

## HYBRID ARTIFICIAL INTELLIGENCE MODEL FOR PREDICTION OF HEATING ENERGY USE

by

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*Currently, in the building sector there is an increase in energy use due to the increased demand for indoor thermal comfort. Proper energy planning based on a real measurement data is a necessity. In this study, we developed and evaluated hybrid artificial intelligence models for the prediction of the daily heating energy use. Building energy use is defined by significant number of influencing factors, while many of them are difficult to adequately quantify. For heating energy use modelling, the complex relationship between the input and output variables is hard to define. The main idea of this paper was to divide the heat demand prediction problem into the linear and the non-linear part (residuals) by using different statistical methods for the prediction. The expectations were that the joint hybrid model, could outperform the individual predictors. Multiple linear regression was selected for the linear modelling, while the non-linear part was predicted using feedforward and radial basis neural networks. The hybrid model prediction consisted of the sum of the outputs of the linear and the non-linear model. The results showed that both hybrid models achieved better results than each of the individual feedforward and radial basis neural networks and multiple linear regression on the same dataset. It was shown that this hybrid approach improved the accuracy of artificial intelligence models.*

*Key words: heating energy use prediction, hybrid model, neural networks, artificial intelligence*

### Introduction

Considering that the building sector in Europe is entitled for 40% of total energy use and 36% of total CO<sub>2</sub> emission [1] the energy efficiency in buildings is still one of the top priorities for engineers and researchers worldwide. The growing interest in improving and designing energy efficient buildings has highlighted the importance of adequate energy use analysis. This task has proven to be highly challenging, due to the complexity of the building itself, its thermal properties, weather conditions, as well as building systems, occupants behavior and other often correlated impact factors. The white-box models based on physical principles require detail knowledge about a building and high quality of input data related to internal loads and occupant behavior. Many of the building energy performance simulation softwares have challenges for model calibration based on the real-time data [2].

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Data-driven models, or often referred as black-box models, have proven to be very successfully used for solving various problems in building energy sector, mainly load estimation and prediction, energy use mapping, benchmarking, defining various energy efficiency strategies and guidelines, *etc.* [3]. The implementation of artificial intelligence models, has shown to be very effective in building energy use predictions and classification, while the recent reviews can be found in [3, 4]. By using these techniques, the modelling of building energy behavior comes down to defining relationship between variables based on the significant amount of high quality historical data. There is wide range of applied methodologies, starting from the more simple algorithms such as statistical regression, over ANN with different architectures, support vector machines, and finally most complex hybrids and multistage ensembles. The most often used influencing parameters for building energy demand and different methods for the prediction of energy use in large-scale buildings were presented in [5].

At early stages of the development, Bauer and Scartezzini [6] presented a regression model used for the prediction of heating and cooling load taking into account internal and solar heat gains. Another regression model was developed for estimating the monthly heating need of the single-family houses in [7]. The multiple regression model showed to be successful predictor of heating energy demand in [8]. The multiple non-linear regression model was used for cooling load prediction in [9]. Comparing to other, far more complex statistical models, multiple regression can be simple, practical solution offering satisfactory prediction results. These models often result poorly when it comes to outliers (untypical input values). Among numerous artificial intelligence (AI) models, the ANN stand out as the most prominent and most often used models for the building energy use prediction. Similar as other AI methods, ANN was first used for solving classification problems, while later its application was extended to the regression, which made these models good candidates for different prediction problems. The ANN proved to be highly successful for the prediction of building energy use as shown in [10]. Support vector machine regression model was developed for the prediction of daily heating energy use in [11]. It has shown that this AI model can achieve similar results and even outperform other, more often used ANN. The importance of having accurate model for the prediction of district heating load is highlighted in [12]. Based on the measured data, the authors developed ANN model for short-term prediction, while using particle swarm optimization for adjusting the parameters. The deep learning recurrent neural network has been successfully developed for the short-term building energy prediction in [13]. The geometric semantic genetic programming has been used for the prediction of heating and cooling building energy load in [14]. On six real-world benchmark datasets, the authors have proven that the regression tree algorithm can successfully tackle the prediction problems in [15]. The random forest model developed for the prediction of energy use in multiple buildings in [16] outperformed even the decision tree and random tree models on the same database. It has shown that this model can be effective tool to help building owners and managers in understanding building energy performance in order to improve energy efficiency. The authors developed gradient boosting machine for modelling the energy consumption based on a large dataset of 410 commercial buildings in [17], showing that it can outperform linear regression and random forest algorithm. It can be used for further development of the model-based predictive controls, diagnostics and malfunction detection.

Hybrid modelling is based on the fact that linear models are very successful in describing linear relationships between variables, while resulting poorly while operating with higher number of variables, as well as with outliers. However, ANN excel in modelling non-linear relationship among variables, and they are often used to solve various non-linear problems,

such as predictions in different engineering fields. By combining the advantages of linear and non-linear models, the hybrid approach has the potential to outperform the individual models. The methodology that combines auto-regressive moving average (ARIMA) model with ANN and successfully uses it for time-series prediction, while using empirical data was suggested in [18]. There were many other successful examples of solving various engineering problems using hybrid approach. The hybrid model combining the seasonal auto-regressive integrated moving average (SARIMA) with ANN is used for the assessment of the yearly energy cost budget in the educational buildings in [19]. The hybrid ARIMA-NN model shows to be successful also for the prediction of wind speed [20] while the achieved results are better than each of the individual models. Alawi *et al.* [21] apply similar algorithm for the prediction of the ground-level ozone while using measured data. They used principal component regression for the linear model and combined it with neural network for the better accuracy. It can be seen that the hybrid models increase the prediction accuracy of the individual AI models. The main idea for this paper was to analyze whether the linear modelling could help ANN in solving complex engineering problems, such as prediction of the heating energy use. With the rapid development of ANN with different architecture, the simple statistical tools, such as multiple linear regression, are being pushed aside and often wrongfully neglected. However, if the problem is divided into the linear and the non-linear part, the combined model could outperform the results of the individuals [18].

### **Problem formulation**

Prediction of multi-building energy use at campus or even on district scale has become more interesting to the researchers recently, since it has been highlighted that the focus on analyzing and modelling large-scale building can provide more insights into energy use patterns and opportunities to save energy [22]. These models can be used also for benchmarking, detecting meter malfunction, creating heating bills for tenants, planning energy savings, *etc.* The authors have developed various AI models to solve the complex task of energy use prediction and proposed several upgrades in order to improve the prediction accuracy. The main goal of this paper is to elaborate the possible improvements of the individual models for the campus energy use prediction by applying the hybrid approach. As it has shown to be successful in solving other prediction problems, the main idea is to use the similar methodology as proposed by Zhang [18], on the daily district heating use prediction. For this case study, the selected input variables are meteorological (based on an outdoor temperature, wind speed, relative humidity and solar radiation) and categorical (day in the week and month in the year).

### **Case study and previous work**

In this study, the database consisting of measured meteorological and daily heating energy use data for the University campus in Trondheim, Norway, was used for the analysis and predictions. The analyzed campus consists of 35 buildings, with the total area of approximately 300000 m<sup>2</sup>. More details can be found in [23]. This case study may be treated as a very effective in analyzing heating energy use patterns of group of mixed-use buildings, representing a small-scale town. The accurate energy use estimation or prediction for large-scale buildings, based on the adequate analysis of the main influencing factors, can help in solving environmental problems, as well as conserve energy [5]. Building energy use data may be available at different frequency and quality depending on the monitoring system and a practical need. Specific annual energy use is relevant for energy planning and policy development, while hourly energy use is relevant for control, operation and maintenance. Daily

energy use data may be also used for operation and maintenance, as well as energy billing. Since some parts of this campus is being leased to other users, the daily heating energy use models can be used for defining heating costs for tenants. If the reading of the meter is significantly different from the value predicted by the model, it can point out the meter malfunction, or indicate excessive consumption, so the management could investigate possible reasons. For the prediction of the daily heating energy use of this campus ANN with different architectures: feedforward neural network (FFNN), radial basis function network (RBFN), and adaptive network based fuzzy inference system (ANFIS) were developed in [23]. The accuracy of individual ANNs is further improved by creating ensemble as shown in [23]. It can be concluded from the presented results that the simple combination of the single networks (simple-SAV, weighted-WAV or median based averaging-MAV) leads to the improvement of the prediction quality. The idea of using  $k$ -means clustering to select the ensemble members is proposed in [24]. First, clustering has been used to divide networks in groups, and then the most accurate individual network is selected for the ensemble, while FFNN and ANFIS networks in the second level are used to create the multistage ensemble. The RBFN is proposed for the second stage in [25]. The  $k$ -means clustering is used for creating subsets used to train individual RBFN in [26]. In the second step the outputs of the individual networks separately trained on different training dataset are aggregated. Another successfully used AI model, the support vector machine, is developed in [11] and it has proven that it can achieve similar and even better results than more popular neural networks. All of the proposed complex algorithms applied to the same dataset has shown improvement in prediction quality compared to the single ANN. The idea of this paper was to investigate another potential improvement of the ANN prediction, by combining it with the linear model.

## Method

The readings of the main meter in the university campus, which is installed by the district heating supplier, for the four years period were collected. The meteorological data were gathered from the local weather station and their correlation with the heating energy use was analyzed [23]. The analysis of the minimum, maximum, and average daily temperatures for the observed period suggested that the database should be divided into three parts: cold, mild, and warm period. For the models development the working days in the cold period (from January 1<sup>st</sup> until March 31<sup>st</sup> and from November 1<sup>st</sup> until December 31<sup>st</sup>) were selected. More details on the selection of input variables and database pre-processing can be found in [17]. In this study, the input variables were: month in year  $M$  [-], day in week  $D$  [-], mean daily outdoor temperature,  $t_m$  [°C], maximum daily outdoor temperature,  $t_{\max}$  [°C], minimum daily outdoor temperature,  $t_{\min}$  [°C], total daily solar radiation,  $SD$  [Whm<sup>-2</sup>], mean daily wind speed,  $w_m$  [ms<sup>-1</sup>], and mean daily relative humidity,  $\varphi_d$  [%]. Month in year, as nominal categorical variable, was represented with values 1-12, and day in week with numbers 1-5. The output variable was campus daily district heating use.

In this case the daily heating energy use of the previous day was not used as additional input variable, so the prediction can be done more accurately for several days ahead (if the energy use of the previous day is also predicted, the error of the model is accumulated), as elaborated in [23]. The dataset was divided in training (years 2009-2011) and testing dataset (year 2012). In total, there were 318 samples for training and 100 for testing. A linear scaling function was used to normalize all input and output variables to the interval (0, 1). The prediction accuracy was measured by the coefficient of determination,  $R^2$ , root mean square error (RMSE), and mean absolute percentage error (MAPE), as suggested in [27] and defined:

$$R^2 = 1 - \frac{\sum_{k=1}^N (y_k - \bar{y})^2}{\sum_{k=1}^N (y_k - \hat{y}_k)^2} \quad (1)$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{k=1}^N (y_k - \hat{y}_k)^2} \quad (2)$$

$$MAPE = \frac{1}{N} \sqrt{\sum_{k=1}^N \frac{|y_k - \hat{y}_k|}{y_k}} \times 100 \quad (3)$$

where  $y_k$  is measured (real) output for  $k^{\text{th}}$  sample,  $\hat{y}_k$  – model output for  $k^{\text{th}}$  sample,  $\bar{y}$  – the mean value of  $y$ , and  $N$  – total number of samples.

#### Hybrid models

Hybrid model may be described as suggested in [13]:

$$y_t = G_t + N_t \quad (4)$$

where  $y_t$  is the output variable,  $G_t$  – the linear component (predicted by the linear model, and  $N_t$  – the non-linear component (predicted by the non-linear model). After the prediction using the linear model, the residuals are defined:

$$e_t = y_t - \hat{G}_t \quad (5)$$

where  $e_t$  is the residual and  $\hat{G}_t$  – the result of the linear model. The residuals are then predicted using the non-linear model and finally the hybrid prediction is defined:

$$\hat{y}_t = \hat{G}_t + \hat{J}_t \quad (6)$$

where  $\hat{J}_t$  is the result of the non-linear model. For the linear model, MLR was selected, while the non-linear part (residuals) were predicted using FFNN and RBFN.

#### Multiple linear regression

Multiple linear regression (MLR) is a linear multivariate regression technique proposed by Galton in 1886 to develop a relationship linking the output to the contributing inputs plus an error term [5]:

$$Y_i = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \dots + \beta_n X_n + \varepsilon \quad (7)$$

where  $Y_i$  is a response variable,  $X_1, X_2, \dots, X_n$  are the predictors,  $n$  – the number of variables,  $\beta_0, \beta_1, \dots, \beta_n$  – the regression coefficients, and  $\varepsilon$  – the error. The MLR analyzes the relationship between predictors and the result is formulated as a prediction equation with the estimated parameters:

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 \hat{X}_1 + \hat{\beta}_2 \hat{X}_2 + \hat{\beta}_3 \hat{X}_3 + \dots + \hat{\beta}_n \hat{X}_n \quad (8)$$

where  $\hat{Y}_i$  is the predicted value and  $\hat{\beta}$  are estimates of the regression coefficients. Similar to other data-driven methods used for prediction, developing statistical regression equations requires significant number of measured data in order to estimate the involved parameters [3]. The advantage of this model is that it is easy-to-use, while offering relatively fair prediction, which

has made it very popular at the early stages of energy use modelling. However, the achieved short-term prediction results show lower accuracy comparing to other, more complex blackbox models, such as ANN and SVM [3].

#### *Feedforward backpropagation neural network*

The ANN are the most commonly used AI models for solving prediction problems. The extensive reviews on the application ANN for building energy use prediction may be found in [10, 28]. The ANN are information processing systems that are inspired by interconnected neurons of biological systems. In significant number of recently published papers they have proven their ability to approximate non-linear relationships between the input and the output variables of various complicated systems. One of the most popular ANN architecture is a feed-forward neural network (FFNN) with one input layer that accepts signals from outside world, an output layer (as a result) and one or more hidden layers. The layers consist of neurons that are connected using adaptable weighted connections. In the training phase, the weights are adjusted, so that the error between the ANN output and the target output is minimized. The activation functions used in this study for the hidden and output layers were the hyperbolic tangent and linear functions, respectively. The Levenberg-Marquardt (LM) algorithm was adopted as the learning algorithm. For the FFNN models, one hidden layer was selected and the number of neurons in the hidden layer was varied, while the optimal number was selected by trial and error method. For the purpose of this study, two FFNN models were created. Firstly, the single FFNN model was used for the prediction of the campus heating energy use (eight input variables). After that, as a part of hybrid modelling, the second, residual FFNN was developed using the same method and input variables to predict the residuals.

#### *Radial basis function network*

A radial basis function (RBF) network consists of three layers – input layer, a single hidden layer and an output layer. The nodes in the input layer are linked with the neurons placed in the hidden layer. For the transformation of data from the input to the hidden space, this network applies the non-linear function. Specific to the RBFN is that the radially symmetrical function, Gaussian function, is used as the activation function. For a RBFN with an  $n$ -dimensional input  $x \in \mathcal{R}^n$ , the output of the  $j^{\text{th}}$  hidden neuron is given:

$$h_j(x) = \varphi_j(\|x - c_j\|), \quad j = 1, 2, \dots, m \quad (9)$$

where  $c_j$  is the center (vector) of the  $j^{\text{th}}$  hidden neuron,  $m$  – the number of neurons in the hidden layer, and  $\phi(\cdot)$  – the radial basis function. The linear transfer function is used for the neurons in the output layer. The outputs of the neurons in hidden layer linked with the  $k^{\text{th}}$  output neuron are multiplied by the weight factor and their sum represents the  $k^{\text{th}}$  output of the RBFN:

$$\hat{y}_k(x) = \sum_{j=1}^m w_{kj} h_j(x) + w_{k_0} \quad (10)$$

where  $w_{kj}$  is the weight between the  $j^{\text{th}}$  hidden neuron and the  $k^{\text{th}}$  output neuron,  $w_{k_0}$  – the bias and,  $m$  – the number of the hidden layer neurons.

In this study, the training of RBFN consisted of setting the centers and widths of the Gaussian functions, while the least mean square algorithm was used for weights optimization. The setting parameters, number of neurons in the hidden layer and the spread (radius value) of the radial basis function were varied in order to achieve the best accuracy of the RBF network. As for the FFNN model, the two RBFN networks were developed:

- single model for the prediction of heating energy use, and
- the RBFN for residuals prediction, as a part of the hybrid model.

**Hybrid FFNN and hybrid RBFN models for the prediction of heating energy use**

As the first step MLR model for the prediction of daily heating energy use was developed. Based on the training dataset, the equation that described the relationship between daily heating energy use and input parameters:

$$\begin{aligned}
 HC_d = & 189.701 - 405.92M - 2126.87D - 5167.69t_m - \\
 & - 2484.62t_{max} - 1625t_{min} - 5.22SD + 191.91w_m - 356.08\phi_d
 \end{aligned}
 \tag{11}$$

In this study, the possibility to combine MLR with ANN using eight input variables in order to improve the accuracy of the individual models was proposed. After performing predictions with individual MLR, FFNN, and RBFN models, the hybrid algorithm was analyzed. First the prediction of the MLR model was used for the linear part of the problem, where there were eight input variables and one output variable (daily heating energy use). Then the residuals were calculated (as the difference between training output values and the results of the MLR prediction for training period) and these values were used as output parameter for training the FFNN network. So, the second (residual) FFNN network had eight input variables ( $M, D, t_m, t_{max}, t_{min}, SD, w_m,$  and  $\phi_d$ ) and one output variable (the residuals). After training the networks, choosing the optimal number of neurons in hidden layer by trial and error method, the best network was evaluated on test dataset., respectively, for the test dataset, the prediction had been firstly done with MLR model (linear part) and the residuals were predicted using FFNN model (as part of the hybrid). The final output (heating energy use) was calculated as the sum of the MLR output and residual FFNN output. The same procedure was performed for the RBFN model and the prediction results of the hybrid models were evaluated and compared with single MLR, FFNN and RBFN models. The prediction accuracy was measured by the  $R^2$ , RMSE, and MAPE.

**Results and discussion**

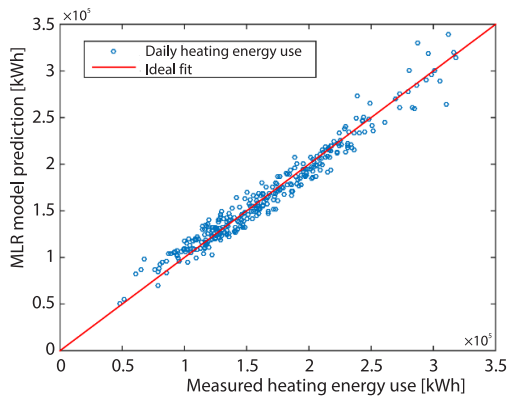
Equation (11) obtained by the MLR model based on the training dataset was used to evaluate the heating energy use of the test dataset. The prediction accuracy indices are shown in tab. 1. The MAPE of 8.1540% could be considered relatively satisfactory for early stages and estimation of daily energy use. By applying very simple procedure, it is possible to roughly estimate daily heating energy use for different values of the input parameters. This model achieves RMSE of 11389 kWh for the training period and 11430 kWh for the test dataset. The MLR model with  $R^2 = 0.9701$  showed acceptable prediction accuracy.

**Table 1. Prediction indices for Hybrid FFNN model**

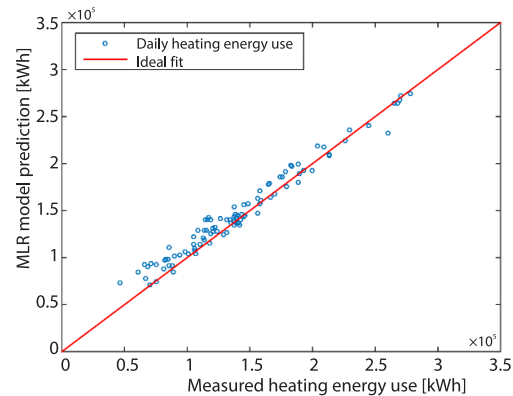
Model	$R^2$ [-]		RMSE [kWh]		MAPE [%]	
	Training	Test	Training	Test	Training	Test
MLR	0.9574	0.9701	11389	11430	5.7850	8.1540
FFNN	0.9739	0.9740	9086	9492	4.5430	6.3438
Hybrid FFNN	0.9719	0.9768	9659	8448	4.8441	5.5137

The comparisons of the measured values and the predictions of the MLR model for the training and test period are shown in figs. 1 and 2, respectively. The results showed that the MLR model tends to *overestimate* heating energy use (predicts higher values than measured).

The training dataset had fewer days with low heating energy use, so the model was not accurately trained for the prediction of lower heat demand. Therefore, more significant deviations were found for the days with heating energy use lower than 150000 kWh, fig. 2.



**Figure 1.** The MLR model vs. measured data for the training period



**Figure 2.** The MLR model vs. measured data for the test period

The quality of the prediction for the hybrid FFNN model is presented in tab. 1. The FFNN model, as one of the most often used ANN models for prediction, achieved MAPE of 6.3438% which is significantly better than the MLR model. However, Hybrid FFNN model improved this accuracy with  $MAPE = 5.5137\%$ . In this case study it can be seen that the hybrid approach offers better prediction quality than the single MLR or FFNN model. The RMSE for the test period has been lowered from 9492-8448 kWh when combining the FFNN with the MLR model. Considering that the range of the daily heating consumptions for the whole dataset is between 46600 kWh and 320366 kWh, with the average value of 160251 kWh, the achieved  $RMSE = 8448$  kWh for the test period can be considered as a good result.

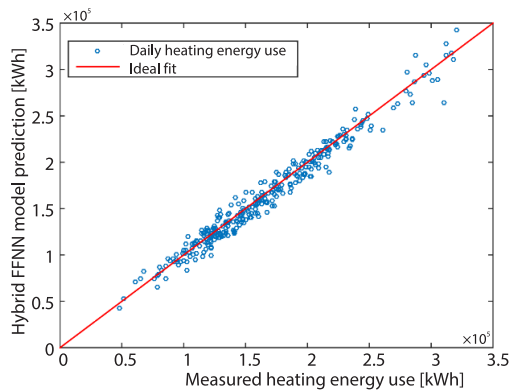
The Hybrid FFNN model outputs compared to the measured data for the training and test period are shown in figs. 3 and 4, respectively. Similar trend of overestimate for lower daily heating energy use in the test period, as in the case of MLR model, can be seen for Hybrid FFNN model, but with smaller error.

In fig. 5 the Hybrid FFNN model outputs for the test period are shown. It can be seen that the Hybrid FFNN model offers great matching with measured data, with some more significant deviation around 60<sup>th</sup> sample. In tab. 2 the prediction accuracy for hybrid RBFN model is shown. This hybrid achieved better results than both linear model and single RBFN, while having MAPE 3.3579% on the training data and 6.0488% on the test data. The achieved RMSE was also lower when using hybrid RBFN model and for this case it was 9230 kWh for test period, comparing to the same quality indicator for single RBFN model of 10196 kWh. In this case study there were obvious improvement in prediction accuracy by using the hybrid approach.

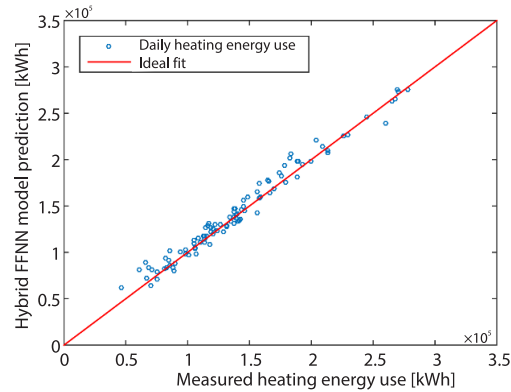
**Table 2.** Prediction indices for Hybrid RBFN model

Model	$R^2$ [-]		RMSE [kWh]		MAPE [%]	
	Training	Test	Training	Test	Training	Test
MLR	0.9574	0.9701	11389	11430	5.7850	8.1540
RBFN	0.9686	0.9766	9766	10196	5.3309	6.5084
Hybrid RBFN	0.9825	0.9703	7377	9230	3.3579	6.0488



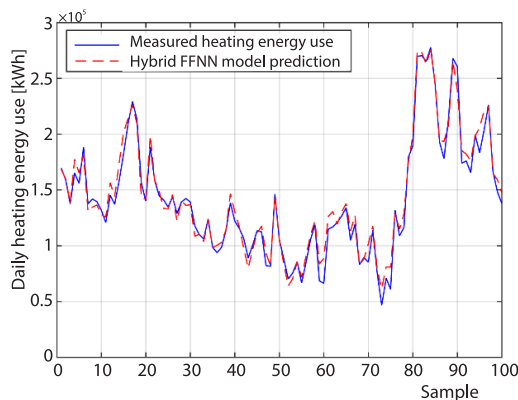


**Figure 3. Hybrid FFNN model vs. measured data for the training period**

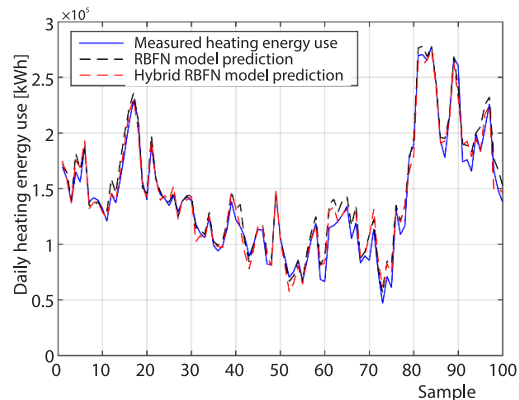


**Figure 4. Hybrid FFNN model vs. measured data for the test period**

In fig. 6 it can be seen that the hybrid RB, FN model predicted values that were closer to the measured values than those achieved with single RBFN model. The comparison of the measured values and the prediction of the hybrid RBFN model for the training and test period is shown in figs. 7 and 8, respectively.

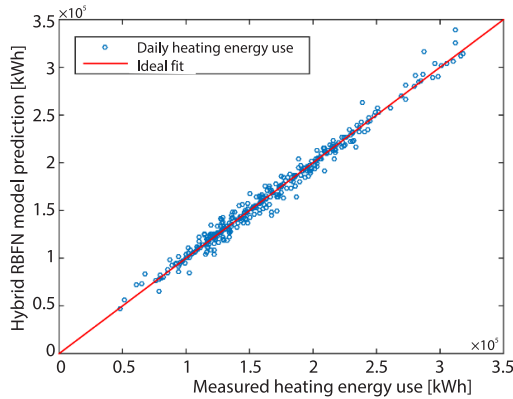


**Figure 5. Prediction results of the hybrid FFNN model for the test period**

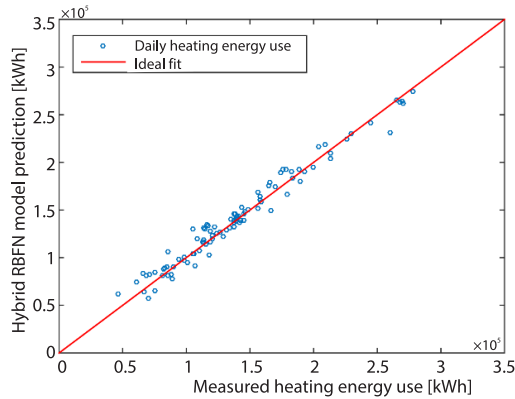


**Figure 6. Prediction results of the hybrid RBFN model for the test period**

The developed models showed more significant deviation around 60<sup>th</sup> sample (beginning of November 2012). The training dataset consists of 318 samples of the coldest period for the years 2009-2011 and the test datasets has 100 samples of the same period in 2012. The analysis of these samples showed that the November of 2012 was unusually cold (lower outdoor temperatures than in November of previous years). This untypical input value can cause more significant prediction error in this case. Other reason for this deviation can be measurement error (meter malfunction) or inadequate data pre-processing (outliers not removed). Besides for the prediction of heating energy use, the AI models can be used for the meters control, pointing out the excessively big difference between the expected and measured values. Further improvements could be achieved by using other non-linear models, such as recurrent neural networks, that are very successful in solving prediction tasks.



**Figure 7. Hybrid RBFN model vs. measured data for the training period**



**Figure 8. Hybrid RBFN model vs. measured data for the test period**

## Conclusion

In this paper, hybrid modelling approach for the prediction of daily heating energy use of the university campus was investigated. The main idea was to divide this complex problem into the linear and the non-linear part. The database for the training and testing the developed models consists of real, measured data. The input variables were: month in year  $M$  [-], day in week  $D$  [-], mean daily outdoor temperature  $t_m$  [°C], maximum daily outdoor temperature  $t_{max}$  [°C], minimum daily outdoor temperature  $t_{min}$  [°C], total daily solar radiation  $SD$  [Whm<sup>-2</sup>], mean daily wind speed  $w_m$  [ms<sup>-1</sup>], and mean daily relative humidity  $\phi_d$  [%]. The FFNN and RBFN models were developed for the daily heating energy use prediction, achieving MAPE in the test period of 6.3438% and 6.50584%, respectively. The RMSE for the test period when using FFNN network is 9492 kWh, while the RBFN makes the error of  $RMSE = 10196$  kWh. The MLR model was used for solving the linear part of the task, while the non-linearity was expected to be captured in the residuals. These residuals were calculated as the difference between the measured data and the MLR outputs. For the prediction of the residuals (using the same eight input variables as for the individual models), the residual FFNN and the RBFN networks were created. The hybrid prediction was the sum of the MLR and residuals FFNN (or RBFN) outputs. The results showed that the hybrid approach improved the prediction quality. The hybrid FFNN and the hybrid RBFN models were compared to the single FFNN and single RBFN models, respectively. In the test period the hybrid FFNN model achieved MAPE of 5.5137%, outperforming the single FFNN with 6.3438%. The RMSE has also shown significant decrease from 9492-8448 kWh. These results can be considered as good comparing to the mean value of the daily heating consumption in the whole dataset of 160251 kWh (the minimum value is 46600 kWh, and the maximum is 320366 kWh). Combining the RBFN and the MLR model also showed the improvement with  $MAPE = 6.0488\%$  comparing to the single RBFN network (6.5084%). The RMSE is in this case lowered from 10196-9230 kWh. This is relatively simple method, comparing to significantly more complex models of improvements presented in the previous research papers using the same database. However, it shown to be very effective in improving the FFNN and RBFN prediction accuracy, by overcoming the shortages of the linear modelling, but with keeping its advantages. The building energy use modelling is very demanding task, considering that the complex relationship between variables is hard to define. Therefore, on this case study, it was proven that the combination of the linear and the non-lin-

ear models might be very successful algorithm for improving prediction accuracy. The hybrid modelling has proven to improve the accuracy of the individual AI models for the prediction of yearly energy cost budget in educational buildings, wind speed, ground-ozone level, and in this case study, also daily heating energy use. These results can encourage other researchers to apply similar methodology for solving prediction problems in their engineering fields. Further improvement of the model accuracy can be achieved by applying some methodology for optimal ANN parameter selection, or using other AI model for the residuals, such as recurrent neural networks, which can be subject of the future work.

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