OPTIMAL CONTROL AND CONSTRAINT REACTIONS OF MECHANICAL SYSTEMS

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1. INTRODUCTION

In the theoretical field of optimal control, the motion optimisation of systems with limited phase state has been discussed in detail, but the mathematical models of those limitations do not show their physical essence. The basic problem of the determination of the optimal control which provides the motion along the optimal trajectori according to the restrictions imposed (theorems 22-25, [1]) is being solved. The immediate application of such solving procedure of mechanical system control problems is sensible if the phase limitations belong to some subjective requirements (insurance against undesirable behaviour of the system, for example). But, if the motion of mechanical system is limited by material constraints, then, independent of the control, the constraint occur which, according to the procedure mentioned, would remain "hiden" in the solutions for optimal control. Another important fact (which is not considered in the mathematical theory of optimal control) is the difference between the holonomous and unholonomous constrains, although some authors [2] call holonomous constraints all phase limitations.

The constraint reactions in practical problems present the load of the system with useful of harmful consequences, and thus it is neccesary to have a possobility of influencing their behaviour during the motion control proces. Therefore in this paper, the mathematical model of a mechanical system will be written in such a way that it provides the explicit presence of constraint reactions. Thereby, if the consideration of all constraint reactions is unnecessary, the structure and dimension of the phase space will depend upon which and how many of them are the object of our interest.

2. FUNDAMENTALS OF THE PROBLEM

Without diminishing the generality of the method which will be presented in this paper, we shall confine ourselves to the consideration of a scleronomic mechanical system the state of which is, in 2n-dimensional phase space, determined by generalised coordinates q^α and generalised momentums p_α ($\alpha=1,2,...,n$). Hamilton's function of such a system presents the total mechanical energy and has the following form:

$$H = T + \Pi = \frac{1}{2}a^{\alpha\beta}(q)p_\alpha p_\beta + \Pi(q) \qquad (2\text{-}1)$$

where: T - kinetic energy, Π - potential energy, $a^{\alpha\beta}$ - contravariant metric tensor of configuration space.

Beside the potential force, let a non-potential force act upon the system:

$$Q_\alpha = Q_\alpha(q, p, u) \qquad (2\text{-}2)$$

where u is the control vector with coordinates u_i ($i=1, 2, ..., r$) from the vector space U_r. In addition, let the motion the system be limited by the mechanical holonomous constraints:

$$\varphi^\nu(q) = 0 \quad (\nu = 1, 2, \ldots, l_1) \qquad (2\text{-}3)$$

and mechanical non-holonomous constraints:

$$b^\rho_\alpha(q)\,\dot q^\alpha = 0 \quad \left(\frac{\partial b^\rho_\alpha}{\partial q^\beta} \neq \frac{\partial b^\rho_\beta}{\partial q^\alpha}\right) \quad (\rho = 1, 2, \ldots, l_2) \qquad (2\text{-}4)$$

The constraints are smooth and continuous where functions $\varphi^\nu(q)$ possess continuous second differentials. In that case, system is, beside the potential and non-potential forces, exposed to the action of forces-constraint reactions (2.3) and (2.4):

$$R_\alpha = \lambda_\nu \frac{\partial \varphi^\nu}{\partial q^\alpha} + \mu_\rho b^\rho_\alpha \qquad (2\text{-}5)$$

so that, for the description of the motion such a system in the phase space, we have the equation [3]:

$$\begin{aligned} \dot q^\alpha &= \frac{\partial H}{\partial p_\alpha} \\ \dot p_\alpha &= -\frac{\partial H}{\partial q^\alpha} + Q_\alpha(q, p, u) + \lambda_\nu \frac{\partial \varphi^\nu}{\partial q^\alpha} + \mu_\rho b^\rho_\alpha \end{aligned} \qquad (2\text{-}6)$$

where λ_ν and μ_ρ are undetermined multipliers from the space U_l ($l=l_1+l_2$). Let in the general case, the admissible controls be limited, i.e.:

$$u_i \in G_u \subset U_r \qquad (2\text{-}7)$$

and let the reactions (2.5) be exposed to certain limitations which can be expressed as:

$$\lambda_\nu \in G_\lambda \subset U_l, \quad \mu\rho \in G\mu \subset U_l \qquad (2\text{-}8)$$

The areas G_u, G_λ and G_ν can be open or closed sets, constant or variable, and admissible controls ui and admissible multipliers λ_ν and μ_ρ can be a piecewise continuous function with a finite number of interuptions in the interval $[t_0, t_1]$.

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Considering (2.6) the non-holonomous constraints (2.4) can be written as:

$$b^{\rho\alpha}\, p_\alpha = 0 \quad (b^{\rho\alpha} = a^{\alpha\beta}\, b^\rho_\beta)$$

If, in addition to the constraints (2.3) and (2.9) following limitations are imposed on the system:

$$g^k(q, p) = 0 \quad (k=1, 2, \ldots, m) \tag{2-10}$$

which do not present mechanical constraints, the structure of Eq. (2.6) will not be changed. This fact indicates a crucial difference between constraints (2.3) and (2.9) on the one hand and limitations (2.10) on the other. Relations (2.3) and (2.9) are fulfilled allways regardless of controls u_i, whereas limitations (2.10) hold only for the optimal solution.

The condition for optimality can be written as:

$$\int_{t_0}^{t_1} F^0(q, p, u)\, dt \rightarrow \inf \tag{2-11}$$

where the state of the system on the interval ends $[t_0, t_1]$, is presented with manifold:

$$Q_\sigma[q(t_0), p(t_0), q(t_1), p(t_1),] = 0 \quad (\sigma = 1, 2, \ldots s, \leq 2n) \tag{2-12}$$

It is important to note that, if constraints (2.5) are also included in some optimality requirements the subintegral function in (2.11) can depend upon multipliers λ_v and μ_ρ.

Basing upon the above, the task of the optimal control of the motion of a constrained mechanical system is defined by relations (2.3), (2.6)-(2.12).

3. SOLUTION OF THE OPTIMAL CONTROL TASK

The problem previously defined problem should be formulated in a form convenient for the solution by applying the method of the optimal control theory. For that purpose let us introduce into the consideration vector space U_z of vector v with coordinates v_γ $(\gamma=1, 2, \ldots, z<n)$ and perform the following transformation:

$$Q_\alpha(q, p, u) + \lambda_v \frac{\partial \varphi^v}{\partial q^\alpha} + \mu_\rho\, b^\rho_\alpha = d^\gamma_\alpha\, v_\gamma \tag{3-1}$$

where, in the general case, $d^\gamma{}_\alpha$ (q, p) are certain known functions. Let:

$$\text{rang}\left\{\frac{\partial Q_\alpha}{\partial u_i}, \frac{\partial \varphi^v}{\partial q^\alpha}, b^\rho_\alpha\right\} = k = \inf(n, r+l) \tag{3-2}$$

In the case when r+l>n (k=n), we can assume that:

$$d^\gamma_\alpha = \begin{cases} 1, & \alpha = \gamma \quad (1, 2, \ldots n) \\ 0, & \alpha \neq \gamma \end{cases} \tag{3-3}$$

where by for r+l= n transformations (3.1) are unique and can be expressed as:

$$\begin{aligned} u_i &= u_i(q, p, v) \quad (i = 1, 2, \ldots, r) \\ \lambda_v &= \lambda_v(q, p, v) \quad (v = 1, 2, \ldots, l_1) \\ \mu_\rho &= \mu_\rho(q, p, v) \quad (\rho = 1, 2, \ldots, l_2) \end{aligned} \tag{3-4}$$

For r+l>n transformations are not unique thus r+l-n the quantities λ_v, μ_ρ and u_i can arbitrarely be choosen from (2.7) and (2.8).

In the case when r+l<n in relations (3.1), γ=1, 2, ..., z=r+l should be taken. Coefficients $d_\alpha{}^\gamma$ should be choosed in such a way that the n-(r+l) relations are excessive.

After the transformations (3.1) equations (2.6) obtain the form:

$$\begin{aligned} \dot{q}^\alpha &= \frac{\partial H}{\partial p_\alpha} \\ \dot{p}_\alpha &= -\frac{\partial H}{\partial q^\alpha} + d^\gamma_\alpha\, v_\gamma \end{aligned} \tag{3-5}$$

where the admissible control are:

$$v_\gamma \in G_v \subset U_z \tag{3-6}$$

The set G_v from the space U_z is determined by the restrictions (2.7) and (2.8), as well as the transformations (3.1).

The optimality condition (2.11) now becomes:

$$\int_{t_0}^{t_1} f^0(q, p, v)\, dt \rightarrow \inf \tag{3-7}$$

where $f^0(q,p,v)=F^0\,[q, p, u\,(q, p, v)]$.

In this manner, the problem previously defined, by means of the transformations (3.1) is reduced in the form determined with the relations (2.3), (2.9), (2.10), (3.5), (3.6) and (3.7). For the solution of the problem, the Theorem 22 [1] can be used, if some of the relations mentioned are transformed into convenient form. For that purpose, let us introduce l+m dimensional vector function

$$\Phi^\xi(q, p, v) = \begin{cases} \dfrac{d^2}{dt^2}\left[\varphi^v(q)\right] \\ \dfrac{d}{dt}\left[b^{\rho\alpha}\, p_\alpha\right] \quad (\xi = 1, 2, \ldots, l+m) \\ \dfrac{d}{dt}\left[g^k(q, p)\right] \end{cases} \tag{3-8}$$

where by the differentiations are performed on the trajectory q(t), p(t) which is the solution of eq. (3.5) for the appropriate control v(t). The conditions:

$$\Phi^\xi(q, p, v) = 0 \quad \forall\, t \in [t_0, t_1] \tag{3-9}$$

are equivalent to the constraints (2.3) and (2.9) and limitations (2.10) if at the begining moment t_o the phase point is on the manifolds:

$$\left[\varphi^v(q)\right]_{t_0} = 0, \qquad \left[\frac{\partial \varphi^v}{\partial q^\alpha} \frac{\partial H}{\partial p_\alpha}\right]_{t_0} = 0,$$

$$\left[b^{\rho\alpha} p_{\alpha}\right]_{t_0} = 0, \quad \left[g^k(q,p)\right]_{t_0} = 0 . \qquad (3\text{-}10)$$

In other words, conditions (3.9) and (3.10) provide the motion of the phase point along the complete interval $[t_0, t_1]$ according to the constraints (2.3) and (2.9) and limitations (2.10).

By separating among the manifolds (2.120 and (3.10) those with the independentgradients on the interval $[t_o, t_1]$ ends we obtain conditions:

$$\omega_{\lambda}=[q(t0), p(t0), q(t1), p(t1)]=0$$

$$(\lambda=1, 2, ..., p, ..., s \le p \le 2n) \qquad (3\text{-}11)$$

In that way, the optimal control task, defined with the equations (3.5), (3.6), (3.7), (3.9) and (3.11), is transformed into the form applicable for the Theorem 22 [1] which provides the necessary conditions for a determination of the optimal control and the corresponding trajectories $v^*=v^*(t), q^*=q^*(t), p^*=p^*(t)$. Returning to the initial problem, the optimal

$$u^*= u^* (t) \quad \lambda^*= \lambda^* (t) \quad \mu^*= \mu^* (t) \qquad (3\text{-}12)$$

solutions follow from (3.4), so that we obtain:

4. EXAMPLE. HOLONOMOUS CONSTRAINT WITH RESTRICTED REACTION

A slider with the mass m=1 (values of all physical quantities are given in this example in basic units) movies in a vertical plane on a smooth constraint under the action of the control force u of the horizontal direction and position A(0,0) reaches the position B(1,1) at the time t_1 (Fig.1).

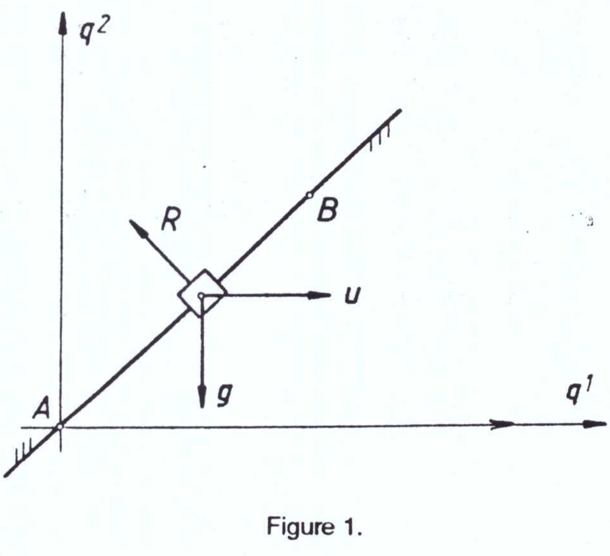

Figure 1.

If, in the initial and the end position, the speeds of the slider equal to zero, and if the constraint reaction is limited $|R| \le 4g\sqrt{2}$, determine control u from the condition that the motion time is minimal.

Optimality task has the form:

$$\dot{q}^1 = p_1 \quad \dot{p}_1 = u - \lambda$$
$$\dot{q}^2 = p_2 \quad \dot{p}_2 = -g + \lambda \qquad (4\text{-}1)$$

$$\int_{t_0}^{t_1} dt \rightarrow \inf \qquad (4\text{-}2)$$

$$|R| \le 4g\sqrt{2} \Rightarrow |\lambda| \le 4g, \ \forall t \in [0, t_1] \qquad (4\text{-}3)$$

$$\varphi(q^1, q^2) = q^2 - q^1 = 0 \qquad (4\text{-}4)$$

$$q^1(0) = q^2(0) = p_1(0) = p_2(0) = p_1(t_1) = p_2(t_2) = 0$$
$$q^1(t_1) = q^2(t_1) = 1 \qquad (4\text{-}5)$$

where λ is undefined constraint multiplier (4.4).

By transformations:

$$u = v_1 + v_2 + g, \quad \lambda = v_2 + g$$

we obtain:

$$\dot{q}^1 = p_1 \quad \dot{q}^2 = p_2 \quad \dot{p}_1 = v_1 \quad \dot{p}_2 = v_2 \qquad (4\text{-}7)$$

$$-5g \le v_2 \le 3g \qquad (4\text{-}8)$$

$$\Phi(v_1, v_2) = v_2 - v_1 = 0 \qquad (4\text{-}9)$$

On the basis of (4.2),(4.7) and (4.9) we can form the the function:

$$\kappa = \Psi'_0 + \Psi'_1 p_1 + \Psi'_2 p_2 + \Psi'_3 v_1 + \Psi' v_2 + \eta(v_2 - v_1) \qquad (4\text{-}10)$$

According to the Pontryagin's maximum principle, we have:

$$\sup_{v \in G_v} \kappa = 0 \qquad (4\text{-}11)$$

$$\dot{\Psi}_1 = 0, \quad \dot{\Psi}_2 = 0, \quad \dot{\Psi}_3 = -\Psi'_1, \quad \dot{\Psi}_4 = -\Psi'_2, \quad \Psi'_0 \le 0 \qquad (4\text{-}12)$$

Considering (4.8) from (4.11) it follows that:

$$\frac{\partial \kappa}{\partial v_1} = \Psi'_3 - \eta = 0$$
$$v_2^* = \begin{cases} -5g, & \Psi'_4 < 0 \\ 3g, & \Psi'_4 > 0 \end{cases} \qquad (4\text{-}13)$$

i.e. considering (4.7), (4.9) and (4.12):

$$v_1^* = v_2^* = \begin{cases} 3g, & \forall t \in [0, \tau], \quad \tau = \frac{1}{2}\sqrt{(\frac{5}{3g})} \\ -5g, & \forall t \in [\tau, t_1], \quad t_1 = \frac{4}{5}\sqrt{(\frac{5}{3g})} \end{cases} \qquad (4\text{-}14)$$

and thus, according to the transformations (4.6):

$$u^* = \begin{cases} 7g, & \forall t \in [0, \tau] \\ -9g, & \forall t \in [\tau, t_1] \end{cases} \qquad (4\text{-}15)$$

5. EXAMPLE. MECHANICAL SYSTEM WITH UNHOLONOMOUS CONSTRAINTS

Body 1 which can along its rotation axis transmits the rotation from disc 2 to disc 3 (Fig.2). Force F ($F \le 1$) acts on body 1, and the torque M acts on body 2, which rotates with the constant angle speed $\Psi=1$. The internal matrix of the system is the unit matrix.

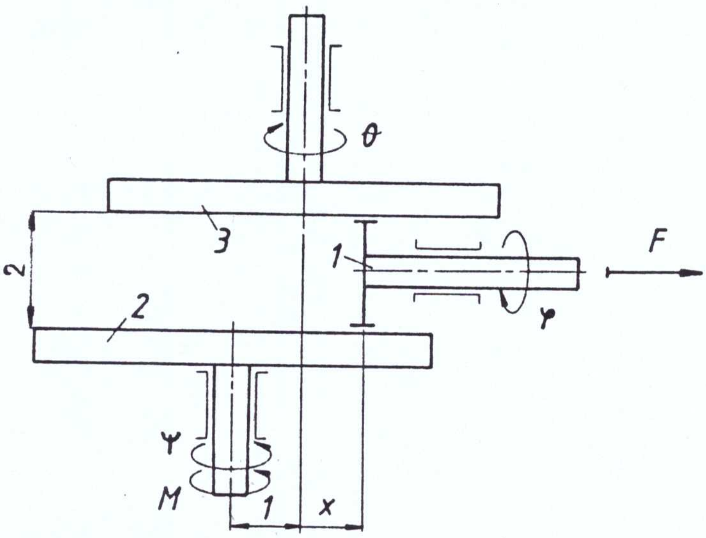

Figure 2.

It is necessary to solve following problem:

$$\dot{x}(t_0) = 0, \quad \dot{\theta}(t_0) = 2, \qquad \int_{t_0}^{t_1} dt \to \inf \qquad (5\text{-}1)$$
$$\dot{x}(t_1) = 0, \quad \dot{\theta}(t_1) = 4,$$

The internal system constraints are:

$$(1+x)\dot{\psi} - \dot{\varphi} = 0$$
$$\dot{\varphi} - x\dot{\theta} = 0 \qquad (5\text{-}2)$$

and the given limitation is:

$$\dot{\psi} - 1 = 0 \qquad (5\text{-}3)$$

By intoducing the variables:

$$q^1 = x, \quad q^2 = \varphi, \quad q^3 = \psi, \quad q^4 = \theta$$
$$p_1 = \dot{x}, \quad p_2 = \dot{\varphi}, \quad p_3 = \dot{\psi}, \quad p_4 = \dot{\theta} \qquad (5\text{-}4)$$

considering (5.2) and (2.6), we obtain the motion equations:

$$\dot{q}^1 = p_1, \quad \dot{q}^2 = p_2, \quad \dot{q}^3 = p_3, \quad \dot{q}^4 = p_4$$
$$\dot{p}_1 = F, \quad \dot{p}_2 = -\mu_1 + \mu_2, \qquad (5\text{-}5)$$
$$\dot{p}_3 = M + \mu_1(1+q^1), \quad \dot{p}_4 = -\mu_2 q^1$$

and the limitations:

$$(1+q^1)\,p_3 - p_2 = 0, \quad p_2 - q^1 p_4 = 0, \quad p_3 - 1 = 0 \qquad (5\text{-}6)$$

where μ_1 and μ_2 are the constraint (5.2) multipliers.

By transforming:

$$v_1 = F, \quad v_2 = \mu_1 + \mu_2, \quad v_3 = M + \mu_1(1+q^1), \quad v_4 = -\mu_2 q^1 \qquad (5\text{-}7)$$

Eq. (5.5) can now be written as:

$$\dot{q}^i = p_i, \quad \dot{p}_i = v_i \quad (i = 1, 2, 3) \qquad (5\text{-}8)$$

basing upon which, the limitations (5.6) now become:

$$p_1 p_3 + (1+q^1)\,v_3 - v_2 = 0$$
$$v_2 - p_1 p_4 - q^1 v_4 = 0 \qquad (5\text{-}9)$$
$$v_3 = 0$$

where:

$$|v_1| \leq 1 \quad \forall\, t \in [t_0, t_1] \qquad (5\text{-}10)$$

The conditions on the boundaries of the interval $[t_0, t_1]$, considering (5.1) and (5.6), are:

$$q^1(t_0)=1, \; p_1(t_0)=0, \; p_2(t_0)=2, \; p_3(t_0)=1, \; p_4(t_0)=2$$
$$q^1(t_1)=\frac{1}{3}, \; p_1(t_1)=0, \qquad p_4(t_1)=4 \qquad (5\text{-}11)$$

On the basis (5.1), (5.8) and (5.9) we have the function:

$$\kappa = \Psi_0 + \sum_{i=1}^{4} (\Psi_i p_i + \Psi_{4+i} v_i) + \eta_1\left[p_1 p_3 + (1+q^1) v_3 - v_2\right] +$$
$$+ \eta_2\left[v_2 - p_1 p_4 - q^1 v_4\right] + \eta_3 v_3 \qquad (5\text{-}12)$$

where:

$$\Psi_0 \leq 0$$
$$\dot{\Psi}_1 = -\eta_1 v_3 + \eta_2 v_4 \qquad \dot{\Psi}_5 = -\dot{\Psi}_1 - \eta_1 p_3 + \eta_2 p_4$$
$$\dot{\Psi}_2 = 0 \qquad \dot{\Psi}_6 = -\dot{\Psi}_2 \qquad (5\text{-}13)$$
$$\dot{\Psi}_3 = 0 \qquad \dot{\Psi}_7 = -\dot{\Psi}_3 - \eta_1 p_1$$
$$\dot{\Psi}_4 = 0 \qquad \dot{\Psi}_8 = -\dot{\Psi}_4 + \eta_2 p_1$$

Transversality conditions, considering (5.11), are:

$$\Psi_2(t_0) = \Psi_3(t_0) = \Psi_4(t_0) = 0,$$
$$\Psi_2(t_1) = \Psi_3(t_1) = \Psi_4(t_1) = \Psi_6(t_1) = \Psi_7(t_1) = 0 \qquad (5\text{-}14)$$

Using the Maximum principle, it is shown that all controls v_i cannot be singular [4], therefore on the basis of (5.10) and (5.12) we obtain necessary conditions for optimality:

$$v_1 = \operatorname{sign}\Psi_5, \quad \Psi_6 - \eta_1 + \eta_2 = 0,$$
$$\Psi_7 + \eta_1(1+q^1) + \eta_3 = 0, \quad \Psi_8 - \eta_2 q^1 = 0 \qquad (5\text{-}15)$$

wherefrom, considering (5.6), (5.9), (5.11), (5.13) and (5.14), we obtain:

$$v_1^* = \begin{cases} -1, & t \in [0, \tau] \\ 1, & t \in (\tau, t_1] \end{cases}, \quad v_2^* = p_1, \quad v_3^* = 0, \quad v_4^* = \frac{-p_1}{(q^1)^2} \qquad (5\text{-}16)$$

and finally, considering (5.5) and (5.7):

$$F^* = \begin{cases} -1, & t \in \left[0, \sqrt{\frac{2}{3}}\right] \\ -1, & t \in \left(\sqrt{\frac{2}{3}}, 2\sqrt{\frac{2}{3}}\right] \end{cases}, \quad M^* = \frac{p_1((q^1)^3 - 1)(q^1+1)}{(q^1)^3}$$

$$\mu_1^* = \frac{p_1(1-(q^1)^3)}{(q^1)^3}, \quad \mu_2^* = \frac{p_1}{(q^1)^3} \qquad (5\text{-}17)$$

$$p_1 = -t, \quad q^1 = -\frac{1}{2}t^2 + 1, \quad \forall t \in \left[0, \sqrt{\frac{2}{3}}\right]$$

$$p_1 = t - 2\sqrt{\frac{2}{3}}, \quad q^1 = \frac{1}{2}t^2 - 2\sqrt{\frac{2}{3}}\,t + \frac{5}{3}, \quad \forall t \in \left(\sqrt{\frac{2}{3}}, 2\sqrt{\frac{2}{3}}\right]$$

6. CONCLUSION

The method presented in this paper has been imposed by the need analyze rezctions of either or internal constraints in the optimal control of the motion of a system with mechanical constraints. It was necessary to form applicable mechanical and mathematical models so that the results of the optimal control theory could be applied to a previously defined problem. Different approaches to the problem are possible, i.e. it is possible to solve a problem considering the undefined constraint multipliers as a part of the control function or with introduction of new control function which contain, as components, constraint reactions. besides, it is possible to transform the mechanical constraint equations as limitations for control functions, according to the initial and final system state. However, all such approaches give the same number of conditions for the definite problem solution, and which of them should be used depends exclusively on the convenience of the solution model. In mechanical system as, for example, closed loop kinematic chains, it is much more convenient to use mathematical model in which a constraint multiplisre exist explicitely, in relation to the model obtained by the elimination of the multipliers, even in those problems where constraint reactions have no influence.

Examples which were used to illustrate the methodology in the present paper, have been choosen because of the possibility of obtaining an analytical solution. However, field of the application of this paper includes several important problems such as the optimal control of the manipulator where the gripper performs a limited motion, and where the constraint reactions must be restricted.

The fact that only ideal mechanical constraints have been considered in this paper, does not exclude the possibility of application of this method to be extendid to systems with real costraints.

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ОПТИМАЛЬНОЕ УПРАВЛЕНИЕ И РЕАКЦИИ СВЯЗЕЙ МЕХАНИЧЕСКИХ СИСТЕМ

J.Вуковић, А.Обрадовић

Решается задача оптимального управления и определения реакции связей для механической системы с механическими (голономными и неголономными) связами. Метод базируется на принципиальном отличии механических связей от других ограничений движения управляемой системы. В рамках математической модели составляются уравнения состояния, которые явно содержит множители связи. Проблема сформулированна так что на ее непосредственно можно применить современную математическую теорию оптимальных процессов при иследовании практических задач.

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OPTIMALNO UPRAVLJANJE I REAKCIJE VEZA MEHANIČKIH SISTEMA

J.Vuković, A.Obradović

Rešava se problem optimanlog upravljanja uz odredjivanje reakcija veza neslobodnog mehaničkog sistema. Metod je zasnovan na činjenici da postoji suštinska razlika izmedju materijalnih veza i drugih vrsta ograničenja kretanja upravljanog sistema. U okviru matematičkog modela problema formiraju se jednačine stanja upravljanog sistema koje u sebi eksplicitno sadrže reakcije veza. Problem je doveden na oblik koji omogućava neposrednu primenu rezultata teorije optimalnog upravljanja za rešavanje praktičnih zadataka.

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