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A CONTRIBUTION TO OPTIMIZATION OF MOTION OF A RIGID BODY SYSTEM

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1. Introduction

Complexity of controlled motion of system with multiple degrees of freedom problem is enlarged with request for certain optimality criterion to be satisfied. However, in some cases, motion control of entire system is subordinated to behaviour of his single part. That is often case with robots and manipulators where entire system motion is determined with demands that the last segment should satisfy. Therefore, naturally imposed idea is to separate that part from the system and to separately to solve the problem of optimal control of its motion with condition that it completely fulfill the task he have as a part of the system. In this paper is used the method that authors have already used in their papers [1] and [2] for certain optimal control problem solution. The method is based on "setting free" part of the system from constraints with the rest of the system and introduction of constraint reactions as control functions. Such procedure, with properly chosen coordinate system for separated part, can significantly simplify solution of given problem.

2.Basic optimal control demand

Let the position of controlled mechanical system with n degrees of freedom in configuration space R_n be determined by coordinates $q^\alpha(\alpha = 1,2,\dots,n)$, thus Lagrange's equations of second kind having form:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}^\alpha}\right) - \frac{\partial T}{\partial q^\alpha} = -\frac{\partial V}{\partial q^\alpha} + Q_\alpha(q,\dot{q}) + u_\alpha \qquad (\alpha = 1,2,\dots,n), \tag{1}$$

where:

$T(q,\dot{q})$ is kinetic energy,

$V(q)$ is potential energy,

Q_α is generalized nonpotential force and

$u = (u_1, u_2, \dots, u_n)$ is control vector from space U.

Let the basic optimal demand be to determine among allowable controls $u \in G_u \subset U$ ones leading mechanical system from initial state:

$$A_\sigma(q(t_0), \dot{q}(t_0)) = 0 \quad \sigma = 1,2,\ldots, a \leq 2n, \tag{2}$$

to final state:

$$B_\sigma(q(t_1), \dot{q}(t_1)) = 0 \qquad \vartheta = 1,2,\ldots,b \leq 2n\ , \tag{3}$$

with optimality condition:

$$\int_{t_0}^{t_1} f^0(q,\dot{q},u)dt \to \inf_{u\in G_u} . \tag{4}$$

3. System separation and optimal demand solution

The basic optimal control demand is defined by relation (1) - (4). For direct application of maximum principle [3] it is necessary to replace equation (1) with corresponding system of 2n differential equations in normal form, by introducing system state coordinates. This definition of the problem is very complex, especially for systems with multiple degrees of freedom and it is, in general, practically impossible to solve. However, industrial robots and manipulators represent systems of rigid bodies with simple interrelations. In most cases, they are kinematics chains with pairs of fifth order having entire motion determined by given behavior of the last segment in chain. Method for optimal demand solution for general example of one branch chain with kinematical pairs of fifth order is presented here where a certain optimal behaviour of the last body in chain is required. The body can have arbitrary position in limited part of space. In that case, the number of degrees of freedom is $n \geq 6$ (for redundant system [4] is $n > 6$) and it equals number of bodies in chain. Generalized coordinates q^α individually determine position of corresponding body relative to previous one for simpler application of symbolic programming [5]. Kinetic energy can be presented in form:

$$T(q,\dot{q}) = \overline{T}(q^1,\ldots,q^{n-1},\dot{q}^1,\ldots,\dot{q}^{n-1}) + T^{(n)}(q^1,\ldots,q^n,\dot{q}^1,\ldots,\dot{q}^n)\ , \tag{5}$$

where

$\overline{T}$ is kinetic energy of system without last body,

$T^{(n)}$ is kinetic energy of the last, n-th body in chain.

Instead of generalized coordinates q^α it is convenient for determination of position of n-th body to introduce coordinates $z^i (i = 1,2,\ldots,6)$ where z^1, z^2, z^3 are coordinates of center of the body mass in stationary Cartesian system and z^4, z^5, z^6 is Euler's angles determining body orientation [6]. That is orientation of coordinate system attached to the body having axes that are main central axes of inertia. For that, z^i and q^α are related:

$$\varphi^i(z^1,\ldots,z^6,q^1,\ldots,q^n)=0,\ rank\left\{\frac{\partial\varphi^i}{\partial q^\alpha}\right\}=6,(i=1,\ldots,6),(\alpha=1,\ldots n\geq 6). \quad (6)$$

Controlled motion of n-th body is described by equations:

$$\dot{z}^i = a^{ij}(z)p_j\ ,\ \dot{p}_i = F_i(z,p)+v_i\ ,\ (i=1,2,\ldots,6)\ , \quad (7)$$

where

p_1, p_2, p_3 are projection of momentum of n-th body on axes of stationary Cartesian system and

p_4, p_5, p_6 are projection of momentum on axes of moving system, attached to the body.

In equations (7) v_i represent control forces and moments from certain set G_v of allowable controls leading given body from initial state $z^i(t_0), p_i(t_0)$ to final state $z^i(t_1), p_i(t_1)$ with optimality condition:

$$\int_{t_0}^{t_1} F_0(z,p,v)dt \to \inf_{v\in G_v}. \quad (8)$$

Considering to (7) and (8), this optimal demand can be solved by direct use of maximum principle, where simplicity of equations (7) compared to equations (1) greatly simplify solving procedure.

If problem can be solved i.e. if optimal control and corresponding motion can be determined in form of functions:

$$v_i^* = v_i^*(t)\ ,\ z^{*i} = z^{*i}(t) \quad (9)$$

further procedure consists in determination optimal controls $u_\alpha^*(t)$ from initial optimal demand (1) - (4).

One way is to directly determine corresponding optimal controls $u_\alpha^*(t)$ using differential equations (1), constraint equations (6) and equations of motion (9). In case $n=6$ system is non-redundant, there is only one solution.

In case $n>6$, system is redundant, and to obtain only one solution it is necessary to solve redundancy problem by introducing additional criteria [4] depending on degree of redundancy ($n-6$).

The other way is based on a fact that controls v_i represent interaction between "separated" n-th body and the rest of the system, which can be, considering (6), expressed in following form:

$$v_i = \lambda_j \frac{\partial \varphi^j}{\partial z^i} \quad (i, j = 1,2,\dots,6) \ , \tag{10}$$

where λ_i are unknown multiplicators.

In that case, motion of the rest of the system having kinetic energy $\overline{T}$ (5), is described by equations:

$$\frac{d}{dt}\left(\frac{\partial \overline{T}}{\partial \dot{q}^{\mu}}\right) - \frac{\partial \overline{T}}{\partial q^{\mu}} = -\frac{\partial \overline{V}}{\partial q^{\mu}} + \overline{Q}_{\mu} + u_{\mu} + \lambda_i \frac{\partial \varphi^i}{\partial q^{\mu}} \quad (\mu = 1,\dots,n-1), \tag{11}$$

being much simpler than (1). Optimal controls $u^*_{\mu}(t) \quad (\mu = 1,2,\dots,n-1)$ depending on degree of redundancy can be determined directly from equations (11), by using constraint equations (6), optimal solutions (9) and relations (10). Control u^*_n is implicitly contained in solution (9) and readily to determined.

Presented optimal demand solution method by separating the system considers optimal behaviour of single body from the system and direct dynamics problem of rest of the system, and significantly simplify the entire procedure. However, this setting of optimal control demand should be equivalent to initial demand (1)-(4). Demands are equivalent if criterions of optimality (4) and (8) are equivalent which is not always allowable. Besides, range of allowable controls G_v cannot always allow existence of corresponding optimal controls u^* within the range G_u. Although these deficiencies narrow the application of presented method it can solve significant class of optimal control problems.

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