IDENTIFICATION AND CONTROL OF A HEAT FLOW SYSTEM BASED ON THE TAKAGI-SUGENO FUZZY MODEL USING THE GREY WOLF OPTIMIZER

by

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Even though, it is mostly used by process control engineers, the temperature control remains an important task for researchers. This paper addressed two separate issues concerning model optimization and control. Firstly, the linear models for the three different operating points of the heat flow system were found. From these identified models a Takagi-Sugeno model is obtained using fixed membership functions in the premises of the rules. According to the chosen objective function, parameters in the premise part of Takagi-Sugeno fuzzy model were optimized using the grey wolf algorithm. Furthermore, by using the parallel distributed compensation a fuzzy controller is developed via the fuzzy blending of three proportional + sum controllers designed for each of the operating points. In order to evaluate performance, a comparison is made between the fuzzy controller and local linear controllers. Moreover, the fuzzy controllers from the optimized and initial Takagi-Sugeno plant models are compared. The experimental results on a heat flow platform are presented to validate efficiency of the proposed method.

Key words*: temperature control, grey wolf optimization algorithm, parallel distributed compensation, fuzzy control, Takagi-Sugeno, discrete-time systems*

Introduction

The temperature control has an extensive range of applications in petrochemical, biochemical and pharmaceutical industries. Modelling and control of the heat flow experiment (HFE), which is the plant in this research as well, was done in papers [1-3]. The proposed method in [1] is based on a set of frequency-domain data to design fixed-order controllers capable of providing satisfactory performance profiles and constrained control inputs. The design problem is formulated as a constrained optimization problem, which has been solved using the genetic algorithm (GA) to find suitable parameter values of a selected controller to achieve satisfactory change in temperature. Ionesi *et al*. [2] examines the on-line implementation of the modulating function method, for parameter and state estimation, for the model of an air-handling unit, central element of HVAC systems. In [3] authors have conveyed and stressed the issue of performance analysis of fractional-order controller designs for integer first-order plus time delay systems.

In this paper, fuzzy control based on Takagi-Sugeno (TS) fuzzy model is applied. The fuzzy model proposed by Takagi and Sugeno [4] is described by fuzzy IF-THEN rules which depict local linear input-output relations of a non-linear system. Fuzzy logic has many

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varieties that can be implemented for control purposes. For instance, one of them is the parallel distributed compensation (PDC). Sadeghi *et al.* [5], a fuzzy controller is constructed based on a PDC method and it is implemented in an experimental tank level control system. Yordanova [6] suggests a procedure used to make two-variable fuzzy logic controllers. In [7] a novel modification the original PDC method is submitted, so that, besides the stability issue, the closed-loop performance of the system can be considered at the design stage. On the other hand, Taniguchi *et al*. [8], gave a unified approach to a non-linear model following control that contains the regulation and servo control problems as distinctive cases.

The fuzzy design can be considered as an optimization problem, where the structure, antecedent, and consequent parameters are required to be identified. Global optimization problems are difficult to be solved efficiently because of their high non-linearity and multiple local optima. Nature has been a major source of inspiration for researchers in the field of optimization [9]. Metaheuristic methods as global optimization algorithms can deal with non-convex, non-linear, and multimodal problems subjected to linear or non-linear constraints with continuous or discrete decision variables. Numerous papers concerning the TS discrete-time fuzzy models are given in the literature such as in [10, 11]. The optimization of TS fuzzy models is to determine the structure and parameters of a model. In order to obtain an optimal fuzzy model, many nature algorithms have been used. The framework for designing Takagi-Sugeno-Kang fuzzy rule-based systems using GA was proposed in [12]. Ilić *et al*. [13] uses a GA to select the best inputs for different multiple linear regression models. A multitude of improved particle swarm optimization algorithms are presented in variations of works [14-17]. In combination with fuzzy control systems, other techniques such as ant colony [18] and a novel method called the cuckoo search [19] can be used.

Grey wolf optimizer (GWO) has proven to be outstanding at resolving a variety of modes, multimodal, and problems that are not linear. The foremost supremacies of this algorithm, and all metaheuristic algorithms in general, are that it avoids getting stuck in the local minimum because of random distribution. The GWO was first suggested by Seyedali Mirjalili, Andrew Lewis in their paper [20]. A improved GWO (IGWO) can be used for optimization fuzzy aided PID controller [21]. Based on [21] this technique illustrates its supremacy with a controller which design has been made for power system frequency control. The GWO applied to the optimal tuning of the parameters of TS proportional-integral (PI) fuzzy controllers was studied in [22]. In [23] TS model was optimized using whale optimizer and PDC based controllers were implemented in order to maintain the desired water level in the tank. Also, there are some hybrid controllers which are optimally tuned in a model-based manner by a GWO algorithm [24]. In control engineering, there are a significant number of publications that investigate the application of GWO in tuning the parameters of different type controllers, mostly PID type, both classical and fuzzy. This paper examines and discusses the application of this algorithm in another task: optimization of TS model coupled with PDC controller. In general, modelling rule-based TS fuzzy systems consists of two parts: structural modelling and parameter optimization. Mostly, the structure and parameters of the TS fuzzy models are determined separately, and this is the case in this paper as well. The structure, which includes the number of rules and the variables involved in the premise of the rules, is determined first, and the parameters are optimized while the structure is fixed. The content and key findings of this study are:

- An initial TS fuzzy model was built of three linear models that describe the behavior of the plant around three nominal points. The membership functions are evenly distributed with centers at these nominal points.

- The initial TS model was optimized using GWO algorithm in such way that the parameters in the premises of the rules have optimal values.
- Synthesis of PDC control system was done. The experimental data with comparison of the responses of the plant controlled by the local PS controller, PDC controller which uses initial TS model, as well as PDC controller which uses optimal widths of membership functions, are given.

Takagi-Sugeno fuzzy model

The main idea of the TS fuzzy modelling method is to partition the non-linear system dynamics into several locally linearized subsystems, so that the overall non-linear behavior of the system could be captured by fuzzy blending of such subsystems. The fuzzy rule associated with the i-th linear subsystem, can then be defined as i^{th} rule:

$$\begin{gathered}\text{IF } z_1(k) \text{ is } M_{i1}, \text{ and } z_2(k) \text{ is } M_{i2},..., \text{ and } z_p(k) \text{ is } M_{ip} \text{ THEN} \\ \mathbf{x}(k+1) = A_i\mathbf{x}(k) + B_i\mathbf{u}(k),\ i=1,2,...,r,\ \mathbf{y}(k) = C_i\mathbf{x}(k),\ i=1,2,...,r\end{gathered} \tag{1}$$

where $\mathbf{x}(k)\in\mathbb{R}^n$ is the state vector, $\mathbf{u}(k)\in\mathbb{R}^M$ is the input vector, $\mathbf{y}(k)\in\mathbb{R}^N$ is the output vector and $A_i\in\mathbb{R}^{n\times n}$, $B_i\in\mathbb{R}^{n\times M}$, $C_i\in\mathbb{R}^{N\times n}$. Here $\{z_1(k), z_2(k), \ldots, z_p(k)\}$ are some non-linear functions of the state variables obtained from the original non-linear equation and $M_{ij}(z_i)$ are the degree of membership of z_i in a fuzzy set M_{ij}. Whenever there is no ambiguity, the discrete time variable k in $z(k)$ is dropped. The overall output, using the fuzzy blend of the linear subsystems, will then be as follows.

It is also true, for all k that:

$$\sum_{i=1}^{r} w_i(z) > 0,\ \ w_i(z) \geq 0,\ \ i=1,2...,r$$

$$\mathbf{x}(k+1) = \frac{\sum_{i=1}^{r} w_i(z)\left\{A_i\mathbf{x}(k) + B_i\mathbf{u}(k)\right\}}{\sum_{i=1}^{r} w_i(z)} = \sum_{i=1}^{r} h_i(z)\left(A_i\mathbf{x}(k) + B_i\mathbf{u}(k)\right) \tag{2}$$

$$\mathbf{y}(k) = \frac{\sum_{i=1}^{r} w_i(z)C_i\mathbf{x}(k)}{\sum_{i=1}^{r} w_i(z)} = \sum_{i=1}^{r} h_i(z)C_i\mathbf{x}(k) \tag{3}$$

$$w_i(z) = \prod_{j=1}^{p} M_{ij}(z_j),\ \ h_i(z) = \frac{w_i(z)}{\sum_{i=1}^{r} w_i(z)} \tag{4}$$

Parallel distributed compensation

The history of the purported PDC was set in motion with a model-based design procedure proposed by Kang and Sugeno, [25]. The design procedure was denominated PDC in [26]. Nonetheless, the design procedure was improved and the stability of control systems was analyzed in [27]. It is stressed that many real (non-linear) systems can be and have been represented by TS fuzzy models. Furthermore, each control rule is designed from the corresponding rule of a TS fuzzy model during the PDC design. As a consequence, the designed fuzzy controller shares the same fuzzy sets as the fuzzy model in the premise parts. The following fuzzy controller for the fuzzy models eq. (1) is designed. Control rule i:

IF $z_1(k)$ is M_{i1}, and $z_2(k)$ is M_{i2},…, and $z_p(k)$ is M_{ip} THEN

$$\mathbf{F}(k) = -\mathbf{F}_i \mathbf{x}(k), \ i = 1, 2, ..., r \tag{5}$$

In the subsequent parts the fuzzy control rules have a linear controller. Instead of the state feedback controllers different controllers can be used, for example, output feedback controllers or dynamic output feedback controllers, [28]. Additionally, the overall output signal of the fuzzy controller is represented:

$$\mathbf{F}(k) = -\frac{\sum_{i=1}^{r} w_i(z)\mathbf{F}_i\mathbf{x}(k)}{\sum_{i=1}^{r} w_i(z)} = -\sum_{i=1}^{r} h_i(z)\mathbf{F}_i\mathbf{x}(k) \tag{6}$$

The fuzzy controller design is to determine the local feedback gains $\mathbf{F}_i$ in the consequent parts.

The grey wolf optimizer

The grey wolf optimization algorithm mimics the hunting mechanism, as well as the social hierarchy of the grey wolves in nature. The leader of the pack is the alpha wolf, his/her main obligations are to impose dictatorship and leadership to the other members of the pack. Right below the alpha is the beta wolf, though not as wise as the alpha, he/she helps in the decision making process and gives feedback to the alpha. The deltas are scouts, who watch the boundary of the territory. Finally, there are omegas. Furthermore, all of the wolves are involved in the main activity, the hunting of the prey, which consists of many phases the first being the tracking, chasing and approaching the prey. The pursuing, encircling and harassing of the prey is continued up until the prey is motionless. Once this happens the group attacks. In order to provide a mathematical model of the social hierarchy the following solutions are considered: the fittest solution set as α; the second best solution set as β; the third best solution set as δ and the rest of the solutions are assumed to be ω. Since the optimization and the hunting is guided by α, β, and δ, in each iteration, the solutions of α, β, and δ are observed and if there is a better one updated, otherwise they remain the same. So as to achieve a mathematical model of the encircling behavior, the following equations are ensued [20] (the distance vector and a vector to update the position):

$$\mathbf{D} = \left| \mathbf{C}\mathbf{X}_p(t) - \mathbf{X}(t) \right|, \ \mathbf{X}(t+1) = \mathbf{X}_p(t) - \mathbf{A}\mathbf{D} \tag{7}$$

where $\mathbf{A}$, $\mathbf{C}$, are the coefficient vectors, t is the current iteration, $\mathbf{X}_p$ is the position of the prey and $\mathbf{X}$ is the position vector of the grey wolf/agent. The coefficient vectors are the main reason why the GWO is considered to be a stochastic algorithm. Additionally, to mathematically simulate the hunting behaviour of the grey wolves, an assumption is made that α, β, δ have a finer knowledge about the potential location of the prey. In turn, three of the finest solutions that are obtained so far are saved and therefore, oblige the other search agents, ω, to update their position according to the position of the best search agent. All that was formerly mentioned may be expressed:

$$\mathbf{D}_\alpha = \left|\mathbf{C}_1\mathbf{X}_\alpha - \mathbf{X}\right|,\ \mathbf{D}_\beta = \left|\mathbf{C}_2\mathbf{X}_\beta - \mathbf{X}\right|,\ \mathbf{D}_\delta = \left|\mathbf{C}_3\mathbf{X}_\delta - \mathbf{X}\right| \tag{8}$$

$$\mathbf{X}_1 = \mathbf{X}_\alpha - \mathbf{A}_1\mathbf{D}_\alpha,\ \mathbf{X}_2 = \mathbf{X}_\beta - \mathbf{A}_2\mathbf{D}_\beta,\ \mathbf{X}_3 = \mathbf{X}_\delta - \mathbf{A}_3\mathbf{D}_\delta,\ \mathbf{X}(t+1) = \frac{\mathbf{X}_1 + \mathbf{X}_2 + \mathbf{X}_3}{3} \tag{9}$$

Simply put, the agents diverge from each other to search for the prey, whilst they converge to attack the prey. In closing, this is exactly what emphasizes exploration and allows the GWO algorithm to search globally, per say have a broad search [20]. All of this assists the GWO to exhibit a more random behavior throughout the optimization process, endorsing exploration and the local optima avoidance.

System description

The quanser HFE shown in the fig. 1 consists of a chamber equipped with a coil-based fan blowing over an electric heating coil. The air temperature inside the chamber is measured by three temperature sensors positioned equidistantly along the duct. Fan speed is measured using a tachometer. The fan is operating with a constant speed during the whole experiment in order to provide uniform air-flow rate through the enclosed chamber. We will assume that the room temperature is unknown and constant during the experiment because the HFE is located in a closed indoor environment and the experiment is conducted on short time interval. In this paper, we are interested in designing a controller to control the temperature, T, measured by the sensor which is closest to the heater. As modelling of the elements constituting the platform is complex to be done, using identification techniques, the plant is approximated by a first-order model. In order to identify the mathematical model of the heat flow system, an open-loop experiment is performed.

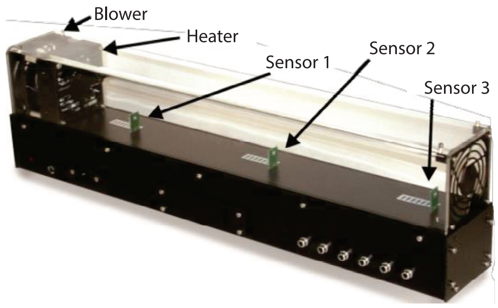

Figure 1. The HFE set-up

Takagi-Sugeno modelling

Takagi-Sugeno model based on linearized models (TS-INITIAL)

The local linear models in the consequent rules, eq. (1), are obtained by utilizing the methods of identification in accordance with the measured input-output data using MATLAB *System Identification toolbox*. The identification methods were used based on the step response of the plant. In this article a non-linear TS model is obtained by combining three linear models around three nominal points: 33, 49.4, 68.5 °C. Nominal temperatures T_{Ni}, heater nominal voltages V_{hNi} and corresponding identified transfer functions are given in tab. 1.

Table 1. Nominal values and linear models

I	T_{Ni} [°C]	V_{hNi} [V]	$G_i(z)$	a_i	b_i
1	33	2	$\frac{0.0026}{z-0.99977}$	0.99977	0.0026
2	49.4	3.1	$\frac{0.006939}{z-0.99965}$	0.99965	0.006939
3	68.5	4.2	$\frac{0.003929}{z-0.99976}$	0.99976	0.003929

Voltage deviation represent control deviations as $u(k) = v_h(k)$. The constants for the plant's state space model are determined from tab. 1 based on $G_i(z)$. As example, a procedure for determining the constants a_1, b_1:

$$G_1(z) = \frac{T_1(z)}{U(z)} = \frac{0.0026}{z-0.99977} \tag{10}$$

$$t(k+1) - 0.99977t(k) = 0.0026u(k) \tag{11}$$

As the state variable the output variable is chosen, $x(k) = y(k) = t(k)$. By substituting the state variable into the previous discrete equation, the discrete state equation and the discrete output equation of the plant are obtained:

$$x(k+1) = 0.99977x(k) + 0.0026u(k) = a_1x(k) + b_1u(k) \tag{12}$$

$$y(k) = x(k) \tag{13}$$

The same was done for the other two discrete equations. Constants for the state space plant model are shown in the tab. 1. The following fuzzy model is constructed based on the linear subsystems.

Model rule i:

$$\text{IF } x(k) \text{ is } M_i \text{ THEN } \begin{cases} x(k+1) = a_ix(k) + b_iu(k), \ i = 1,2,3 \\ y(k) = x(k) \end{cases} \tag{14}$$

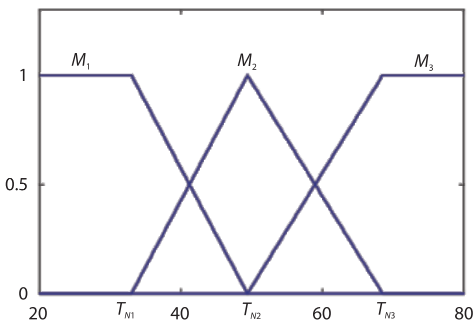

Figure 2. Membership functions

Assumed membership functions, which are corresponding to the operating points T_{Ni}, are shown in fig. 2.

Takagi-Sugeno model optimization (TS-OPT)

In order to improve the accuracy of the model, the parameters in the premises of the rules are optimized using the GWO. In the aim of optimization it is necessary to provide experimental input-output data. Therefore, in order to cover as large a working range as possible, the plant is supplied with the input voltage with a shape as

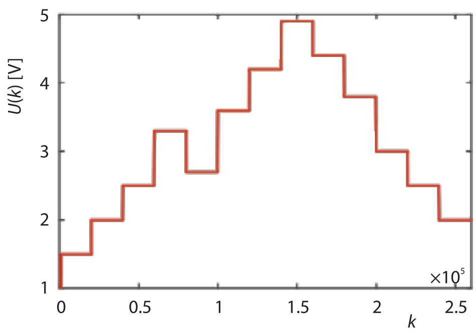

Figure 3. Input signal used for model optimization

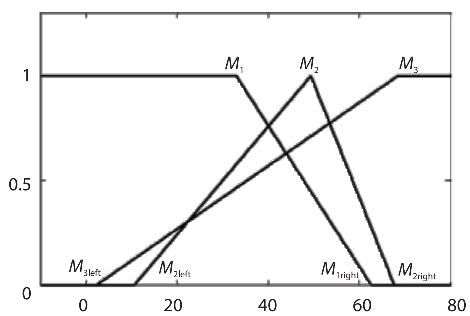

Figure 4. Optimized membership functions

depicted in fig. 3. The structure of the fuzzy TS model was fixed, while in the consequents of the rules there were local linear models. Furthermore, the model is optimized by changing the premise parameters: the width of the membership functions. Moreover, the mentioned TS parameters are all coded into one grey wolf, per say one agent, that is presented with a vector which contains the premise parameters. The membership function in the center contains two, and functions from the ends contain one parameter each, as depicted on fig. 4. That makes a total of 4 unknown parameters. Parameters for the GWO were taken from the original paper [20], while the population is set to 20 and the total number of iterations is set to 30. Furthermore, in this optimization method, one agent represents one potential optimal fuzzy model. The sum of squared errors (SSE) is taken as an objective function and it can be calculated:

$$J = \sum_{i=1}^{n} \left[y(i) - y_m(i) \right]^2$$

where $y(i)$ is the measured output of the plant, $y_m(i)$ is the output of the model. This is constrained optimization task. The lower and upper constraints are determined by the physical constraints of the HFE. Also, it was necessary to introduce lower and upper limits so that at each individual moment at least one rule is active *i.e.* to exclude the possibility that in some iteration no rule will be active. This would cause division by zero and singularities in the TS model. Optimized membership functions are shown on fig. 4 where $M_{3left} = 2.19853$, $M_{2left} = 10.5573$, $M_{1right} = 62.718$, and $M_{2right} = 67.7686$.

Comparisons of the TS model based on initial membership functions and the TS model based on optimized membership functions with experimental results are shown on fig. 5. The experimental data shown in fig. 5 are derived after the plant is supplied with the input voltage with a shape as depicted in fig. 3. The values of the SSE of two models, are given as a measure of their accuracy in relation the experimentally recorded data

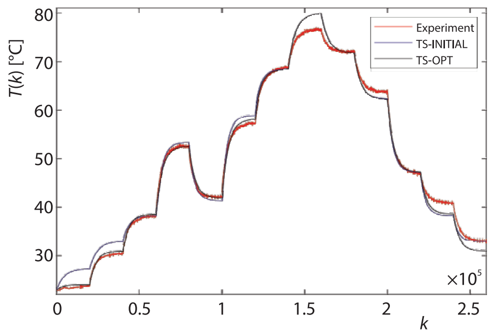

Figure 5. Comparison of initial and optimized TS model (*for color image see journal web site*)

$$SSE_{TS\text{-}INITIAL} = 702810, \quad SSE_{TS\text{-}OPT} = 385910$$

The SSE, as a measure of performance, has a much lower value, and the matching of experimental and simulation results is significantly better in the case of TS-OPT, which can be seen in fig. 5.

Control systems design

The PDC proposes a procedure to design a fuzzy controller from a given TS fuzzy model. For each of the linearized models a linear PS controller is defined. The control rule i of the fuzzy controller via the PDC is: IF $x(k)$ is M_i, THEN the controller is C_i. The overall output signal of the fuzzy controller:

$$C = \frac{\sum_{i=1}^{3} w_i[x(k)]C_i}{\sum_{i=1}^{3} w_i[x(k)]} = \sum_{i=1}^{3} h_i[x(k)]C_i \tag{15}$$

where C_i are PS controllers defined in a complex domain as follows. A zero-order PS control system is used:

$$u(k)=K_P e(k)+K_S T\sum_{i=0}^{i=k-1} e(i)/\Delta \tag{16}$$

$$u(k+1)-u(k)=K_P e(k+1)-K_P e(k)+\ K_S Te(k)/Z \tag{17}$$

$$zU(z)-U(z)=K_P zE(z)-K_P E(z)+K_S TE(z) \tag{18}$$

With that, the expression for the Z transfer function of the controller C_i from the eq. (15), was obtained:

$$C_i=\frac{K_{Pi}z+K_{Si}T-K_{Pi}}{z-1},\ i=1,2,3 \tag{19}$$

A discrete-time controller is constructed indirectly from a continuous design. The design is carried out exactly as with continuous systems, the only change due to the digital implementation being the extra step of discretizing the resulting continuous controller. The control objective is to maintain the room temperature at the set value by varying the current supply to the heating coil. The requirement is that the control systems for all three operating points should satisfy the following specifications: the steady-state error should be zero; the percentage overshoot has to be less than 5%, the $\Pi \leq 5\%$; the settling time should be less than 30 seconds, $T_s \leq 30$ seconds. Percent overshoot and settling time requirements for the closed-loop system responses can be transformed into the desired natural frequency ω_n and damping coefficient ζ. If the i-th plant model is represented by $G_i(s)=\beta_i/(s-\alpha_i)$ [29]:

$$b_i=\left|\ln\left(\frac{PO_i}{100}\right)\right|,\ \zeta_i=\frac{b_i}{\sqrt{b_i^2+\pi^2}},\ \omega_{ni}=\frac{4}{T_{Si}\zeta_i} \tag{20}$$

$$K_{Pi}=\frac{1}{\beta_i}\left(2\zeta_i\omega_{ni}+\alpha_i\right),\ K_{Si}=\frac{\omega_{ni}^2}{\beta_i},\ i=1,2,3 \tag{21}$$

Gain values for linear models are obtained: K_{P1} = 0.93768, K_{P2} = 0.33312, K_{P3} = 0.61816, K_{S1} = 0.14357, K_{S2} = 0.053788, K_{S3} = 0.095009.

Further experimental results

The evaluation of the synthesized controllers was done by using the following criteria: step response, tracking control and disturbance rejection.

Step response

The PDC-OPT controller was compared with the specifically designed PS controller (C1) for the nominal Point 57 °C. The linear discrete model of the plant around this nominal point is $G(z)=0.01088/(z-0.9998)$. Heater nominal voltage is 3.65 V. As to say, that the most onerous challenge for the PDC-OPT is precisely this, because that point is the most further from the operation points of local linear controllers, which are designed to operate around 33, 49.4, and 68.5 °C. The requirements for this local linear PS controller are the same. In the same way parameters K_P = 0.2256 and K_S = 0.0343 were obtained. A juxtapose of the operation of this local PS controller with the PDC-OPT is shown in fig. 6. As can be seen in fig. 6 the PDC-OPT achieves a better performance than the local PS controller which can be clearly seen based on the value of the SSE, in correlation the set point, which is $SSE_{C1} = 3578$, $SSE_{PDC-OPT} = 1773$. The PDC-OPT controller uses the same fuzzy sets M_1, M_2, and M_3 depicted in the fig. 4 as the TS fuzzy model of the plant. Based on the fig. 4 and the appearance of the fuzzy sets M_i, it is concluded that for the temperature between 54 °C and 60 °C there is the grade of membership of the temperature $T(k)$ in the fuzzy set M_i, *i.e.* $M_i(T(k)) \neq 0$, $i = 1,2,3$. As seen in fig. 6 when the temperature changes in the range from 54-60 °C, the first, second and third fuzzy rule will be active, in which the fuzzy sets M_1, M_2, and M_3 appear. A smaller overshoot was obtained when the plant was controlled using a PDC that contains information about the optimized model (TS-OPT), than when the plant was controlled by a PDC with initial membership functions (TS-INITIAL). In order for the results to be observed better, the filtered responses are shown in fig. 7. The same moving average filter with a span of 30 data points has been used for both of the signals.

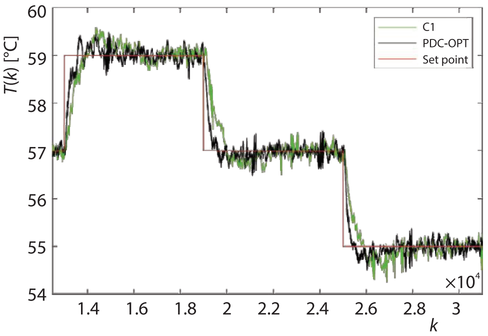

Figure 6. Comparison of PDC-OPT and PS controller around 57 °C (*for color image see journal web site*)

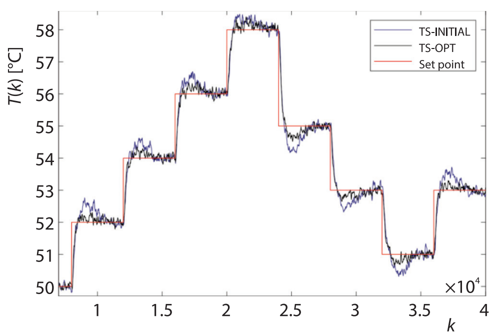

Figure 7. Comparison of PDC with an initial and optimized plant model (*for color image see journal web site*)

Comparison of system response percentage overshoots for the initial and optimized TS model are shown in tab. 2.

Table 2. Percentage overshoot for different step responses

Step [°C]	50-52	52-54	54-56	56-58	58-55	55-53	53-51	51-53
TS-INITIAL [%]	38	31.25	34	24	28	34	34	35
TS-OPT [%]	12.5	17	17.5	20	16	17.5	17.5	15

Tracking control

In this section, the intention is to indicate that the PDC with optimized fuzzy sets achieves a *better* behavior of the object compared to the PDC that uses the initial fuzzy sets and compared to the local linear controller designed to work around 49.4 °C. A desired trajectory that is not too demanding (*e.g.*, slow time-varying sine-like input around 49.4 °C) is used. The comparison of tracking responses for the desired trajectory is given in fig. 8. A comparison of control signals of TS-INITIAL, TS-OPT and local linear PS controller is given in fig. 9.

Again, the PDC-OPT controller achieved better object behavior than when using a local linear controller, as was the case in fig. 6. On the other hand, it can be seen that the local linear controller is better than the PDC controller using the TS-INITIAL model. This justifies

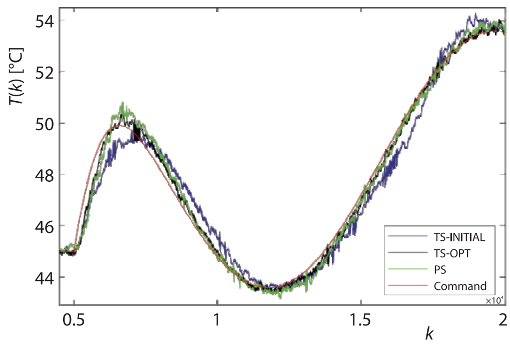

Figure 8. Comparison of PDC with the local linear PS controller (*for color image see journal web site*)

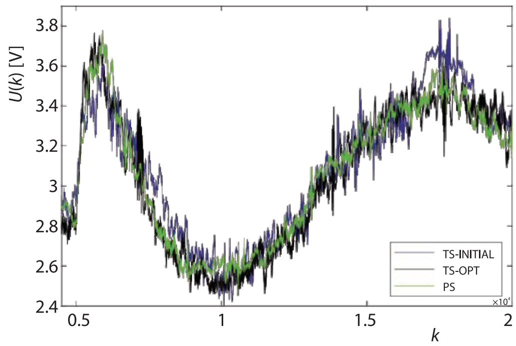

Figure 9. Control signals (*for color image see journal web site*)

and gives importance to the optimization done. This improvement can be explicitly seen based on the values of the SSE, in correlation the given trajectory, which are $SSE_{TS\text{-}INITIAL} = 6587$, $SSE_{TS\text{-}OPT} = 1384$, and $SSE_{PS} = 2602$.

Disturbance rejection

In the following section the disturbance rejection problem is being evaluated. In HFE the fan speed can be used as an disturbance because it greatly affects the duct temperature. Assume that the initial temperature is 56 °C and it is in a steady-state. In the 50th second the fan speed is suddenly increased for 5 seconds, which is supposed to imitate a step disturbance. Experimental results obtained are presented in fig. 10 which shows that the temperature has dropped down to about 55.5 °C due to the disturbance introduced, before the implemented PDC controllers quickly reacts to reject the disturbance and bring the actual system temperature to the desired value. Hence, the designed PDC has played its role in achieving the desired performance specifications as well as in disturbance rejection. In this section, the goal was to convey that by optimizing the widths of the fuzzy sets an improved behavior of the PDC was obtained in comparison the PDC that uses the initial fuzzy sets that are common the TS fuzzy model and its corresponding controller.

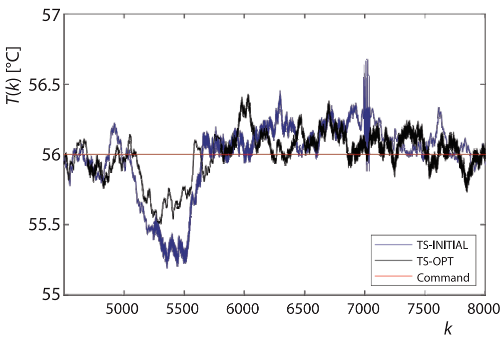

Figure 10. Comparison of PDC with an initial (TS-INITIAL) and optimized (TS-OPT) plant model (*for color image see journal web site*)

Conclusion

Initially, in this paper, the mathematical model of the heat flow system was obtained experimentally. Further, TS fuzzy model was obtained based on three identified local linear models. Regardless of the superiority in *catching* the non-linear behavior of the plant, TS model was optimized using GWO. Optimization was implemented only on the unknown parameters in the premises of the rules. Moreover, verification was done by comparison of these models. Then, by using the PDC method, fuzzy controllers were developed based on initial and optimized TS model. The TS models are improved by GWO which in turn enlarges the efficiency of

TS based PDC controller. So, in the synthesis of a fuzzy controller using the PDC technique, the developed fuzzy controller's effectiveness is related to the previously designed system model with fuzzy structure. System model with fuzzy structure was created to *catch* the non-linearity in the real object. Finally, tracking control and disturbance rejection problems were evaluated. Future research will focus on exploiting these possibilities in terms of using more fuzzy rules and metaheuristic algorithms.

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