NUMERICAL EVALUATION OF THERMAL STRESSES GENERATED IN LAMINATED COMPOSITE STRUCTURE EXPOSED TO LOW TEMPERATURES

by

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This paper presents numerical calculation of thermal stresses generated in a carbon fiber-epoxy laminated composite structure which is operating at low temperatures. The thermal stress evaluation is based on the finite element method. A sample problem involving generated thermal stresses in a real laminated composite structure, exposed to low temperatures (four different temperature levels), is analyzed and discussed. Obtained results suggest that these thermal stresses remarkably decrease the strength of the laminated composite structure.

Key words: *finite element method, carbon fiber-epoxy composite, reserve of strength, thermal stresses*

Introduction

In laminated composite structures, during manufacturing, sources of thermal loads include cooling down from processing temperatures. Stresses generated as a result of curing during the manufacturing process are called residual stresses. Thermal expansion coefficient mismatch with respect to different directions is the main cause of residual stresses arising after the manufacturing process is finished.

The second source of thermal loads can be attributed to environmental factors (extreme operating temperatures) resulting in so-called thermal stresses. A typical example is an aircraft, having structural parts made of laminated composite material, exposed to extremely low temperatures while flying at high altitudes. These conditions contribute to the reduction of composite structure strength by the fact that additional thermal stresses can significantly descrease estimated composite structure strength. Composite lamina strengths, such as: tensional, compressional and shear strength, are ussualy determined by standard labaratory tests conducted at labaratory temperature (usually 23 °C). Since operating temperatures could be different from standard labaratory tests temperature, it is clear that thermal stresses should be considered in composite structure design.

A number of researchers investigated this phenomenon in a number of published papers. These studies can generally be divided into two groups. The first group of authors investigated thermally induced stresses at micro-level, where they are generated locally in vicinity of fiber and matrix contact surface due to a difference in thermal expansion coefficients between matrix and fiber [1, 2]. The other group focused their work on macro-level, where mismatch

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of thermal expansion coefficients in different directions generates so-called thermal stresses within a composite [3-6].

Thermal stresses usually occur due to exposing real composite structures to extreme operating temperatures, for instance, higher than 50 °C or lower than –40 °C. In order to investigate possible thermal effects, a thermal stress analysis of carbon fiber-epoxy composite (CFC) exposed to different levels of low temperatures (below 0 °C) should be performed. For a simple geometry structure (*e.g.* flat plate), this analysis can be done analytically, using classical lamination theory (CLT). In case of a complex geometry structure, this analysis has to be done numerically using the finite element method (FEM). This method was successfully used for solving different problems involving thermal stresses [7, 8].

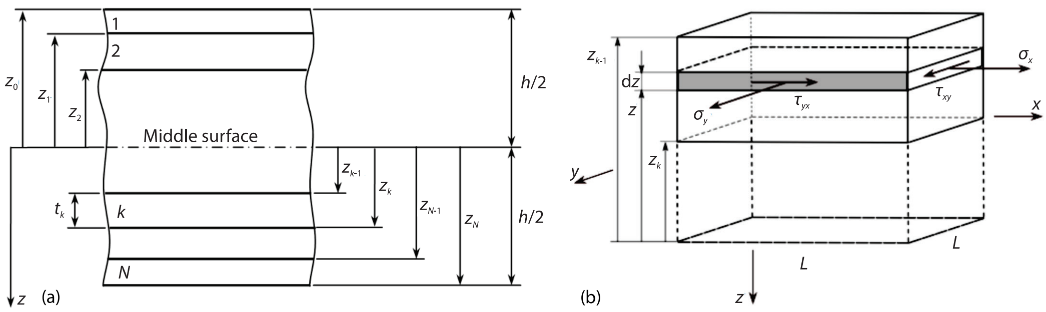

Figure 1. (a) Laminated plate geometry with numbering and (b) stresses in the k^{th} lamina

Analytical modelling

In mechanics of composite materials, stress analysis is usually done using principles of CLT [9]. According to these principles, under temperature loads, each *ply* in a laminate gets stressed by the deformation differences of adjacent lamina. These additional stresses appear when the actual strains of a lamina – $[\varepsilon]$ differ from the thermal strains – $[\varepsilon^T]$ of the unrestricted lamina (free thermal strains). These strain differences are called mechanical strains – $[\varepsilon^M]$ and the stresses caused by them are called mechanical stresses under temperature loads or so-called thermal stresses – $[\sigma]$. Typical geometry of laminated plate used for stress analysis is shown in fig. 1(a). According to CLT, in a laminate with N lamina, fig. 1(a), and with a known temperature change, ΔT, the thermal stresses in the k^{th} lamina are given by:

$$\begin{bmatrix} \sigma_x^T \\ \sigma_y^T \\ \sigma_{\text{xy}}^T \end{bmatrix}_{\text{k}} = \left[\bar{Q}\right]_k \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \end{bmatrix} + z\left[\bar{Q}\right]_k \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix} - \left[\bar{Q}\right]_k \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k \Delta T \tag{1}$$

where matrix $[\bar{Q}]_k$ is the reduced transformed stiffness matrix of the k^{th} lamina, ε_x^0, ε_y^0, ε_{xy}^0 are midplane strains, K_x, K_y, and K_{xy} are midplane curvatures, and α_x, α_y, and α_{xy} are the coefficients of thermal expansion along global axes x and y. Usually, coefficients of thermal expanssion along local axes 1 and 2 (α_1, α_2) are known. Then, coefficients α_x, α_y, and α_{xy} for global axes x and y can be calculated:

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy}/2 \end{bmatrix} = [T]^{-1} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{bmatrix} \tag{2}$$

Matrix $[T]$ is called the transformation matrix and is defined:

$$[T]=\begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2-s^2 \end{bmatrix} \tag{3}$$

where $s = \sin\theta$, $c = \cos\theta$. Angle θ is the angle between global x-axis and direction of fibers in the *ply*-axis 1. The stresses in each lamina can be integrated through the laminate thickness to give resultant forces and moments (or applied external forces and moments):

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix}=\int_{-h/2}^{h/2}\begin{bmatrix} \sigma_x^T \\ \sigma_y^T \\ \tau_{xy}^T \end{bmatrix}_k \mathrm{d}z=\sum_{k=1}^{N}\int_{z_{k-1}}^{z_k}\begin{bmatrix} \sigma_x^T \\ \sigma_y^T \\ \tau_{xy}^T \end{bmatrix}_k \mathrm{d}z \tag{4}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix}=\int_{-h/2}^{h/2}\begin{bmatrix} \sigma_x^T \\ \sigma_y^T \\ \tau_{xy}^T \end{bmatrix}_k z\,\mathrm{d}z=\sum_{k=1}^{N}\int_{z_{k-1}}^{z_k}\begin{bmatrix} \sigma_x^T \\ \sigma_y^T \\ \tau_{xy}^T \end{bmatrix}_k z\,\mathrm{d}z \tag{5}$$

The forces and moments applied to a laminate are known, which in case of absense of external forces and moments can be set to be zero. Substituting thermal stresses in the k^{th} lamina from eq. (1) into eqs. (4) and (5), we get:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix}=0=\sum_{k=1}^{N}\int_{z_{k-1}}^{z_k}\left\{[\bar{Q}]_k\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}+z[\bar{Q}]_k\begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix}-[\bar{Q}]_k\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k \Delta T\right\}\mathrm{d}z \tag{6}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix}=0=\sum_{k=1}^{N}\int_{z_{k-1}}^{z_k}\left\{z[\bar{Q}]_k\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}+z^2[\bar{Q}]_k\begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix}-z[\bar{Q}]_k\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k \Delta T\right\}\mathrm{d}z \tag{7}$$

Since $[\bar{Q}]_k$ is constant through the thickness of k^{th} lamina, previous equations becomes:

$$0=\left\{\sum_{k=1}^{N}[\bar{Q}]_k\int_{z_{k-1}}^{z_k}\mathrm{d}z\right\}\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}+\left\{\sum_{k=1}^{N}[\bar{Q}]_k\int_{z_{k-1}}^{z_k}z\,\mathrm{d}z\right\}\begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix}-\left\{\sum_{k=1}^{\mathrm{N}}[\bar{Q}]_k\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k\int_{z_{k-1}}^{z_k}\mathrm{d}z\right\}\Delta T \tag{8}$$

$$0=\left\{\sum_{k=1}^{N}[\bar{Q}]_k\int_{z_{k-1}}^{z_k}z\mathrm{d}z\right\}\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}+\left\{\sum_{k=1}^{N}[\bar{Q}]_k\int_{z_{k-1}}^{z_k}z^2\mathrm{d}z\right\}\begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix}-\left\{\sum_{k=1}^{N}[\bar{Q}]_k\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k\int_{z_{k-1}}^{z_k}z\mathrm{d}z\right\}\Delta T \tag{9}$$

By solving integrals in eqs. (8) and (9), we obtain:

$$0=\left\{\sum_{k=1}^{N}\left[\bar{Q}\right]_k\left(z_k-z_{k-1}\right)\right\}\begin{bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{bmatrix}+\left\{\sum_{k=1}^{N}\left[\bar{Q}\right]_k\frac{1}{2}\left(z_k^2-z_{k-1}^2\right)\right\}\begin{bmatrix}K_x\\ K_y\\ K_{xy}\end{bmatrix}-$$
$$-\left\{\sum_{k=1}^{N}\left[\bar{Q}\right]_k\begin{bmatrix}\alpha_x\\ \alpha_y\\ \alpha_{xy}\end{bmatrix}_k\left(z_k-z_{k-1}\right)\right\}\Delta T \tag{10}$$

$$0=\left\{\sum_{k=1}^{N}\left[\bar{Q}\right]_k\frac{1}{2}\left(z_k^2-z_{k-1}^2\right)\right\}\begin{bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{bmatrix}+\left\{\sum_{k=1}^{N}\left[\bar{Q}\right]_k\frac{1}{3}\left(z_k^3-z_{k-1}^3\right)\right\}\begin{bmatrix}K_x\\ K_y\\ K_{xy}\end{bmatrix}-$$
$$-\left\{\sum_{k=1}^{N}\left[\bar{Q}\right]_k\begin{bmatrix}\alpha_x\\ \alpha_y\\ \alpha_{xy}\end{bmatrix}_k\frac{1}{2}\left(z_k^2-z_{k-1}^2\right)\right\}\Delta T \tag{11}$$

Introducing following members:

$$A_{ij}=\sum_{k=1}^{N}\left[\bar{Q}_{ij}\right]_k\left(z_k-z_{k-1}\right),\ B_{ij}=\frac{1}{2}\sum_{k=1}^{N}\left[\bar{Q}_{ij}\right]_k\left(z_k^2-z_{k-1}^2\right)$$
$$D_{ij}=\frac{1}{3}\sum_{k=1}^{N}\left[\bar{Q}_{ij}\right]_k\left(z_k^3-z_{k-1}^3\right) \tag{12}$$

where $[\bar{Q}_{ij}]_k$ are members of reduced transformed stiffness matrix $[\bar{Q}]_k$ of the k^{th} lamina. Now, previous equations become:

$$0=\begin{bmatrix}A_{11}&A_{12}&A_{16}\\ A_{12}&A_{22}&A_{26}\\ A_{16}&A_{26}&A_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{bmatrix}+\begin{bmatrix}B_{11}&B_{12}&B_{16}\\ B_{12}&B_{22}&B_{26}\\ B_{16}&B_{26}&B_{66}\end{bmatrix}\begin{bmatrix}K_x\\ K_y\\ K_{xy}\end{bmatrix}-\left\{\sum_{k=1}^{N}\left[\bar{Q}\right]_k\begin{bmatrix}\alpha_x\\ \alpha_y\\ \alpha_{xy}\end{bmatrix}_k\left(z_k-z_{k-1}\right)\right\}\Delta T \tag{13}$$

$$0=\begin{bmatrix}B_{11}&B_{12}&B_{16}\\ B_{12}&B_{22}&B_{26}\\ B_{16}&B_{26}&B_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{bmatrix}+\begin{bmatrix}D_{11}&D_{12}&D_{16}\\ D_{12}&D_{22}&D_{26}\\ D_{16}&D_{26}&D_{66}\end{bmatrix}\begin{bmatrix}K_x\\ K_y\\ K_{xy}\end{bmatrix}-\left\{\sum_{k=1}^{N}\left[\bar{Q}\right]_k\begin{bmatrix}\alpha_x\\ \alpha_y\\ \alpha_{xy}\end{bmatrix}_k\frac{1}{2}\left(z_k^2-z_{k-1}^2\right)\right\}\Delta T \tag{14}$$

So-called fictitious thermal loads (forces and moments):

$$\left[N^T\right]=\begin{bmatrix}N_x^T\\ N_y^T\\ N_{xy}^T\end{bmatrix}=\left\{\sum_{k=1}^{N}\left[\bar{Q}\right]_k\begin{bmatrix}\alpha_x\\ \alpha_y\\ \alpha_{xy}\end{bmatrix}_k\left(z_k-z_{k-1}\right)\right\}\Delta T \tag{15}$$

$$\left[M^T\right]=\begin{bmatrix} M_x^T \\ M_y^T \\ M_{xy}^T \end{bmatrix}=\frac{1}{2}\left\{\sum_{k=1}^{N}\left[\bar{Q}\right]_k \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k \left(z_k^2-z_{k-1}^2\right)\right\}\Delta T \tag{16}$$

Now eqs. (13) and (14) can be expressed in short form notation:

$$\left[N^T\right]=\left[\mathrm{A}\right]\left[\varepsilon^0\right]+\left[\mathrm{B}\right]\left[K\right] \tag{17}$$

$$\left[M^T\right]=\left[\mathrm{B}\right]\left[\varepsilon^0\right]+\left[\mathrm{D}\right]\left[K\right] \tag{18}$$

The fictitious thermal loads represent the loads in eqs. (15) and (16), are the loads one can apply mechanically to induce the same stresses and strains as by the thermal load. Thus, if both mechanical and thermal loads are applied, the external mechanical loads can be added to the fictitious thermal loads to find the ply-by-ply stresses and strains in the laminate. However, these loads can be applied even separately and then added producing resulting stresses and strains from the solution of the two problems, *i.e.* mechanical and thermal. As obvious from eqs. (15) and (16), fictitious thermal loads can be calculated and are known.

It can be easily shown, [9], that in the case of symmetrical laminate: $[B] = 0$, $[M^T] = 0$. Therefore, eqs. (17) and (18) become:

$$\left[N^T\right]=\left[\mathrm{A}\right]\left[\varepsilon^0\right] \tag{19}$$

$$0=\left[\mathrm{D}\right]\left[K\right] \tag{20}$$

Now, midplane strains and curvatures can be calculated from eqs. (19) and (20):

$$\left[\varepsilon^0\right]=\left[\mathrm{A}\right]^{-1}\left[N^T\right] \tag{21}$$

$$\left[K\right]=0 \tag{22}$$

Global thermal stresses in the k^{th} *ply* are then calculated by using eq. (1):

$$\begin{bmatrix} \sigma_x^T \\ \sigma_y^T \\ \tau_{xy}^T \end{bmatrix}_k=\left[\bar{Q}\right]_k \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}-\left[\bar{Q}\right]_k \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k \Delta T \tag{23}$$

Local thermal stresses in the k^{th} *ply* (stresses in principal material directions 1 and 2) can be obtained using transformation matrix defined by eq. (3):

$$\begin{bmatrix} \sigma_1^T \\ \sigma_2^T \\ \tau_{12}^T \end{bmatrix}_k=\left[T\right]\begin{bmatrix} \sigma_x^T \\ \sigma_y^T \\ \tau_{xy}^T \end{bmatrix}_k \tag{24}$$

However, if both mechanical and thermal loads are applied, mechanical and thermal stresses can be calculated separately and then added forming the total (or resulting) stress.

To assess the contribution of thermally induced stresses to the reduction of lamina load-carrying capacity, modified Tsai-Hill (mTH) failure criterion is used:

$$\left(\frac{\sigma_1}{X_1}\right)^2 - \left(\frac{\sigma_1\sigma_2}{X_1^2}\right) + \left(\frac{\sigma_2}{X_2}\right)^2 + \left(\frac{\tau_{12}}{X_{12}}\right)^2 < 1 \tag{25}$$

where

$$X_1 = \begin{cases} X_{1C}, \sigma_1 \leq 0 \\ X_{1T}, \sigma_1 > 0 \end{cases} \tag{26}$$

$$X_2 = \begin{cases} X_{2C}, \sigma_2 \leq 0 \\ X_{2T}, \sigma_2 > 0 \end{cases} \tag{27}$$

where X_{1C} and X_{1T} are unidirectional lamina compressive and tensile strengths in direction of the fibers, X_{2C} and X_{2T} – the unidirectional lamina compressive and tensile strengths in direction normal to the direction of the fibers, while X_{12} is the shear strength of unidirectional lamina. These strengths are ussualy determined by standard labaratory tests or adopted from literature. Stresses: σ_1, σ_2 and τ_{12} are lamina local total stresses formed by adding thermal stresses to mechanical ones.

If the left side of in eq. (25) is less than 1, composite lamina is considered to be safe and there is an amount of so-called *strength reserve* (SR), meaning that loads can be increased until left side of inequality approaches unity while satisfying condition given in eq. (25) in every moment. According to the previous definition of *strength reserve* it can be expressed:

$$SR = 1 - \left[\left(\frac{\sigma_1}{X_1}\right)^2 - \left(\frac{\sigma_1\sigma_2}{X_1^2}\right) + \left(\frac{\sigma_2}{X_2}\right)^2 + \left(\frac{\tau_{12}}{X_{12}}\right)^2\right] \tag{28}$$

Values of SR lie between 0 and 1. Hypothetical case when $SR = 1$ means that the lamina is in a stress free state (no external loads are applied). As the loads increase, less reserve of strength remains and the SR value approaches zero. The case of $SR \leq 0$ means failure of lamina. As obvious from eq. (28), linearly added loads result in a linear increase in stresses σ_1, σ_2 and τ_{12} and in a non-linear reduction of SR. Therefore, SR value is not a clear indicator of the allowable load increase. Consequently, for different load cases, separate analyses should be performed.

Numerical modelling

In order to obtain thermal stresses and their impact on laminated composite structure strength, a numerical analysis has been performed using Code_Aster [10], an open source finite element analysis software. Composite material laminates are modeled as a 3-D continuum using FEM and first-order hexahedral elements.

The material is modeled as orthotropic linear elastic and local co-ordinate system orientation is assigned to each element using Euler angles according to lamina orientation and element position in the model. The following material properties are used in all analyses:

$$E_1 = 142\ \text{GPa},\ E_2 = 10.3\ \text{GPa},\ E_3 = 10.3\ \text{GPa}$$

$$G_{12} = 7.67\ \text{GPa},\ G_{23} = 2.5\ \text{GPa},\ G_{31} = 7.67\ \text{GPa}$$

$$\nu_{12} = 0.27,\ \nu_{23} = 0.37,\ \nu_{31} = 0.37$$

$$\alpha_1 = -1.8 \cdot 10^{-6}\,\text{K}^{-1},\ \alpha_2 = 2.7 \cdot 10^{-5}\,\text{K}^{-1},\ \alpha_3 = 2.7 \cdot 10^{-5}\,\text{K}^{-1}$$

$$X_{1T} = 1500\,\text{MPa},\ X_{1C} = 1500\,\text{MPa},\ X_{2T} = 40\,\text{MPa},\ X_{2C} = 246\,\text{MPa},\ X_{12} = 68\,\text{MPa}$$

For validating the numerical model, a simple case of an uncostrained carbon fiber-epoxy $[0/90_2]_s$ laminated plate (6×6 mm) is considered. Each lamina is 0.2 mm thick and the plate is exposed to temperature of –37 °C. Since there are no other external loads, such as external forces and moments, the plate is considered to be stress free at room temperature of 23 °C. In this case temperature change which induce thermal loads is: ΔT = –60 °C. The size of the numerically modeled plate is large enough so that the free-edges effects are negligible at the center of the plate, fig. 2.

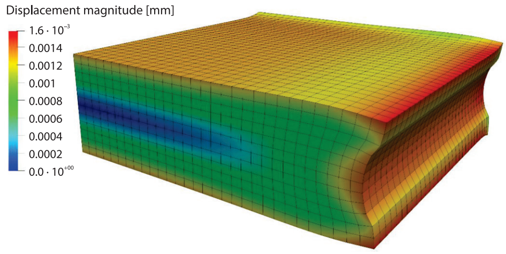

Figure 2. Thermally loaded plate deformed shape (100 displacement scale factor, 1/4 of the model)

Numerically obtained thermal stresses, measured throughout the laminate thickness at the center of the plate, are in excellent agreement with the results obtained analytically using eq. (28), tab. 1. Out-of-plane stresses are negligible far from free edges and thus are not taken into account. Values of SR for each lamina, calculated using eq. (28), are also presented in tab. 1.

Table 1. Stresses and SR of each lamina in thermally loaded plate, ΔT = –60 °C

		σ_x	σ_{xy}	τ_{xy}	SR
0° lamina	Analytical	–30.250 MPa	16.313 MPa	0 MPa	0.8330
	Numerical	–29.818 MPa	16.310 MPa	0 (–0.002) MPa	0.8331
	Relative error	1.43%	0.02%	–	0.01%
90° lamina	Analytical	15.125 MPa	–8.254 MPa	0 MPa	0.9573
	Numerical	15.139 MPa	–8.206 MPa	0 (–0.006) MPa	0.9578
	Relative error	–0.10%	0.58%	–	0.05%

Analysis of a mechanically and thermally loaded cylinder

A carbon fiber-epoxy laminated cylinder (80 mm) (in diameter and 80 mm long), representing a part of a more complex structure, is considered here. The laminate is symmetric $–[0/+45/–45/90]_s$ and each lamina is 0.25 mm thick. Thickness t = 0 mm corresponds to the inner surface, while t = 2 mm corresponds to the outer surface of the cylinder, fig. 3.

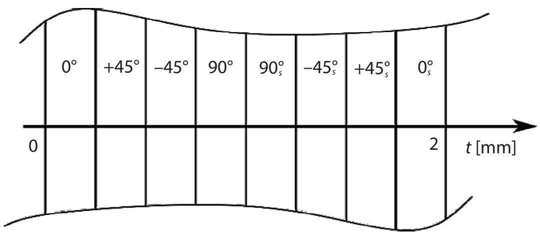

Figure 3. Laminae orientation

All nodes at left end of the cylinder are fixed, fig. 4. At the right end, displacements along the axial direction and all relative displacements of the nodes are constrained, allowing only uniform rotation of the right end. Two load cases are considered:

– load Case A no-mechanical loads and temperature differences ΔT = (0, –20, –40, –60) °C and
– load Case B – a torque of 2500 Nm resulting in 0.64° rotation of the right end and temperature differences: ΔT = (0, –20, –40, –60) °C.

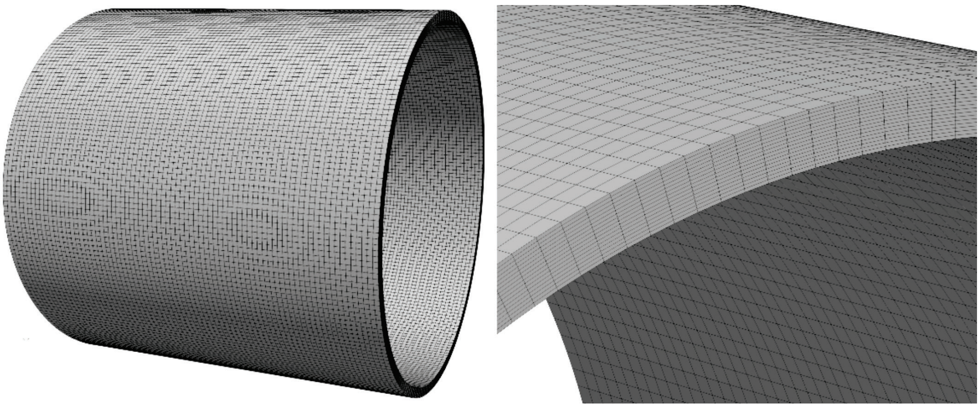

Figure 4. Finite element mesh for the cylinder analysis

For load case A, it has been shown that a $\Delta T = -60$ °C temperature difference reduces the strength reserve (load-carrying capacity) of the modeled structure by 21.2% (+45° oriented inner lamina), as shown on fig. 5. As obvious from fig. 5, the rise of the failure criterion value with respect to an increase in negative temperature difference is exponential.

For load case B, the initial mechanical load reduces the strength by 41.4%, as shown on fig. 6. By including a temperature load, $\Delta T = -60$ °C, the strength reserve is dramatically reduced from 0.5857 (state of no thermal stresses) to 0.0731 (state with thermal stresses). As shown on fig. 6, strength reserve values in other laminae are either not so significantly decreased or even slightly increased.

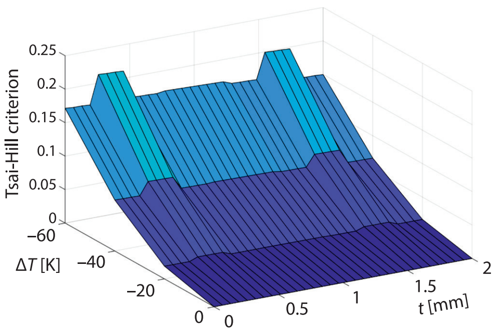

Figure 5. The mTH failure criterion values for load Case A

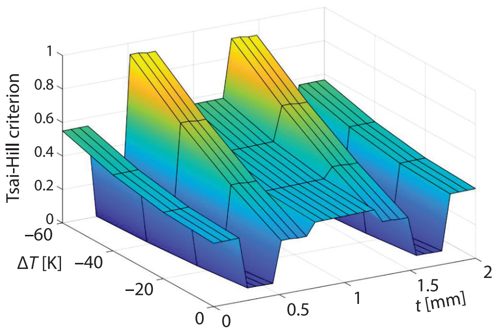

Figure 6. The mTH failure criterion values for load Case B

Numerically obtained minimum SR values for each lamina, calculated using eq. (28), and both load cases, are presented in tabs. 2 and 3.

Table 2. Minimum SR values in each lamina for load Case A

	0°	+45°	–45°	90°	90°_s	-45°_s	$+45^\circ_s$	0°_s
0 °C	1	1	1	1	1	1	1	1
–20 °C	0.9809	0.9764	0.9812	0.9812	0.9813	0.9820	0.9779	0.9832
–40 °C	0.9238	0.9054	0.9247	0.9246	0.9253	0.9279	0.9114	0.9327
–60 °C	0.8285	0.7872	0.8306	0.8304	0.8319	0.8377	0.8007	0.8486

Table 3. Minimum SR values in each lamina for load Case B

	0°	45°	–45°	90°	90°_s	-45°_s	$+45^\circ_s$	0°_s
0 °C	0.6185	0.9257	0.6788	0.6037	0.5988	0.6782	0.9177	0.5837
–20 °C	0.5997	0.9599	0.5129	0.5850	0.5802	0.5162	0.9528	0.5671
–40 °C	0.5432	0.9836	0.3095	0.5289	0.5246	0.3183	0.9782	0.5172
–60 °C	0.4491	0.9969	0.0685	0.4353	0.4319	0.0845	0.9939	0.4341

Numerically obtained SR values presented in tabs. 2 and 3 suggest the fact that linearly added temperature loads result in a non-linear reduction of SR as shown on figs. 7 and 8. That can be attributed to the complex interaction of stresses: σ_1, σ_2 and τ_{12}, as well as the quadratic form of the mTH failure criteria, eqs. (25) and (28).

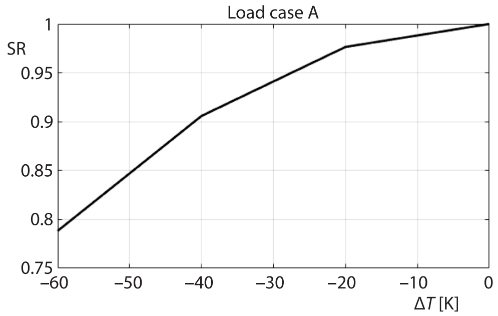

Figure 7. Minimum SR values in the laminate for load case A

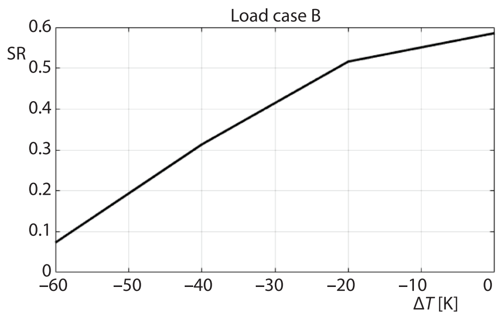

Figure 8 – Minimum SR values in the laminate for load case B

From results presented in tab. 3 it can be concluded that in case of further increase of temperature load, failure of –45° oriented inner lamina is expected to occur. This means that for considered mechanical and temperature loads, *first ply failure* of laminate is expected at –45° oriented inner *ply*.

It has been shown that, compared to a structure subjected to temperature change only, the decrease of laminate strength reserve due to thermal stresses is significantly greater in a mechanically loaded structure.

Conclusions

In order to investigate thermal effects on strength of CFC structure exposed to different levels of low temperature, a thermal stress analysis has been performed. This analysis has been done numerically using the FEM. Numerical analysis aproach is verified using CLT.

The mTH failure criterion was used for assessing the contribution of thermally induced stresses to the reduction of composite structure strength. Results of this analysis showed that strength of a mechanically loaded composite structure can be considerably reduced by additionall thermal stresses generated at low temperatures. According to mTH failure criterion, in case of $\Delta T = -60$ °C temperature difference, for the considered torsionally loaded composite structure, strength reserve is dramatically reduced from 0.5837 (state of no thermal stresses) to 0.0685 (state with thermal stresses), which represents a reduction of strength reserve by 88%.

Obtained results clearly suggest that thermal stresses generated at low temperatures can be of the order of magnitude of the mechanical ones. Lower temperatures produce higher thermal stresses which contribute to the reduction of strength reserve in non-linear manner.

Therefore, these stresses must not be neglected and have to be involved in composite design regarding the composite structure strength.

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Nomenclature

[A] – extensional stiffness matrix, [MPa]
[B] – coupling stiffness matrix, [MPa]
[D] – bending stiffness matrix, [MPa]
E – elastic modulus, [MPa]
G – shear modulus, [MPa]
K – midplane curvature,
M – resultant moment,
M^T – fictitious thermal moment, [Nmm]
N – resultant force, [N]
N^T – fictitious thermal force, [N]
$[\overline{Q}]$ – reduced transformed stiffness matrix, [MPa]
SR – strength reserve, [–]
$[T]$ – transformation matrix, [MPa]
ΔT – temperature change, [K]
X_1 – unidirectional lamina strength in the longitudinal direction, [MPa]
X_{1T} – unidirectional lamina tensile strength in the longitudinal direction, [MPa]
X_{1C} – unidirectional lamina compressive strength in the longitudinal direction, [MPa]
X_2 – unidirectional lamina strength in the transversal direction, [MPa]
X_{2T} – unidirectional lamina tensile strength in the transversal direction, [MPa]
X_{2C} – unidirectional lamina compressive strength in the transversal direction, [MPa]
X_{12} – unidirectional lamina shear strength, [MPa]

Greek symbols

α – coefficient of thermal expansion, [K^{-1}]
ε – strain, [–]
ε^0 – midplane strain, [–]
ε^M – mechanical strain, [–]
ε^T – thermal strain, [–]
ν – Poisson's ration, [–]
σ – stress, [MPa]
σ^T – thermal stress, [MPa]

Acronyms

CFC – carbon fiber-epoxy composite
CLT – classical lamination theory
FEM – finite element method
mTH – modified Tsai-Hill criterion

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