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Singular Control in Time Minimization of System of Bodies Motion

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Abstract

The problem of determination of restricted controls (generalized forces with restricted coordinates) in transfer of systems of rigid bodies with holonomous constraints between two given states in minimum time is considered. It is shown that optimal control on a boundary of allowable controls area with inevitable consideration of singular controls existence possibility in a part of control vector coordinates. Review of solutions of problems of singular control determinations which includes results obtained by authors of this paper, along with some failed attempts of other authors is given. Authors point to difficulties in computational determination of optimal control , to corresponding necessary conditions of optimality and to contemporary numerical methods used in solution of this kind of problems. One of problems is unknown optimal control structure and the fact that optimal controls lie partially on the boundary and partially inside of allowable controls area. Therefore, solution of two point boundary value problem of Pontryagin's principle is complicated. Besides, two point boundary value numerical unstable problem , appearing in this kind of problems, makes every successfully solved task worth-while. Therefore, in contemporary literature some works are dedicated to solution of concrete optimal control problem. Example of system of bodies with two degrees of freedom with optimal control which contains singular part illustrate mentioned method.

Key words: singular optimal control, bodies system

1. Introduction

The applying of modern mathematical theory of optimal control [1,2] to concrete mechanical systems is limited by difficulties connected with computation of optimal control. The solving of two-point boundary value problem of maximum principle nowadays doesn't have the general algorithm and adequate computer programs. The applying of existing programs, for example [3], isn't always successful, primarily because of numerical instability [4]. The applying of discrete variant of dynamic programming is limited mostly to mechanical systems which, in some way, can be reduced to one-degree of freedom [5] because of famous "curse of dimensionality". Some of more important problems of variable mass body control have been solved [6] only in recent years, where it is prominent that every successfully solved two-point boundary value problem is worth-while. During the computation of optimal control for rigid body systems when the controls are non-potential generalized forces with restricted coordinates, special problems appear. They are caused by the fact that the controls partially lie at the boundary of allowable control area, and partially inside of it, and then they are most often singular [7]. The widest survey of results from optimal control of bodies systems with application primarily in robotics can be found in [8] and the latest results in [9,10]. It can be seen from the review that the difficulties of computing nature caused the giving up of computing of optimal controls. Instead of that, the sub-optimal controls are computed. The question of singular control computing for robotic systems in the task of time minimization is made for the first time by the authors of paper [11]. However, as it was shown in [12], their solution isn't optimal because it doesn't satisfy necessary conditions of optimality [13,14]. For manipulator of the same type as in [11], in the paper [15] optimal controls with singular parts are calculated till the final solution. As in paper [16] the problem of numerical instability is overcome by determining of moments of getting to and getting of the point on the singular surface in phase space instead of unknown initial value of costate vector. The further exploration in [17] consisted of investigation of possibility of singular control existence for the systems with cyclical coordinate, where the physical explanations of singular control are given. The task with unfixed final and initial states of rest for two-degrees of freedom robot was considered in [18] where the proper algorithm was given for the case when the structure of optimal control can be presumed. In paper [19] optimal controls for the same type of manipulator as in [18] are calculated by using the fact that the solution is symmetric. The possibility of phenomenon of chattering in joining with singular parts of optimal trajectory is particularly emphasized. In the second part of this paper the proper task of optimal control and analysis of solution on the basis of necessary optimality condition is formulated. The computing method of optimal controls with singular part is illustrated in the third part of this paper on example of one system bodies [20,21] with two-degrees of freedom.

2. The Solution Analysis of the Time Minimization Task

Let's consider the time minimization task (1) for rigid bodies systems, where q^i are the generalized coordinates, p_j are generalized momentums, a^{ij} is contravariant metric tensor of configuration space, Π is potential energy , n is number of degrees of freedom, u_j are non-potential generalized forces (controls) and constants $A_j > 0$ determine available controls area.

$$\dot{q}^i = a^{ij} p_j \qquad i,j,k = 1,\dots,n$$

$$\dot{p}_j = -\frac{1}{2}\frac{\partial a^{ik}}{\partial q^j} p_i p_k - \frac{\partial \Pi}{\partial q^j} + u_j$$

$$u_j = u_j(t) = ?\,,\; -A_j \le u_j \le A_j \qquad (1)$$

$$\int_0^{t_1} dt \rightarrow \inf$$

$$q^i(0) = q^{i0}, q^i(t_1) = q^{i1}, p_j(0) = p_j(t_1) = 0$$

Introducing Pontryagin's function [2] :

$$H = \lambda_0 + \lambda_i a^{ij} p_j + \nu^j\left(-\frac{1}{2}\frac{\partial a^{ik}}{\partial q^j} p_i p_k - \frac{\partial \Pi}{\partial q^j} + u_j\right) \qquad (2)$$

and applying maximum principle, the non-singular optimal controls $u_j = A_j$ for $\nu^j > 0$ and $u_j = -A_j$ for $\nu^j < 0$ are achieved while in time intervals in which $\nu^j = 0$ singular controls u_j are achieved by condition :

$$\frac{d^{2r}}{dt^{2r}}\frac{\partial H}{\partial u_j} = 0 \qquad (3)$$

where r determines order of singular controls. Costate vector coordinates satisfy the system of differential equations :

$$\dot{\lambda}_i = -\lambda_k \frac{\partial a^{kj}}{\partial q^i} p_j + \nu^j\left(\frac{1}{2}\frac{\partial^2 a^{lk}}{\partial q^i \partial q^j} p_l p_k + \frac{\partial^2 \Pi}{\partial q^i \partial q^j}\right) \qquad (4)$$

$$\dot{\nu}^j = -\lambda_k a^{kj} + \nu^l \frac{\partial a^{jk}}{\partial q^l} p_k$$

$$i,j,k,l = 1,\dots,n$$

The case of singular control in all coordinates of control vector, when $\nu^j = 0, j = 1,\dots,n$, brings to the basis (4) condition $H = 0$ as well as in $\det\left[a^{kj}\right] > 0$ to the unacceptable case [2] of zero costate vector. Singular control u_j must satisfy Kelley's conditions[13] :

$$\frac{\partial}{\partial u_j}\left[\frac{d^2}{dt^2}\frac{\partial H}{\partial u_j}\right] \ge 0 \qquad (5)$$

if it is first order (r = 1). Beside that , if it joins with non-singular part on which $\nu^j \ne 0$ and if at the point of joining control u_j have discontinuity, the fulfilling of condition (5) brings to the satisfaction of proper necessary condition of singular and non-singular parts joining (when the optimal controls belong to in a part continuous functions). Singular controls of second order (r = 2) cannot join with in a part continuous controls in optimal trajectory. The fulfilling of necessary high order conditions of optimality (in this case conditions of Kopp-Moyer, which are analog to conditions (5)) brings to non-fulfilling joining conditions [14,7]. Singular controls can then be joined with controls which in finite time interval have infinite number of discontinuity (chattering), although such controls are not suitable for technical realization. On the basis of above mentioned, when solving concrete tasks of optimal control, first, singular control which tend to be optimal by fulfilling proper necessary conditions, are calculated. Then we try to add them non-singular parts. Instead of unknown initial value of costate vector, moments of the beginning and the end of singular parts are determined which is by far simpler. After that adjoint system of differential equations are integrated with aim to check fulfilling of maximum principle conditions.

3. Example

Let us solve the problem of optimal control determining on the example of time minimization problem for bodies system with two-degree of freedom. System consists of homogenous bars OA and AB (Fig.1) of the identical masses m and lengths L and 2L respectively.

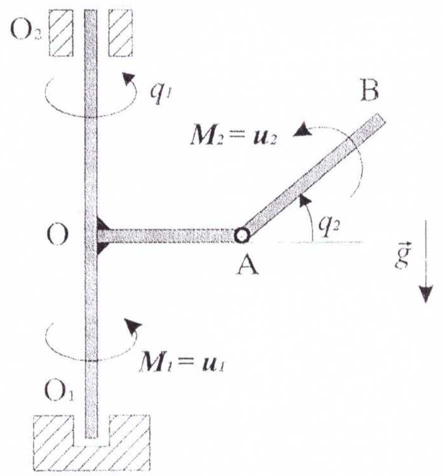

Fig.1

Parameters of task (1) are:

$$a^{11} = \frac{3}{2mL^2(3 + 3\cos q^2 + \cos(2q^2))},$$

$$a^{22} = \frac{3}{4mL^2}, \qquad a^{12} = a^{21} = 0,$$

$$\Pi = mgL\sin q^2 , A_1 = A_2 = 2mgL, \qquad (6)$$

$$m = 1kg, L = 1m$$

$$q^{10} = q^{20} = 0, q^{11} = \frac{\pi}{9} = 20^0, q^{21} = \frac{\pi}{2} = 90^0$$

Non-singular optimal controls are :

$$u_i = 2mgL\, sign\, \nu^i \;, i = 1,2 \qquad (7)$$

Taking in consideration initial and final conditions (6) and reasons for not-considering second order singular control u_2 (exposed in the second part) let us determine first order singular control u_1 . Kelley's conditions (5) have the form :

$$\frac{3v^2(3\sin q^2+2\sin(2q^2))}{2mL^2(3+3\cos q^2+\cos(2q^2))^2}\geq 0 \qquad (8)$$

and in this task they are reduced to condition $v^2 \geq 0$, which implies that singular control u_1 can be at the same time only with $u_2 = 2mgL$. That has physical explanation in undesirable "action" of proper inertial forces. Above mentioned considerations show the possibility of singular control u_1 at the beginning of movement time interval. Since then $\dot{v}^1 = 0$ and $p_1(0) = p_2(0) = 0$, from (4) it is concluded that $\lambda_1(0) = 0$, i.e. $\lambda_1(t) = 0$ in whole movement time interval. Using the condition $\ddot{v}^1 = 0$ on singular part, singular control is then $u_1 = 0$. On basis of previous analyses the following optimal control structure can be suggested :

$$u_1 = \begin{cases} 0, & t\in[0,t') \\ +2mgL, & t\in[t',t''') \\ -2mgL, & t\in[t''',t_1] \end{cases} \quad u_2 = \begin{cases} +2mgL, & t\in[0,t'') \\ -2mgL, & t\in[t'',t_1] \end{cases} \qquad (9)$$

4. The Numerical Solution

During the numerical solution it is necessary to determine previously four unknown moments (9) which provide that integration of differential equations of motion (1) satisfies the four final conditions for state variables. During that proper integrals of system (1) can be used, taking in consideration (9) :

$$p_1(t_1) - p_1(0) = \int_0^{t_1} u_1 dt \qquad (10)$$

$$E_k(t_1) - E_k(0) = -mgL + \int_0^{\frac{\pi}{9}} u_1 dq^1 + \int_0^{\frac{\pi}{2}} u_2 dq^2$$

which implies :

$$q^1(t''') = \frac{1}{4} + \frac{11\pi}{36} - q^2(t'') \quad , \quad t_1 = 2t''' - t' \qquad (11)$$

The characteristic moments are :

$$t' = 0.401953\,s \qquad t'' = 0.531459\,s$$

$$t''' = 0.589703\,s \qquad t_1 = 0.777454\,s \qquad (12)$$

and their determining is much simpler than determining of unknown initial value of costate variables. With purpose of checking Pontryagin's principle conditions it is necessary to integrate adjoint system (4). First the basic system (1), with condition (9) is integrated in interval $[0, t'']$. Then, the basic system (1) and proper

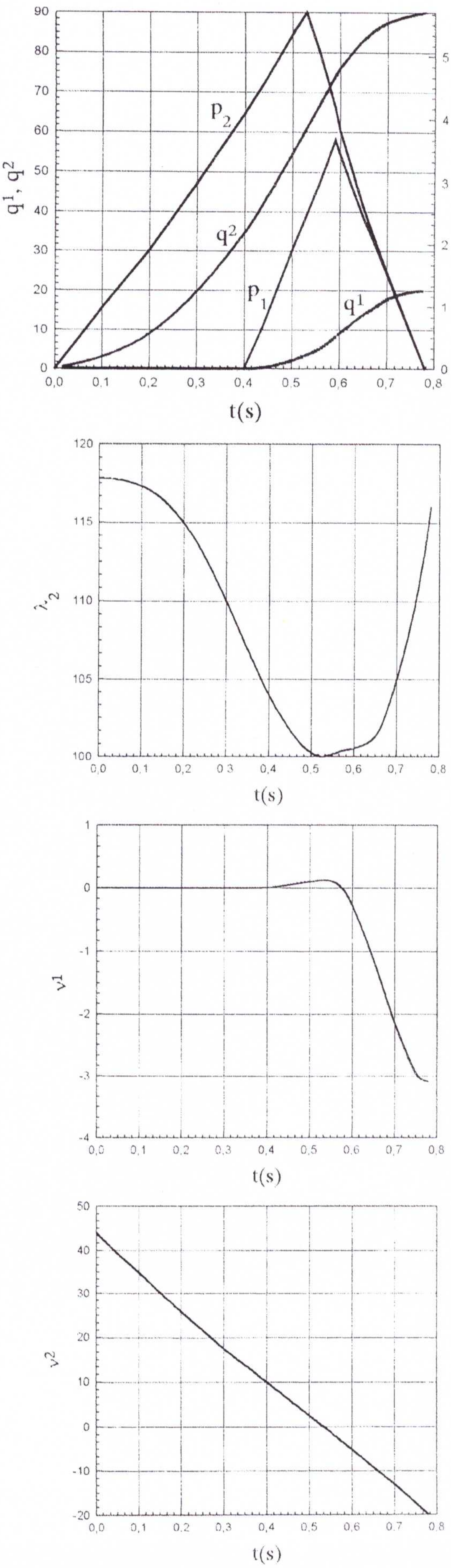

Fig.2

equations of adjoint system (4) in the same interval integrate "backwards" using the conditions $\nu^2(t'') = \lambda_1(t'') = 0$ and $\lambda_2(t'') = 100$ (unspecified value, because system (4) is homogenous). At last, with condition $\nu^1(0) = 0$, basic (1) and adjoint system (4) are integrated on the whole time interval. On Fig.2 diagrams of state and costate variables are given. The numerical computation is derived by Runge-Kutta-Werner's method of the fifth and sixth order with relative error tolerance 10^{-6}.

By analyzing the results it can be noticed :

$$\nu^1 \begin{cases} = 0 & , t \in [0, t') \\ > 0 & , t \in [t', t''') \\ < 0 & , t \in [t''', t_1] \end{cases} \qquad \nu^2 \begin{cases} > 0, t \in [0, t'') \\ < 0, t \in [t'', t_1] \end{cases} \tag{13}$$

which, considering (9) completely responds to conditions (7) of Pontryagin's principle. Beside that, with condition H=0 deducted from unspecified moment t_1, it can be calculated :

$$\lambda_0 = -mgL\nu^2(0) < 0 \tag{14}$$

and thus all conditions of maximum principle are fulfilled.

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