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ERRATUM

Erratum to: Brachistochrone with limited reaction of constraint in an arbitrary force field

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In the original publication, Eq. (15) should read as following:

$$\begin{aligned} \frac{d\lambda_y}{dx} &= -\frac{\partial H}{\partial y} = -\lambda_v \frac{\partial \Psi_v}{\partial y} + \nu \frac{\partial N_n}{\partial y}, \\ \frac{d\lambda_p}{dx} &= -\frac{\partial H}{\partial p} = \frac{\partial \Psi}{\partial p} - \lambda_y - \lambda_v \frac{\partial \Psi_v}{\partial p} + \nu \frac{\partial N_n}{\partial p}, \\ \frac{d\lambda_v}{dx} &= -\frac{\partial H}{\partial v} = \frac{\partial \Psi}{\partial v} - \lambda_v \frac{\partial \Psi_v}{\partial v} + \nu \frac{\partial N_n}{\partial v}, \end{aligned} \tag{15}$$

where for $N^\star < N_n < N^{\star\star}$ it is $\nu \equiv 0$ and $\nu = \lambda_p/(\partial N_n/\partial u)$ otherwise (see e.g. [1]).

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This correction implies the following corrections in Sect. 4.

1 Section 4.1

The part "ℓ_0 is the free length of spring" bellow Eq. (28) should read: ℓ_0 is the length of the part above x-axis of the free length of the spring.

The values of the parameters p_f and x_1 bellow Eq. (33) should be: $p_f = 0.6912$ and $x_1 = 2.5203$ m.

The numerical calculations are done for $\ell_0 = 4$ m.

Corrected values in Table 1.

2 Section 4.2

Bellow Eq. (36), the following new sentence should be added:

Now, the multiplier ν is determined by (see [1]):

$$\nu = \frac{\lambda_p + \lambda_v \frac{\partial \Psi_v}{\partial u}}{\frac{\partial N_n}{\partial u}}.$$

Table 1 Numerical values of the parameters of brachistochrone curves for various values of the coefficient of viscous friction k

k [kg/m]	p_f	v_f [m/s]	x_1 [m]
0.6	0.706629	3.47916	2.71499
0.8	0.723655	3.21593	2.93336
0.9	0.732851	3.0908	3.05211
1.0	0.742551	2.96975	3.14159

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The four numerical steps in Sect. 4.2 should be replaced by the following three ones:

- Applying the Runge–Kutta method one solves in the interval $[0, x_2]$ a Cauchy problem of the system of differential equations (9) and (15), with the initial conditions $y(0) = 0$, $p(0) = p_f$, $v(0) = v_f$, $\lambda_v(0) = 0$, $\lambda_p(0) = 0$, $\lambda_y(0) = \lambda_y$, where the nonsingular control (17) was obtained based on boundary value $N^\star = 18$ N of the constraint reaction. Now, applying the conditions (37) at the point x_2, numerical dependencies $f_1(p_f, v_f, x_2, \lambda_y) = 0$ and $f_2(p_f, v_f, x_2, \lambda_y) = 0$ are established.
- The differential equations (24) are integrated in the singular interval $[x_2, x_1]$. The values $y(x_2)$, $p(x_2)$, $v(x_2)$, $\lambda_v(x_2)$ obtained in the first step are taken for initial values, where a singular control was determined from the conditions (20).
- The final step is the integration of the differential equations (9) and (15) in the nonsingular interval $[x_1, x_0]$, the values $y(x_1)$, $p(x_1)$, $v(x_1)$, $\lambda_v(x_1)$ obtained in second step as well as $\lambda_p(x_1)$ and $\lambda_y(x_1)$ determined according to (37), being taken for initial values, where the nonsingular control (17) was obtained on the basis of the boundary value $N^{\star\star} = 0$ of the constraint reaction. Thus, numerical dependencies $y(x_0) = f_3(p_f, v_f, x_1, x_2, \lambda_y)$, $v(x_0) = f_4(p_f, v_f, x_1, x_2, \lambda_y)$, and $\lambda_p(x_0) = f_5(p_f, v_f, x_1, x_2, \lambda_y)$ are established.

Equation (38) should read:

$$\begin{aligned}
0 &= f_1(p_f, v_f, x_2, \lambda_y),\\
0 &= f_2(p_f, v_f, x_2, \lambda_y),\\
y(x_0) &= f_3(p_f, v_f, x_1, x_2, \lambda_y),\\
v(x_0) &= f_4(p_f, v_f, x_1, x_2, \lambda_y),\\
\lambda_p(x_0) &= f_5(p_f, v_f, x_1, x_2, \lambda_y).
\end{aligned} \tag{38}$$

The first sentence bellow Eq. (38) should be replaced with:

Table 2 Numerical values of the parameters of brachistochrone curves for various values of speed v_0 (three-segment brachistochrone)

v_0 [m/s]	x_1 [m]	x_2 [m]	p_f	v_f [m/s]	λ_y [s/m]
5	2.41248	0.585324	0.318344	7.42511	0.027754
5.5	2.30418	0.42243	0.357016	7.79394	0.030944
6	2.19576	0.243516	0.392991	8.17732	0.033317
6.5	2.08774	0.048703	0.426203	8.57323	0.035002

Table 3 Numerical values of the parameters of brachistochrone curves for various values of speed v_0 (two-segment brachistochrone)

v_0 [m/s]	x_1 [m]	p_f	v_f [m/s]
8	1.76849	0.508874	9.81917
10	1.35671	0.585823	11.5744
14	0.573338	0.671996	15.2642
16	0.195741	0.696597	17.1602
17	0.0094198	0.706191	18.1164

Incorporating the conditions (10) and (16) into (38) and solving the obtained system of nonlinear equations for unknowns p_f, v_f, x_1, x_2, λ_y, one obtains the following solution: $p_f = 0.234$, $v_f = 6.7412$ m/s, $x_1 = 2.6254$ m, $x_2 = 0.8626$ m, $\lambda_y = 0.0185$ s/m.

Corrected numerical values are presented in Tables 2 and 3.

The values $(v_0)_{cr1}$ and $(v_0)_{cr2}$ should be replaced with the following values: $(v_0)_{cr1} \approx 6.61902$ m/s and $(v_0)_{cr2} \approx 17.05076$ m/s.

In the last sentence in Sect. 4.2, the part "is higher than" should be replaced with "is the same as".

Note that, as the changes of numerical parameters used to draw graphs in Figs. 3, 4, 8 and 9 are around 10^{-3} in the order of magnitude, the graphs mentioned do not have any visible changes in their form.

References

1. Leitman, G.: An Introduction to Optimal Control. McGraw-Hill, New York (1966)