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First passage of stochastic fractional derivative systems with power-form restoring force

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ABSTRACT

In this paper, the first-passage failure of stochastic dynamical systems with fractional derivative and power-form restoring force subjected to Gaussian white-noise excitation is investigated. With application of the stochastic averaging method of quasi-Hamiltonian system, the system energy process will converge weakly to an Itô differential equation. After that, Backward Kolmogorov (BK) equation associated with conditional reliability function and Generalized Pontryagin (GP) equation associated with statistical moments of first-passage time are constructed and solved. Particularly, the influence on reliability caused by the order of fractional derivative and the power of restoring force are also examined, respectively. Numerical results show that reliability function is decreased with respect to the time. Lower power of restoring force may lead the system to more unstable evolution and lead first passage easy to happen. Besides, more viscous material described by fractional derivative may have higher reliability. Moreover, the analytical results are all in good agreement with Monte-Carlo data.

1. Introduction

First-passage [1], aims to determine the probability that systems response reaches the boundary of a bounded safe domain of state space within its lifetime. Being as one branch of reliability in mathematics, it can exactly describe the response feature and fatigue life of certain structures such as offshore platform, civil construction, etc. In the last several decades, stochastic averaging method has been proved as a powerful technique to solve the first passage problem of stochastic non-linear dynamics. The main feature of this method is that it can leads original systems to a Markovian approximation with systems dimensions reduced as well. Many authors dedicated their efforts to searching for first passage by using the stochastic averaging method. For example, Ariaratnam and Pi [2] explored first passage time for envelope crossing for a linear oscillator. Robert and Spanos [3–5] developed standard stochastic averaging method and applied it to study reliability under evolutionary seismic excitations by transforming random dynamical systems into a partial differential equation. In 1990s, Zhu et al. [6] proposed the stochastic averaging of quasi-Hamilton systems and investigated first passage problem in random dynamical systems under stationary noise excitations. Recently, the corresponding author and his coworker [7] examined the first passage failure of MDOF quasi generalized Hamiltonian systems based on the stochastic averaging method of quasi generalized Hamiltonian systems.

In recent years, with the development of new material called viscoelastic material such as liquid crystal, rubber, polymer, etc., the mathematical theory of fractional derivative to describe viscoelasticity especially viscous damping attracts much attention. In this regard, Makris and Constantinou [8] explored fractional derivative in deterministic Maxwell model for viscous-damper. Mainardi [9] and Rossikhin and Shitikova [10,11] provided an excellent perspective about the research on application of fractional derivative in the field of solid mechanics. Mainardi [12] gave a tutorial survey on fractional calculus in linear viscoelasticity and the time-fractional derivative in relaxation process. Shen et al. [13] studied the primary resonance of Duffing oscillator with two kinds of fractional derivative terms using the averaging method. It should be pointed out that the work mentioned above endowed with fractional derivative were all proceeded in deterministic systems.

At the beginning of new century, however, some researchers have begun to engage in random dynamical system with fractional derivative involved. Agrawal [14] suggested an analytical scheme for stochastic dynamical systems containing fractional derivative. Huang and Jin [15] applied the stochastic averaging method for deriving the stationary response and stability in a quasi-Hamiltonian

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system. Spanos and Evangelatos [16] achieved non-stationary response in a non-linear system with fractional derivative restoring force by utilizing Newmark algorithm and statistical linearization. Paola et al. [17] examined stationary and non-stationary stochastic response of linear fractional viscoelastic system. Chen and his cow-orkers [18] proposed a review on stochastic dynamics and fractional order optimal control of quasi integrable Hamiltonian systems with damping modeled by a fractional derivative. Xu et al. [19,20] developed an perturbation technique by combining the LP method and multiple scales method to investigate the responses of the stochastic Duffing oscillator with fractional damping which shows good agreement with numerical simulations. More recently, Matteo et al. [21] obtained the stochastic response of non-linear oscillator with fractional derivatives elements via the Wiener path integral.

It should be noted that the work with respect to fractional derivative in random dynamical systems mainly dedicates to those restoring forces with integer-power especially odd-integer power restoring force. In fact, restoring force in engineering structures especially in elastic–plastic seismic structures may be modeled as purely non-linear function with arbitrary order of power-form, for example, $g(x)=\mathrm{sgn}(x)|x|^{\beta}$, where β is arbitrary value, and many references [22–29] have considered oscillators with such non-linear restoring force in deterministic systems. In 2003, Gottlieb [22] analyzed the frequencies of oscillators with fractional-power non-linearities and obtained an expression for the exact period. Next year, Pilipchuk [23] considered a class of elastic oscillators with power of non-linear restoring force taking as real fraction, rational or irrational number respectively. Recently, Kovacic and Rakaric [24] applied Ritz method to derive higher-order approximations for oscillators with a fractional-order restoring force. Wang and Yang [25] even proposed a positive-power non-linear restoring force by studying amplitude-frequency and phase-frequency characteristics of forced oscillators. Except that, references [26–29] have emphasized the importance and widely background of this kind of restoring force.

In this paper, the first passage failure of a commonly fractional derivative system with a power-form non-linear restoring force, where the power can be a fraction, is addressed by using the combined method of stochastic averaging method for quasi Hamiltonian systems and diffusion theory of first passage failure. Two cases, namely, purely power-form non-linear restoring force and combination of linear with non-linear restoring forces are considered. Besides, the Monte-Carlo simulation will be employed to examine the efficiency and accuracy of the proposed approaches.

2. Model and formulations

Consider non-linear dynamical systems with fractional derivative and power-form restoring force subjected to Gaussian white-nose excitations, the motion of equation can be expressed in the following form:

$$\ddot{x}+\varepsilon f(x,\dot{x})D^{\alpha}x(t)+g(x)=\varepsilon^{1/2}h_k(x,\dot{x})W_k(t),\quad (k=1,2,\ldots m,\quad m\in Z^{+}) \tag{1}$$

where $x(t)$ is a non-Markov process, usually, denotes generalized displacement, $f(x,\dot{x})$ and $h_k(x,\dot{x})$ are linear or non-linear functions with respect to x and $\dot{x}$. ε is a small parameter, and $W_k(t)$ are uncorrelated Gaussian white- noise with zero means and correlation functions, which satisfy

$$E[W_k(t)W_l(t+\tau)]=2D_{kl}\delta(\tau),\quad k,l=1,2,\ldots m$$

where $\delta(\tau)$ is Dirac Delta function, D_{kl} are constants. $D^{\alpha}x(t)$ is Caputo-type fractional derivative and defined by

$$D^{\alpha}x(t)=\frac{1}{\Gamma(n-\alpha)}\int_0^t\frac{x^{(n)}(t-\tau)}{\tau^{\alpha}}d\tau,$$

$$n=[\alpha]+1,\quad n-1<\alpha\le n \tag{2}$$

$g(x)$ is a non-linear restoring force and characterized by a power-form function

$$g(x)=\sum_{\beta}c_{\beta}\mathrm{sgn}(x)|x|^{\beta} \tag{3}$$

where β is an arbitrary non-negative real number and c_{β} is a constant. To sum up, the dynamical system (1) is characterized by fractional derivative and power-form restoring force with weakly external and (or) parametric random excitations.

Suppose that $x=x_1$ and $\dot{x}=x_2$, system (1) can be rewritten as a set of first-order differential equations, that is

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=-\varepsilon f(x_1,x_2)D^{\alpha}x_1(t)-g(x_1)+\varepsilon^{1/2}h_k(x_1,x_2)W_k(t)\end{cases} \tag{4}$$

Now consider the free vibration of the dynamical system (1) in the case of $\varepsilon=0$, then

$$\ddot{x}_1+g(x_1)=0 \tag{5}$$

Correspondingly, the Hamiltonian function of this system $H(t)\triangleq H$ is a slow-varying variable, which satisfies the following equation:

$$\tfrac{1}{2}x_2^2+V(x_1)=H \tag{6}$$

where $V(x_1)$ is the potential energy of the system and estimated by

$$V(x_1)=\int_0^{x_1}g(t)dt \tag{7}$$

Substituting (3) into (7), it may have

$$V(x_1)=\sum_{\beta}\frac{c_{\beta}}{\beta+1}x_1^{\beta} \tag{8}$$

According to stochastic differential law [6], the equation governing Hamiltonian function satisfies

$$\dot{H}=\varepsilon\left[-x_2f(x_1,x_2)D^{\alpha}x_1(t)+\tfrac{1}{2}\sigma_k(x_1,x_2)\sigma_l(x_1,x_2)\right]+\varepsilon^{1/2}x_2\sigma_k(x_1,x_2)W_k(t) \tag{9}$$

where

$$\sigma_k(x_1,x_2)\sigma_l(x_1,x_2)=2D_{kl}h_k(x_1,x_2)h_l(x_1,x_2)\quad k,l=1,2,\ldots m \tag{10}$$

the Hamiltonian function will weakly converge to an averaged Itô differential equation on the basis of Khasminskii averaging theorem [30] if $\varepsilon\to 0$, which is governed by

$$dH=m(H)dt+\sigma(H)dB(t) \tag{11}$$

where $B(t)$ is standard Wiener process, $m(H)$ and $\sigma(H)$ are drift and diffusion functions, respectively, they can be calculated by stochastic averaging procedure of quasi-Hamiltonian system [6] as follows:

$$m(H)=\langle F(x_1,x_2)\rangle, \tag{12}$$

$$\sigma^2(H)=\langle G_{kl}(x_1,x_2)\rangle,\quad k,l=1,2,\ldots m, \tag{13}$$

in which

$$F=-x_2f(x_1,x_2)D^{\alpha}x_1(t)+\tfrac{1}{2}\sigma_k(x_1,x_2)\sigma_l(x_1,x_2), \tag{14a}$$

$$G_{kl}=x_2^2\sigma_k(x_1,x_2)\sigma_l(x_1,x_2),\quad k,l=1,2,\ldots m, \tag{14b}$$

$$\langle\bullet\rangle=\frac{1}{T(H)}\int_{\Omega}\bullet\, dx_1, \tag{15}$$

$$x_2=\sqrt{2H-2V(x_1)}, \tag{16}$$

$$\Omega=\left\{(x_1,x_2)\middle|H(x_1,0)\le H\right\} \tag{17}$$

In order to get the detail expressions for averaged drift and diffusion coefficients, the joint response process (x_1,x_2) is needed

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to be transformed to a pair of slow varying processes denoted by $(a(t), \theta(t))$. To that end, a generalized Van der Pol transformation is introduced as follows:

$$x_1(t) = a(t)\cos\varphi(t)$$
$$x_2(t) = -a(t)\omega(a)\sin\varphi(t) \tag{18}$$

where $\varphi(t) = \int_0^t \omega(a)d\tau + \theta(t)$, $a(t)$ is envelope process and determined by $a(t) = V^{-1}(H)$, where $V()$ is potential function of system and defined by (8). $\theta(t)$ is phase process, it is slow varying with respect to time as same as envelope process. $\omega(a)$ is system's averaged frequency, and decided by following equation:

$$\omega(a) = \frac{2\pi}{4\int_0^a \frac{1}{\sqrt{2V(a)-2V(x)}}dx} \tag{19}$$

It is seen that the fractional derivative with Caputo definition is essentially a generalized integral with derivative of time-delay in it, usually, it is very difficult to deal with a higher fractional order in mathematics. Herein only the case $0 < \alpha \le 1$ in this paper is considered, other values for α will be discussed further in our future work.

According to formula (18) and suppose that τ is small, we have

$$x_2(t-\tau) \approx -a(t)\omega(a)\sin(\varphi(t) - \dot{\varphi}(t)\tau)$$
$$\approx x_2(t)\cos\omega\tau + x_1(t)\omega\sin\omega\tau \tag{20}$$

Then the main integral part in Caputo fractional derivative definition can be rewritten as

$$\int_0^t \frac{\dot{x}(t-\tau)}{\tau^\alpha}d\tau = \left[x_2(t)\int_0^t \frac{\cos\omega\tau}{\tau^\alpha}d\tau + x_1(t)\omega\frac{\sin\omega\tau}{\tau^\alpha}d\tau\right] \tag{21}$$

It turns out that how to calculate or approximate the integrals appeared in (21) is an important task to replace the complicated Caputo-type fractional derivative in terms of envelope and frequency. Fortunately, the following two generalized integrals can play a role to solve this problem, they are, respectively

$$\int_0^t \frac{\sin\omega\tau}{\tau^\alpha}d\tau = \omega^{\alpha-1}(a)\left[\Gamma(1-\alpha)\cos\frac{\pi\alpha}{2} - \frac{\cos\omega t}{(\omega t)^\alpha} + o(\omega t)^{-\alpha}\right] \tag{22}$$

$$\int_0^t \frac{\cos\omega\tau}{\tau^\alpha}d\tau = \omega^{\alpha-1}(a)\left[\Gamma(1-\alpha)\sin\frac{\pi\alpha}{2} + \frac{\sin\omega t}{(\omega t)^\alpha} + o(\omega t)^{-\alpha}\right] \tag{23}$$

On the basis of integrals of (22) and (23), then Caputo-type fractional derivative can be approximated as

$$D^\alpha x_1(t) = \omega^{\alpha-1}(a)\left[x_2(t)\sin\tfrac{\pi\alpha}{2} + x_1(t)\cos\tfrac{\pi\alpha}{2}\right]$$
$$+ \frac{\omega^{\alpha-1}(a)}{\Gamma(1-\alpha)}\left[\frac{x_1(t)\cos\omega t - x_2(t)\sin\omega t}{(\omega t)^\alpha}\right] + o(\omega t)^{-\alpha-1} \tag{24}$$

After that, the drift function and diffusion function in differential equation (11) can be computed out completely by means of stochastic averaging method mentioned ahead.

3. First-passage failure

Suppose that a safety domain of Hamiltonian function $H(t)$ is an open interval $D = [0, \partial D)$, where ∂D is a smooth boundary of D. As defined in the part of introduction, reliability of a structure depends on the probability of reaching boundary within the system lifetime. Furthermore, the system or the structure will be destroyed once the system response crosses beyond the boundary of safety domain. Therefore, conditional reliability function should be defined as follows:

$$R(t|H_0) = p\{H(s) \in D, s \in [0,t) | H(0) = H_0 \in D\} \tag{25}$$

which satisfies a BK equation in the form of partial differential equation

$$\frac{\partial R}{\partial t} = m(H_0)\frac{\partial R}{\partial H_0} + \frac{1}{2}\sigma^2(H_0)\frac{\partial^2 R}{\partial H_0^2} \tag{26}$$

In which $m(H_0)$ is governed by Eq. (12) and $\sigma^2(H_0)$ is governed by Eq. (13). Except that, boundary condition and initial condition are required in order to solve BK equation successfully. The initial condition is

$$R(0|H_0) = 1, \quad H_0 \in D \tag{27}$$

and two boundary conditions are, respectively

$$R(t|H_0) = 0, \quad \text{if } H_0 \in \partial D$$
$$R(t|H_0) \le 1, \quad \text{if } H_0 = 0 \tag{28}$$

On the other hand, mean first-passage time $E(T)$ is another important variable to measure the reliability of a stochastic dynamical system. First-passage time refers to the special time that the responses of system exceed the boundary of the safe domain at the first time on some certain conditions. It has been proved that the statistical moments of first-passage time fulfill the following GP equation, which has a form of differential equation. Denote $\mu_N(H) = E\left(T^N\right)$, then they are governed by

$$m(H_0)\frac{d\mu_{N+1}}{dH_0} + \frac{1}{2}\sigma^2(H_0)\frac{d^2\mu_{N+1}}{dH_0} = -(N+1)\mu_N$$
$$N = 0, 1, 2\ldots \tag{29}$$

Similarly, boundary conditions are needed to solve the GP equation.

The left boundary condition is

$$\mu_{N+1}(H_0) = \text{finite}, \quad \text{if } H_0 = 0 \tag{30}$$

And the right boundary condition is

$$\mu_{N+1}(H_0) = 0, \quad \text{if } H_0 = \partial D$$

Specifically, the mean first-passage time $E(T)$ is exactly equivalent to μ_1 if the initial value is $\mu_0 = 1$.

4. Examples and numerical results

4.1. Example 1

Consider

$$f(x,\dot{x}) = 1 - x^2, \quad h_1(x,\dot{x}) = 1, \quad h_2(x,\dot{x}) = x^2$$

Then the dynamical system (1) is subjected to external and parametric Gaussian white-noise excitations, the system can be written as

$$\ddot{x} + \varepsilon\left(1 - x^2\right)D^\alpha x(t) + c_\beta^2 \mathrm{sgn}(x)|x|^\beta$$
$$= \varepsilon^{1/2}\left[W_1(t) + x^2 W_2(t)\right] \tag{31}$$

According to formula (12) and (13), in this example

$$\langle F(x_1,x_2)\rangle = \frac{1}{T_1(H)}\int_\Omega \left\{\begin{array}{l} -(1-x_1^2)x_2\omega^{\alpha-1}(a) \\ \left[x_2(t)\sin\frac{\pi\alpha}{2} + x_1(t)\cos\frac{\pi\alpha}{2}\right] \\ +D_{11} + D_{22}x_1^4 \end{array}\right\}dx_1 \tag{32}$$

$$\langle G(x_1,x_2)\rangle = \frac{1}{T_1(H)}\int_\Omega \left(2D_{11}x_2^2 + 2D_{22}x_1^4x_2^2\right)dx_1 \tag{33}$$

where

$$T_1(H) = \int_\Omega \frac{1}{\sqrt{2H - 2c_\beta x_1^{\beta+1}/(\beta+1)}}dx_1$$
$$= 4\sqrt{\frac{2}{\beta+1}}a^{\frac{1-\beta}{2}}\int_0^{\pi/2}(\sin\varphi)^{\frac{1-\beta}{\beta+1}}d\varphi \tag{34}$$

in which $a = \left((\beta+1)H/c_\beta\right)^{1/(\beta+1)}$.

The corresponding BK equation governing the conditional reliability function and the GP equation governing the mean first-passage time are the same as Eqs. (26) and (29), respectively, and they are solved numerically together with suitable boundary and initial conditions.

In Figs. 1–3, some numerical results for the conditional reliability function and mean of the first-passage time have been obtained and shown. It is seen from Fig. 1 that the reliability probability is a decreasing function with respect to time. α is the order of fractional derivative, as a matter of fact, different values of α have small influence on reliability functions in the case of external and parametric excitations on system in Example 4.1. In Fig. 1, the solid lines represent analytical results obtained from solving BK equation (26) with energy boundary values $\partial D=5$, but hollow triangles denote the numerical results obtained from Monte-Carlo Simulation by performing on original dynamical system (1). Fig. 2 displayed the mean first-passage time by solving GP equation (29) when $N=1$ with the same parametric values as in Fig. 1. Note that energy function $H(t)$ stay longer in the safe domain if initial energy is smaller. It is worthy to say that numerical results in Figs. 1 and 2 are all in excellent agreement for parameter values $\varepsilon=1$, $\beta=3.5$, $D_{11}=0.05$, $D_{22}=0.05$, $c_\beta=1$.

In addition, we also examined the change caused by power-form restoring force on reliability in Fig. 3, the rest parametric values are same as in Fig. 1 except for $\alpha=0.1$. It is seen that the strong non-linearity plays a good role in improving system reliability. Restoring force usually depicts the ability that structures restore to the original shape after external loads such as noises are removed. Therefore, the small fractional power of non-linear restoring force in Fig. 3 may lead the system to more unstable evolution. That means first passage is correspondingly easy to happen.

4.2. Example 2

Let

$$\varepsilon f(x,\dot{x})=\beta_0(1-x^2), \quad h(x,\dot{x})=1$$

$$g(x)=\omega_0^2 x+c_\beta \mathrm{sgn}(x)|x|^\beta \tag{35}$$

and rewrite the original system (1) as

$$\ddot{x}+\beta_0(1-x^2)D^\alpha x(t)+\omega_0^2 x+c_\beta \mathrm{sgn}(x)|x|^\beta=W(t) \tag{36}$$

In this example, the system is subjected to external excitation absolutely. We change the expression of restoring force, where $g(x)$ is composed by a linear function and a non-linear power-form function. In this case, the mathematical procedure to obtain averaged differential equation is more complicated since frequency of the system will be difficult to obtain and it should be derived numerically.

According to (7), we have

$$V(x_1)=\frac{1}{2}\omega_0^2 x_1^2+\frac{c_\beta}{\beta+1}|x_1|^\beta.$$

Then by solving from Eq. (6), the generalized velocity will be of form

$$x_2=\sqrt{2H-\omega_0^2 x_1^2-2c_\beta x_1^{\beta+1}/(\beta+1)},$$

Substituting this expression into formula (14a) and (14b) and finishing stochastic averaging procedure (12) and (13), then drift function and diffusion function will be followed. After that, the conditional reliability function and mean of the first-passage time associated with the averaged equation can be also harvested by solving the BK equation and GP equation, respectively.

Fig. 4 shows the reliability function with respect to time in Example 4.2, where parameters are $\beta_0=0.05$, $\beta=3.5$, $\omega_0=1$, $D_{11}=0.05$, $c_\beta=1$, and $\partial D=5$ respectively.

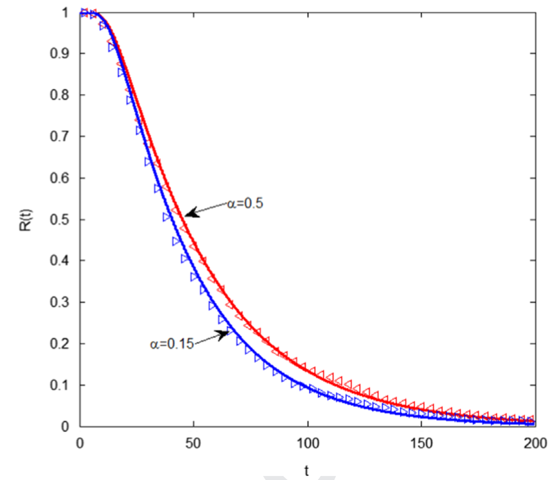

Fig. 1. Reliability functions with respect to time. The parameters are $\varepsilon=1$, $\beta=3.5$, $D_{11}=0.05$, $D_{22}=0.05$, $\partial D=5$, $c_\beta=1$, and $H_0=0.0$.

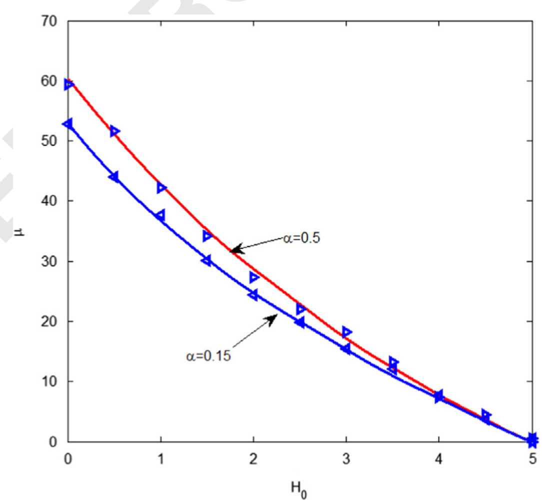

Fig. 2. Mean first passage time with respect to initial energy. The other parameters are the same as those in Fig. 1.

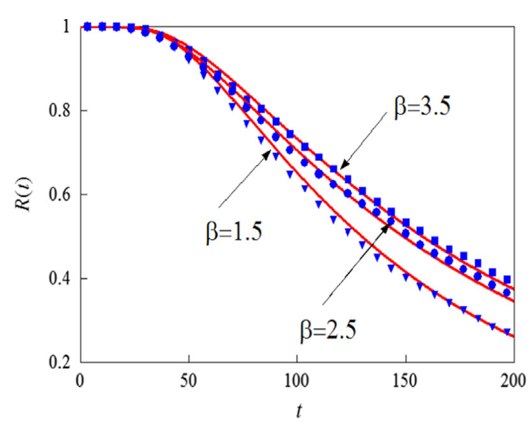

Fig. 3. The change of reliability functions caused by power-form restoring force with $\partial D=5$ and $\alpha=0.1$. The other parameters are the same as those in Fig. 1.

It is founded that, different from Example 4.1, the order of fractional derivative play an obvious role in changing reliability of system. Generally, fractional derivative is used to represent constitutive relation for special viscoelastic material in structure engineering, and the order of fractional derivative is helpful to distinguish the physical feature of viscoelastic material. If α value is bigger, then the material in physical feature tends to viscous,

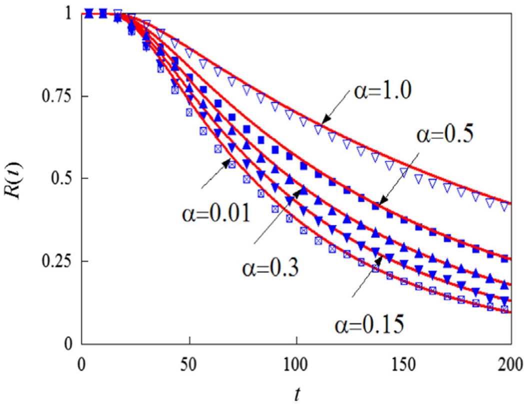

Fig. 4. Reliability functions with respect to time. The parameters are $\beta_0 = 0.05$, $\beta = 3.5$, $\omega_0 = 1$, $D_{11} = 0.05$, $c_\beta = 1$ $\partial D = 5$, and $H_0 = 0.0$.

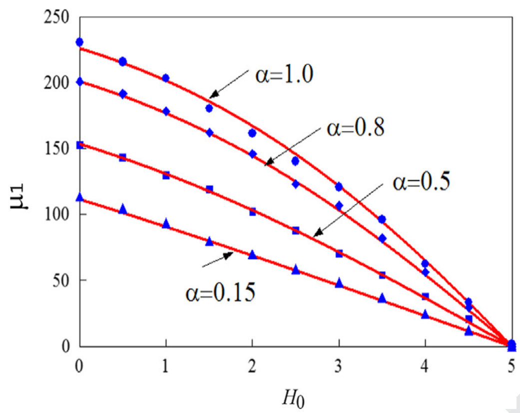

Fig. 5. Mean first-passage times with respect to initial energy in the case of same parameters in Fig.4.

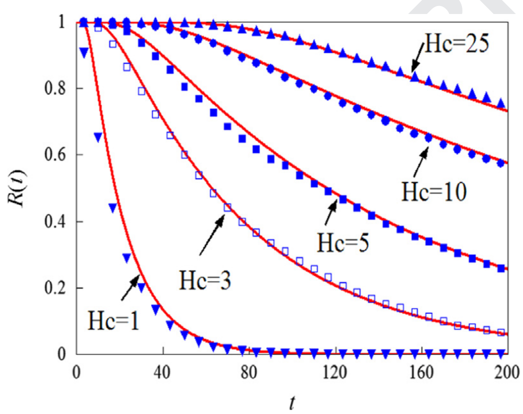

Fig. 6. Reliability functions with respect to time corresponding to different boundary values in safety domain.

otherwise, the material tends to elastic. It is shown in Fig. 4 that the reliability of system is enhanced greatly by the same α values, and integer order of derivative can reach the maximum reliability. This suggests that more viscous viscoelastic material in structural engineering may have higher safety.

Fig. 5 plotted the mean first-passage time with respect to initial energy. Comparatively, the time has been prolonged very much. Fig. 6 showed the effect of safe domain boundary on system reliability, where the parameters are the same as those in Fig. 4 except for $\alpha = 0.5$. Obviously, the larger of boundary value ∂D is, the higher probability of reliability is as well. Fig. 7 gave some numerical results about influence caused by parameter β, in which $\partial D = 10$ and $\alpha = 0.5$. Similarly, the larger β is, the higher reliability probability is.

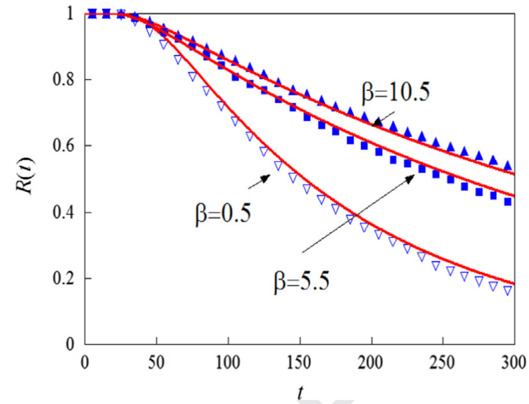

Fig. 7. Reliability functions with respect to time related to different values of β.

Correspondingly, we can choose a suitable value to design restoring force according to this result.

5. Conclusions

To sum up, in this paper, we have investigated the first-passage failure in a fractional derivative system with power-form restoring force. Stochastic averaging method is used to convert the original system into an Itô differential equation. According to the definition of first-passage failure, BK equation and GP equation are derived and solved, respectively. The numerical results tell us that the reliability probabilities are decreased monotonously with respect to the time. Higher order of fractional derivative can lead to higher reliability of the system. Boundary value of safe domain can affect the reliability probability greatly. The larger boundary value it is, the higher probability is. Exponent value of β in power-form function has also influence on reliability. Larger exponent may yield to enhanced reliability.

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