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INFLUENCE OF SECOND ORDER EFFECTS ON PRESSURE DISTRIBUTION IN MICROTUBES

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Extended Abstract

A pressure field of isothermal compressible two dimensional subsonic slip gas flow through microtube with constant cross section is considered in this paper. In gas flow through microtube the ratio of the molecular mean free path $\tilde{\lambda}$ and characteristic dimension of microtube $\tilde{D}$ cannot be ignored. Hence, considered gas flow is characterized as a rarefied gas flow [1]. Parameter that determines level of rarefaction is Knudsen number ($Kn = \tilde{\lambda} / \tilde{D}$). According to the values of Kn we examined continuum and slip gas flow, where values of Knudsen number are $Kn \leq 0.1$.

The gas flow through microtube is caused by the pressure difference between inlet and outlet of the microtube. The slip gas flow is considered by macroscopic approach, so the system of governing equations and slip boundary condition given in dimensional form in cylindrical coordinates ($\tilde{r}$, $\tilde{z}$) are:

$$\tilde{\rho}\left(\tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{r}} + \tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{z}}\right) = \tilde{\mu}\left(\frac{\partial^2 \tilde{u}}{\partial \tilde{r}^2} + \frac{\partial^2 \tilde{v}}{\partial \tilde{r}\partial \tilde{z}}\right) - \frac{\partial \tilde{p}}{\partial \tilde{z}} - \frac{2}{3}\tilde{\mu}\left(\frac{\partial}{\partial \tilde{z}}\frac{1}{\tilde{r}}\frac{\partial(\tilde{r}\tilde{v})}{\partial \tilde{r}}\right) + \frac{4}{3}\tilde{\mu}\frac{\partial^2 \tilde{u}}{\partial \tilde{z}^2} + \frac{\tilde{\mu}}{\tilde{r}}\left(\frac{\partial \tilde{u}}{\partial \tilde{r}} + \frac{\partial \tilde{v}}{\partial \tilde{z}}\right), \quad (1)$$

$$\frac{1}{\tilde{r}}\frac{\partial(\tilde{\rho}\tilde{r}\tilde{v})}{\partial \tilde{r}} + \frac{\partial(\tilde{\rho}\tilde{u})}{\partial \tilde{z}} = 0, \quad (2)$$

$$\tilde{p} = \tilde{\rho} R_g \tilde{T}, \quad (3)$$

$$\tilde{u}\Big|_{\tilde{r}=\tilde{R}} = -\frac{2-\sigma_v}{\sigma_v}\tilde{\lambda}\frac{\partial \tilde{u}}{\partial \tilde{r}}\Big|_{\tilde{r}=\tilde{R}}. \quad (4)$$

where $\tilde{u}, \tilde{v}, \tilde{p}, \tilde{T}, \tilde{\rho}, \tilde{\mu}, \sigma_v$ are velocity components, pressure, temperature, density, dynamic viscosity and accommodation coefficient, respectively.

With the aim of getting analytical solution, all physical values are assumed by perturbation series with two approximations $f = f_0 + Kn_e f_1 + O(Kn_e^2)$. System of governing equations is converted in non-dimensional form with the dimensionless variables:

$$u = \frac{\tilde{u}}{\tilde{u}_e},\ v = \frac{\tilde{v}}{\tilde{u}_e},\ p = \frac{\tilde{p}}{\tilde{p}_e},\ T = \frac{\tilde{T}}{\tilde{T}_e},\ r = \frac{\tilde{r}}{\tilde{R}_e},\ z = \frac{\tilde{z}}{\tilde{L}},\ \mu = \frac{\tilde{\mu}}{\tilde{\mu}_e}, \quad (5)$$

where u_e, p_e, T_e, R_e and μ_e are the values at the outlet cross section. Also, as gas flows at low values of Knudsen and Mach numbers, certain suppositions are made:

$$Kn_e = \eta\varepsilon^n,\ \kappa Ma_e^2 = \gamma\varepsilon^m,\ \kappa Ma_e^2 / Re_e = \beta\varepsilon, \quad (6)$$

where small parameter ε is the ratio between diameter and length of the microtube.

From dimensionless system of equations and boundary condition two systems of equations with boundary condition, of order $O(1)$ and order $O(Kn_e)$, are extracted:

$$\int_0^R 2p_0u_0 r\mathrm{d}r = 0, \qquad p_0\,|_{z=1} = 1 \tag{7}$$

$$4\beta\frac{\partial^2 u_0}{\partial r^2} - \frac{\partial p_0}{\partial z} + \frac{4\beta}{r}\frac{\partial u_0}{\partial r} = 0, \qquad u_0\,|_{r=1} = 0 \tag{8}$$

$$\int_0^R 2(p_0u_1 + p_1u_0) r\mathrm{d}r = 0, \qquad p_1\,|_{z=1} = 0 \tag{9}$$

$$4\beta\frac{\partial^2 u_1}{\partial r^2} - \frac{\partial p_1}{\partial z} + \frac{4\beta}{r}\frac{\partial u_1}{\partial r} = \frac{\gamma}{\eta}p_0\left(V_0\frac{\partial u_0}{\partial r} + u_0\frac{\partial u_0}{\partial z}\right), \quad u_1\,|_{r=1} = -\frac{2-\sigma_v}{\sigma_v}\frac{2}{p_0}\frac{\partial u_0}{\partial r} \tag{10}$$

By solving systems of equations (7)-(10) for known pressure at exit $p_0\,|_{z=1} = 1$ and $p_1\,|_{z=1} = 0$, an analytical solution for pressure distribution along the microtube is obtained:

$$p = p_0 + Kn_e\, p_1 = p_0 + Kn_e\left(\frac{2\gamma \ln p_0}{\eta\, p_0} + 8\left(\frac{1}{p_0} - 1\right)\right), \tag{11}$$

where $p_0 = \sqrt{1 + 64\beta(1-z)}$ is the first approximation which represents continuum (Kn_e=0), p_1 represents the second approximation which include second order effects: slip and inertia, where $Kn_e\dfrac{2\gamma \ln p_0}{\eta\, p_0}$ represents impact of inertia.

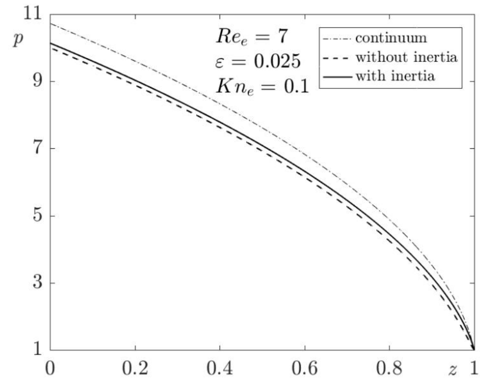

Fig. 1. The pressure distribution in a microtube for 3 cases: continuum, slip without inertia and slip with inertia taken into account.

From the results for the pressure distribution in microtube presented in Fig. 1. it is obvious that for the same mass flow slip effect leads to the pressure decrease while inertia leads to the increase of pressure.

Key words: microtube, pressure, slip gas flow, isothermal, rarefied gas

References

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