960	-3.353	0.006
2.821	2.663	0.006	-0.820	0.756	-9.962	-2.934	0.013
2.863	2.628	0.010	-0.861	1.008	-9.965	-2.516	0.021
2.905	2.591	0.016	-0.900	1.260	-9.970	-2.097	0.030
2.947	2.552	0.024	-0.939	1.512	-9.977	-1.678	0.040
2.989	2.512	0.033	-0.976	1.764	-9.984	-1.259	0.049
3.031	2.470	0.044	-1.010	2.016	-9.992	-0.840	0.057
3.073	2.427	0.057	-1.043	2.268	-9.997	-0.420	0.063
3.115	2.383	0.072	-1.072	2.520	-10.000	0.000	0.065
3.133	2.363	0.079	-1.047	2.630	-9.999	0.182	0.065
3.152	2.344	0.087	-1.022	2.739	-9.997	0.365	-0.063
3.170	2.326	0.095	-0.996	2.849	-9.994	0.547	0.060
3.188	2.308	0.103	-0.969	2.958	-9.988	0.730	0.057
3.206	2.291	0.112	-0.941	3.068	-9.980	0.912	0.051
3.225	2.274	0.121	-0.912	3.177	-9.969	1.094	0.045
3.243	2.257	0.130	-0.882	3.287	-9.955	1.276	0.036
3.261	2.242	0.141	-0.851	3.396	-9.937	1.457	0.026
3.279	2.226	0.151	-0.820	3.506	-9.915	1.638	0.014
3.298	2.212	0.162	-0.787	3.615	-9.889	1.819	0.000
3.358	2.168	0.198	-0.680	3.254	-9.785	2.412	-0.058
3.418	2.130	0.231	-0.581	2.892	-9.674	2.998	-0.126
3.478	2.097	0.261	-0.489	2.530	-9.564	3.578	-0.202
3.539	2.070	0.287	-0.405	2.169	-9.462	4.151	-0.281
3.599	2.048	0.310	-0.327	1.808	-9.377	4.718	-0.361
3.659	2.031	0.329	-0.254	1.446	-9.310	5.281	-0.436
3.719	2.018	0.344	-0.186	1.085	-9.264	5.840	-0.502
3.780	2.008	0.355	-0.122	0.723	-9.238	6.398	-0.554
3.840	2.003	0.361	-0.061	0.361	-9.227	6.954	-0.589
3.900	2.001	0.363	0.000	0.000	-9.226	7.510	-0.601

Table 1. Results of numerical integration

and numerical integration of the base system (11) was made on interval $[t_1, t_2]$. Then, after estimating $\lambda_1(t_2) = -10$ (freely chosen because system (13) is homogeneous) and under conditions $\nu^1(t_2) = 0$ and $\lambda_2 = \text{const} = 0$, numerical integration (backward) of the systems (11), (13a) and (13d) on the interval $[t_1, t_2]$ was done. Thus, values $\nu^1(t_1)$ and $\lambda_1(t_1)$ were obtained. At the end, numerical integration of the system (11) together with (13a), (13d), (13e) was made upon interval $[t_1, \tau]$.

It is obvious from the solution that $\nu^2(t_3) = 0$, as well as u_2 satisfies (16a). Also, over the values obtained and condition $H(t_1) = 0$, λ_0 can be calculated, which confirms that it is non-positive, which is the outcome of Pontryagin's theorem:

$$\lambda_0 = -\lambda_1(t_1)p_1(t_1) + \nu^1(t_1) < 0\,. \tag{21}$$

The solution of the problem satisfies all conditions of optimality expressed through the maximum principle. Is this solution optimal? It is necessary to show if controls u_1 and u_2 of some other structure which satisfy maximum principle exist

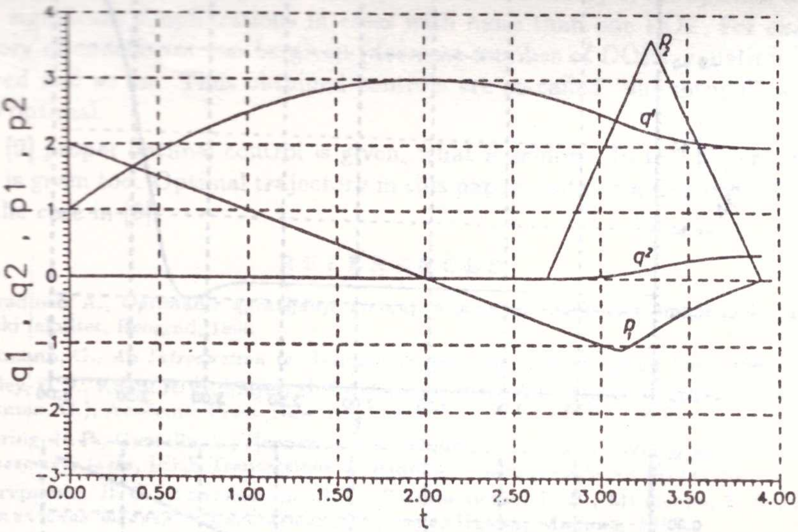

Fig. 2. Diagrams of state variables

and then to determine one of the minimal time among them. If they exist, the number of switching points must be larger, because in the supposed structure this number is the minimal possible to satisfy the final conditions for state variables. However, precise analysis from the mechanical point of view will show that such obtained time must be larger than one here calculated, i.e. calculated controls are optimal.

For different task parameters, the solutions would be of a different approach. For example, let only $q^2(\tau) = 0$ be changed. Then, one more unknown time instant must be obtained (discontinuity of u_2), so we could have five of them. They must satisfy four final conditions for state variables and minimize final instant, which leads to a more complex process of their determination. In this paper $q^2(\tau)$ was sufficiently far from zero. More details can be found in [1].

6. Conclusion

It is shown in this paper that in minimal time control problem, when the control is non-potential generalized force of limited coordinates, at least one of control vector coordinates takes values on its own limit. At the same time, it is necessary to consider possibilities of singular arcs existence, but not in all coordinates. If singular arcs join non-singular, appropriate junction conditions have to be fulfilled.

Optimal control of rigid bodies system motion has large implementation in manipulators control. Authors of [11] give a comprehensive overview of the last twenty years efforts in this field, where it is shown that in problems of here analysed kind the solution with singular arcs has not been found till now. The difficulties

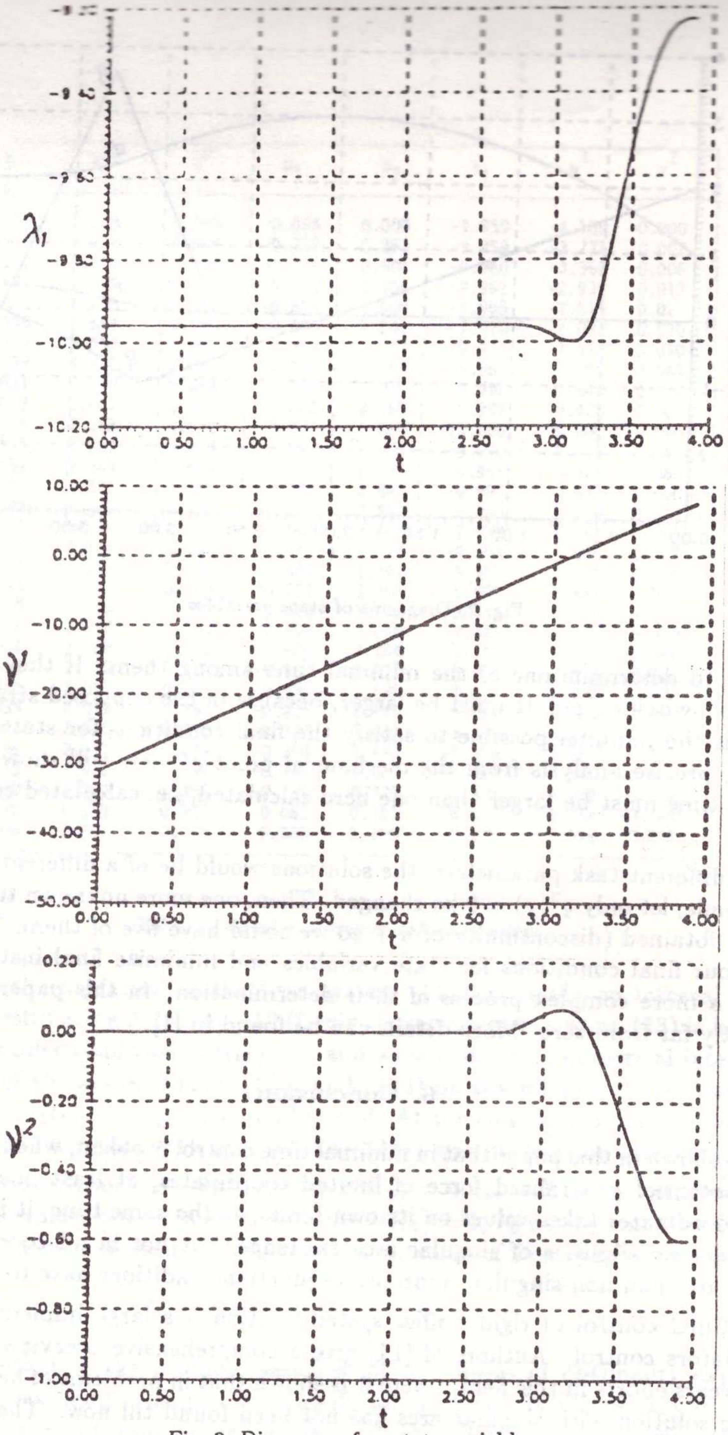

Fig. 3. Diagrams of co-state variables

connected with numerical aspects of the mathematical theory of the optimal control, lead to significant simplifications in cases with more than one DOF. For example, trajectory of endeffector can be given (decrease number of DOF), equation can be linearized and so on. Thus obtained controls are so-called "suboptimal" and are near to optimal.

In [9] proper optimal control is given, what is seldom. In this paper, optimal control is given too. Optimal trajectory in this paper contains a singular arc, which is not the case in [9].

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ОПТИМАЛЬНОЕ УПРАВЛЕНИЕ ДВИЖЕНИЕМ СИСТЕМЫ ТВЕРДЫХ ТЕЛ В ОДНОМ СЛОЖНОМ СЛУЧАЕ

Эта статья посвящена задаче оптимального быстродействия для систем твердых тел. Управлениями являются неконсервативные обобщеные силы, с ограниченными компонентами. Доказана здесь возможность особого управления только по части компонент вектора управления. Рассматривается сопряжение особых и неособых участков оптимальной траектории. Построено численное решение для одного типа манипулятора, где оптимальное управление имеет особый участок.

OPTIMALNO UPRAVLJANJE KRETANJEM SISTEMA KRUTIH TELA U JEDNOM SLOŽENOM SLUČAJU

U radu je rešavan problem minimizacije vremena upravljanog kretanja sistema krutih tela. Za upravljanje je uzeta nepotencijalna generalisana sila ograničenih

koordinata. Pokazana je mogućnost postojanja singularnih upravljanja samo po delu koordinata vektora upravljanja. Razmatrana je problematika sprezanja singularnih i nesingularnih delova optimalne trajektorije. Dato je numeričko rešenje određivanja optimalnih upravljanja kod jednog tipa manipulatora u složenom slučaju, kada postoji singularni deo.

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