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DISCRETE-TIME SYSTEM CONDITIONAL OPTIMISATION IN THE PARAMETER SPACE VIA THE FULL TRANSFER FUNCTION MATRIX

Summary

Dynamic systems operate under the simultaneous influence of both the initial conditions and the input vector. There is neither physical nor mathematical justification for ignoring the initial conditions, e.g., in the control optimisation. This paper gives a response to the following question: Is a set of controller parameters which is optimal for the operation of a control system under zero initial conditions also optimal for its operation under non-zero initial conditions?

The paper presents a new approach to the design of a classical proportional-difference-sum (PDS) controller for a plant in a closed loop control system. The system relative stability with respect to a desired damping coefficient is accomplished. The minimal value of the performance index in the form of the sum of squared errors is the optimality criterion. Unlike the classical approach, the output error used in the performance index is influenced by all actions performed on the system at the same time.

Key words: *discrete-time control systems, three parameters synthesis, relative stability, full transfer function matrix, conditional stabilisation and optimisation*

1. Introduction

Usually, the region of pole location has been determined by using the demanded damping coefficient or/and settling time. Pioneering work on the problem in the parameter plane was conducted by Vishnegradski [1], Neymark [2] and D. Mitrović, who developed a method of graphical analysis for the synthesis of two free parameters in a closed loop, both continuous-time and discrete-time, control system [3, 4, 5]. Later, this method was widely cited in the literature and referred to as Mitrovic's method. The first two coefficients in the system characteristic polynomial were the two free parameters.

Mitrovic's method was generalized firstly by Šiljak who permitted any two characteristic polynomial coefficients to be adjustable parameters [6], and soon afterwards, quite independently, Šiljak [7] and Grujić [8] considered some characteristic polynomial coefficients to be linearly dependent on two adjustable parameters. Furthermore, Šiljak treated linear dependency of all characteristic polynomial coefficients on two unknown parameters [9, 10], which actually evolved into an algebraic method. All these approaches had their drawbacks. They did not consider the position of zeros of the system transfer function, which also influences the quality of the system behaviour.

The disadvantage was overcome by Šiljak in [7], where he established the concept of conditional optimisation in the parameter plane. He did it by introducing one extra criterion for the system, in addition to its relative stability, which is the minimal value of the performance index in the form of the integral of squared errors. Its optimisation is achieved through an appropriate choice of two adjustable parameters. This was designated as the conditional parameter optimisation. In that way, the system was relatively stable with the desired damping coefficient or/and settling time, and had the finer transient state behaviour regarding, e.g., overshoot.

Grujić went further in [11, 12] to permit all coefficients of a system transfer function (denominator and numerator polynomial coefficients) to be dependent linearly on two unknown adjustable parameters. He combined the conditional optimisation with another aspect proposed by Rakić [13], i.e. the system impulse response through the numerical values calculation of its transfer function residues. Compatible recurrent formulas have been developed for mapping from the s-complex plane in the parameter plane, the calculation of transfer function residues, and the calculation of the performance index by means of the residues. The formulas were very simplified and suitable for computer calculation compared to the ones previously used.

On the other side, approximately in the same period of the last century, in a quite different way, Kalman made great progress in the discrete-time linear systems optimisation [14, 15]. He developed a matrix synthesis method of the controller algorithm:

$$\mathbf{u}(k) = -\mathbf{K}\mathbf{x}(k), \mathbf{K} \in \mathbb{R}^{m \times n},$$

by means of Pontryagin's maximum principle and Bellman's continuous dynamic programming, without constraints to the number of the matrix gain **K** coefficients, that is, the number of controls and state variables. His great contribution is that his method is matrix synthesis of all controller coefficients at the same time. The famous Kalman's discrete regulator has resulted from the procedure, which encouraged many researchers to continue research in that direction. Kalman used the optimality criterion in the form of the sum of two quadratic forms:

$$J = \sum_{k=0}^{+\infty} \left[\mathbf{x}^{\mathrm{T}}(k) Q \mathbf{x}(k) + \mathbf{u}^{\mathrm{T}}(k) R \mathbf{u}(k) \right].$$

The first form with the symmetric positive semi-definite matrix Q, $Q=Q^{\mathrm{T}}\geq 0$ expresses the process quality and the other form with the symmetric positive definite matrix R, $R=R^{\mathrm{T}}>0$ expresses the energy consumption by the control to be performed. The main issue regarding matrices Q and R is their proper selection.

Over the decades and to the present day, plant control by using the classical control algorithm is still up-to-date and researchers are dealing with it all the time. Many of the researchers focused on determining the stabilizing proportional-integral-derivative (PID) gain region in the parameter space. For example, Xu et al. [16] treated a digital system on the basis of previously obtained results for continuous-time systems; Keel et al. [17] considered a digital system, grouping its characteristic roots inside an inner circle of unit one and in this regard the smallest possible circle which leads to the almost deadbeat control; Tantaris et al. [18] dealt with three-parameter first-order discrete-time controllers; Li et al. [19] discussed continuous-time systems knowing only frequency response of the plant and its characteristic root number located strictly in the right half of the complex plane; Matušů [20] used Tan's and Kronecker's methods for a continuous-time system and then picked one parameter sample by means of not a novel but a known method. Similarly, other authors found the stability domain in the parameter space: Gryazina and Polyak [21] found it for both continuous-time and discrete-time system in the state space form; Gryazina and Polyak in [22] decomposed the domain in the parameter plane for a continuous-time system in the state space form, in root sign invariant regions; Gryazina et al. [23] described the state of the art, starting in the past from Vishnegradsky to Neymark to the present for the continuous-time (scalar and multivariable) system parameter synthesis with recent extensions and new results related to the stability domain in the parameter space, and considering the aspect of uncertainty, robustness, H_∞ criterion etc.; Kipnis and Nigmatulin [24] treated the trinomial discrete equation with two delays, and established a criterion for testing Lyapunov stability depending on parameter values, as well as generalization of the Lyapunov stability concept by using the new r-stability concept. Some authors considered optimal PID controller tuning: Padula and Visioli [25] used standard and fractional order PID controllers and performance index in the form of the integral of absolute error; Barbosa and Jesus [26] considered a fractional order PID controller and metaheuristic tuning algorithm Cuckoo Search (inspired by the behaviour of living beings in nature), whereas the performance index was in the form of the integral of the sum, weighted time multiplied absolute error and also weighted squared control.

The results based on all mentioned approaches, except Kalman's, were obtained under decades-long controversy between the stability concept and the classical system transfer function. The system stability is by definition the dynamical property of a system in the free working mode under all zero inputs and arbitrary unknown initial conditions. The transfer function is defined for the system in the forced working regime under non-zero input and all zero initial conditions. This controversy has been recently solved by introducing and developing the full transfer function matrix [27, 28, 29]. The conditional optimisation synthesis procedure is carried out herein by using the characteristic polynomial of the full transfer function matrix and not of the classical one. More precisely, it is the characteristic polynomial of the row nondegenerate full transfer function matrix, which is only adequate and appropriate to be used for the objective test of the system stability and optimisation.

In order to design a controller to cope with real working conditions, we use a new form of the performance index. It is the sum of the squared errors that occurred in the most general and realistic circumstances, under all actions performed on the system at the same time: the external nonzero inputs and nonzero initial conditions.

Why to opt for the conditional optimisation and not for the Kalman regulator? Both approaches have their advantages and disadvantages. A significant progress in the proposed new conditional optimisation is that it is now complete, the number of parameters is enlarged to three, and the controller is dynamic whereas the Kalman regulator is static. Besides, the control algorithm is given in terms of measurable physical output variables rather than as the function of unmeasurable mathematical state variables, used in the Kalman regulator. Moreover, the Kalman regulator does not consider the disturbance input. To get the matrix gain $\mathbf{K}$ for the Kalman regulator, it is necessary to solve the matrix nonlinear Riccati equation, which is, despite the present age of digital computers, an aggravating circumstance. Furthermore, related to the Kalman regulator, the right choice of matrices Q and R is necessary. When the state variables are not measurable, then estimation procedures are to be applied. Kalman's method does not yield relative stability, whereas conditional optimisation does. Also, the Kalman regulator is inapplicable to the input-output systems in the following sense: generally a pure mathematical state variable choice algorithm is used, without any physical sense, to pass from the input-output equation to the state space equations. Then, system state **x**(k) contains in itself control **u**(k) as its input, that is, **x**=**x**(**u**); further, Kalman's control algorithm **u**=-**Kx** yields **u**=-**Kx**(**u**)=**f**(**u**) that makes no sense. Kalman's optimality criterion contains two parts, one concerns the process quality and another the energy consumption, which necessarily results in a compromise between the two requirements. Our dynamic controller optimality criterion vs Kalman's criterion fully concerns the process quality, which leads to a higher degree of process quality than in Kalman's case.

Again, the question is: why to opt for the conditional optimisation in the "outdated complex domain" and not for some other among many modern state space methods? The answer is similar as with the Kalman regulator. When the system input-output discrete equation includes the right-hand-side time shifted items, the only way to pass to the state space equation is to use the pure mathematical state variable choice algorithm without any physical sense. Then, system state **x**(k) contains in itself system input **u**(k), i.e. control, which is meaningless. The conclusion is that in such a case the state-space methods are not applicable.

The paper deals with discrete-time systems only because of the contemporary practice, where the systems are almost exclusively controlled by digital computers.

Illustrative examples are given to show the difference in the system behaviour when the system is designed in the classical and in the proposed novel way. The differences are detected by simulations and practical experiment.

2. Problem statement

2.1 Plant

The linear time-invariant discrete-time of a most general single-input single-output (SISO) plant is considered. It is described by its input-output equation

$$\begin{gathered} a_{\nu P} y(k+\nu)+a_{(\nu-1)P} y(k+\nu-1)+\cdots+a_{1P} y(k+1)+a_{0P} y(k)= \\ =b_{0P} u_P(k)+\cdots+b_{(\mu-1)P} u_P(k+\mu-1)+b_{\mu P} u_P(k+\mu), \\ a_{\nu P}=1, \nu \in \mathbb{N}, \mu \in \mathbb{N}_0, \mu \leq \nu, \end{gathered} \tag{1}$$

where $k\in\mathbb{N}_0$, $y(k+j)\in\mathbb{R}$ is the plant output at time $k+j$, $\forall j=0,1,2,\cdots,\nu$, $u_P(k+j)\in\mathbb{R}$ is the plant input at time $k+j$, $\forall j=0,1,2,\cdots,\mu$, $\mu\leq\nu$; $a_{jP}\in\mathbb{R}$, $\forall j=0,1,2,\cdots,\nu$, and $b_{jP}\in\mathbb{R}$, $\forall j=0,1,2,\cdots,\mu$, are real numbers. Equation (1) is generated through the Lyapunov's coordinate transformation process, that is, $y = Y-Y_d$ is the plant output deviation from the desired output Y_d, and $u_P=U_P-U_{PN}$ is the plant input deviation from the nominal input U_{PN}.

The compact form of Equation (1) is as follows [27, 28, 29]:

$$\mathbf{A}_P^{(\nu)}\mathbf{y}^\nu = \mathbf{B}_P^{(\mu)}\mathbf{u}_P^\mu, \tag{2}$$

where $\mathbf{A}^{(\nu),P}$ and $\mathbf{B}^{(\mu),P}$ are the extended coefficient matrices:

$$\mathbf{A}_P^{(\nu)}=\begin{bmatrix} a_{0P} & a_{1P} & \cdots & a_{\nu P}\end{bmatrix}, \mathbf{B}_P^{(\mu)}=\begin{bmatrix} b_{0P} & b_{1P} & \cdots & b_{\mu P}\end{bmatrix},$$

and $\mathbf{y}^\nu$ and $\mathbf{u}_P^\mu$ are the extended output and input vectors:

$$\begin{aligned} \mathbf{y}^\nu &= \begin{bmatrix} y_P(k) & y_P(k+1) & \cdots & y_P(k+\nu)\end{bmatrix}^{\mathrm{T}}, \\ \mathbf{u}_P^\mu &= \begin{bmatrix} u_P(k) & u_P(k+1) & \cdots & u_P(k+\mu)\end{bmatrix}^{\mathrm{T}}. \end{aligned}$$

2.2 Controller

The most general classical linear discrete-time time-invariant controller is considered, whose input-output equation in the difference form is, [30]:

$$T_\eta^\eta \frac{1}{T^\eta}\Delta^\eta u(k)+\cdots+T_1\frac{1}{T}\Delta u(k)+u(k)=K\varepsilon(k)+K_D\frac{1}{T}\Delta\varepsilon(k)+K_S T\sum_{i=0}^{i=k-1}\varepsilon(i). \quad (3)$$

From Equation (3) different special cases arise. For example:

2.2.1 The zero order proportional-sum (PS) controller

$$\mathbf{A}_C^{(1)}\mathbf{u}^1=\mathbf{B}_C^{(1)}\boldsymbol{\varepsilon}^1, \quad (4)$$

$$\mathbf{A}_C^{(1)}=\begin{bmatrix}a_{0C} & a_{1C}\end{bmatrix}, \mathbf{B}_C^{(1)}=\begin{bmatrix}b_{0C} & b_{1C}\end{bmatrix}, a_{0C}=-1, a_{1C}=1,$$
$$b_{0C}=\left(K_S T-K\right), b_{1C}=K.$$

2.2.2 The first order proportional-difference-sum (PDS) controller

$$\mathbf{A}_C^{(2)}\mathbf{u}^2=\mathbf{B}_C^{(2)}\boldsymbol{\varepsilon}^2; \mathbf{A}_C^{(2)}=\begin{bmatrix}a_{0C} & a_{1C} & a_{2C}\end{bmatrix}, \mathbf{B}_C^{(1)}=\begin{bmatrix}b_{0C} & b_{1C} & b_{2C}\end{bmatrix}, \quad (5)$$

$$a_{0C}=\frac{T_1-T}{T}, a_{1C}=\frac{T-2T_1}{T}, a_{2C}=\frac{T_1}{T}; b_{0C}=\left(\frac{K_D}{T}+K_S T-K\right),$$
$$b_{1C}=\left(K-\frac{2K_D}{T}\right), b_{2C}=\frac{K_D}{T}.$$

2.3 Problem definition

The aim of this paper is, in the first step, to synthesize the unknown adjustable system parameters with respect to the desired damping coefficient using the algebraic method in a qualitatively new way. The system relative stability is achieved by using only an adequate and appropriate system characteristic polynomial in the sense already described. This is different from the appropriate classical procedure where the characteristic polynomial of the transfer function matrix $G(z)$ is used, which is not correct in general.

In the second step, unknown parameters should be selected from an acceptable set according to an additional criterion, i.e. the value of the performance index in the form of the sum of squared errors is to be minimal. The performance index is a new developed index because the used error, in the most general and realistic circumstances, is caused by the influence of all actions performed on the system, external non-zero inputs and non-zero initial conditions at the same time.

3. Major results

In this section the solution to the problem posed in the section Problem definition is provided.

3.1 Relative stability

Formally, the procedure of synthesising adjustable and unknown parameters to get relative stability is similar to the classical procedure known from literature [10, 12], but essentially, it is different because of using the system characteristic equation, that is, the characteristic equation of the system row nondegenerate full transfer function matrix. We know from the literature [10, 12] that,

$$\begin{aligned}&\alpha\bar{B}_1(\rho_z,\zeta_z)+\beta\bar{C}_1(\rho_z,\zeta_z)+\bar{D}_1(\rho_z,\zeta_z)=0,\\&\alpha\bar{B}_2(\rho_z,\zeta_z)+\beta\bar{C}_2(\rho_z,\zeta_z)+\bar{D}_2(\rho_z,\zeta_z)=0.\end{aligned} \tag{6}$$

For three unknown parameters we have,

$$\begin{aligned}&\alpha\bar{B}_1(\rho_z,\zeta_z)+\beta\bar{C}_1(\rho_z,\zeta_z)+\gamma\bar{D}_1(\rho_z,\zeta_z)+\bar{E}_1(\rho_z,\zeta_z)=0,\\&\alpha\bar{B}_2(\rho_z,\zeta_z)+\beta\bar{C}_2(\rho_z,\zeta_z)+\gamma\bar{D}_2(\rho_z,\zeta_z)+\bar{E}_2(\rho_z,\zeta_z)=0.\end{aligned} \tag{7}$$

Solving Equations (6), we obtain:

$$\begin{aligned}\alpha&=\frac{\bar{C}_1\bar{D}_2-\bar{C}_2\bar{D}_1}{\bar{B}_1\bar{C}_2-\bar{B}_2\bar{C}_1},\\\beta&=\frac{\bar{B}_2\bar{D}_1-\bar{B}_1\bar{D}_2}{\bar{B}_1\bar{C}_2-\bar{B}_2\bar{C}_1}.\end{aligned} \tag{8}$$

The solution of Equations (7) is not unique because there are three adjustable unknown parameters α, β, and γ, and only two equations. Equations (7) may be solved with respect to any two parameters, e.g., α and β, so that they are the functions not only of ρ_z and ζ_z but also of the third free parameter γ. In that case, the solution is as follows:

$$\begin{aligned}\alpha&=\frac{\bar{C}_1(\gamma\bar{D}_2+\bar{E}_2)-\bar{C}_2(\gamma\bar{D}_1+\bar{E}_1)}{\bar{B}_1\bar{C}_2-\bar{B}_2\bar{C}_1},\\\beta&=\frac{\bar{B}_2(\gamma\bar{D}_1+\bar{E}_1)-\bar{B}_1(\gamma\bar{D}_2+\bar{E}_2)}{\bar{B}_1\bar{C}_2-\bar{B}_2\bar{C}_1}.\end{aligned} \tag{9}$$

It means that any value of the free parameter γ may be chosen, and after that the procedure continues as in the previous case when there are only two adjustable unknown parameters.

Now, we can map the loci of constant value damping coefficient ζ from the s-complex plane into the $\alpha\beta$ parameter plane like a curve, or into the $\alpha\beta\gamma$ parameter space like a surface. In doing so, the numerical value of natural frequency ω_n is changed with a certain step. When using Expressions (9), each time a numerical value of the parameter γ should be selected, with a certain numerical step.

3.2 Performance index

The proposed form of performance index I is the sum of the squared output errors:

$$I=\sum_{k=0}^{k=\infty}\varepsilon^2(k),\varepsilon(k)=r(k)-y(k), \tag{10}$$

where error $\varepsilon(k)$ occurred under the influence of all actions performed on the system, external non-zero input and non-zero initial conditions, at the same time. It is more natural and realistic in practice than error $\varepsilon(k)$ that has been used in the literature so far is influenced only by non-zero input but for all zero initial conditions. The proposed optimisation procedure is completely new.

The full block diagram [27] of the system is shown in Fig. 1.

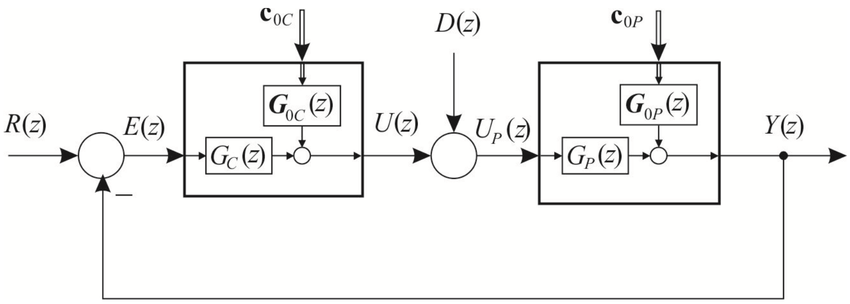

Fig. 1 Full block diagram of the overall closed loop system with unity feedback

From the block diagram, it is easy to obtain:

$$Y(z)=\mathbf{F}(z)\mathbf{V}(z)=\begin{bmatrix}\frac{G_P(z)G_C(z)}{1+G_P(z)G_C(z)}\\ \frac{G_P(z)}{1+G_P(z)G_C(z)}\\ \frac{G_P(z)}{1+G_P(z)G_C(z)}\mathbf{G}_{0C}^{\mathrm{T}}(z)\\ \frac{1}{1+G_P(z)G_C(z)}\mathbf{G}_{0P}^{\mathrm{T}}(z)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\mathbf{I}(z)\\ \mathbf{c}_{0C}\\ \mathbf{c}_{0P}\end{bmatrix}= \tag{11}$$

$$=\begin{bmatrix}\mathbf{G}(z) & \mathbf{G}_0(z)\end{bmatrix}\begin{bmatrix}\mathbf{I}(z)\\ \mathbf{c}_0\end{bmatrix},\mathbf{I}(z)=\begin{bmatrix}R(z)\\ D(z)\end{bmatrix},$$

$$E(z)=R(z)-Y(z)=\begin{bmatrix}\frac{1}{1+G_P(z)G_C(z)}\\ -\frac{G_P(z)}{1+G_P(z)G_C(z)}\\ -\frac{G_P(z)}{1+G_P(z)G_C(z)}\mathbf{G}_{0C}^{\mathrm{T}}(z)\\ -\frac{1}{1+G_P(z)G_C(z)}\mathbf{G}_{0P}^{\mathrm{T}}(z)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\mathbf{I}(z)\\ \mathbf{c}_{0C}\\ \mathbf{c}_{0P}\end{bmatrix}. \tag{12}$$

From (11) the equivalent full block diagram of the system follows, as shown in Fig. 2.

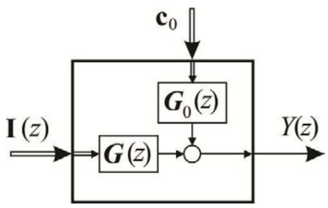

Fig. 2 Equivalent full block diagram of the system

4. Simulation example

In this section, a mathematical example illustrates the presented results.

Let us consider a plant:

$$\mathbf{A}_P^{(1)}\mathbf{y}^1=\mathbf{B}_P^{(1)}\mathbf{u}_P^1,\mathbf{A}_P^{(1)}=\begin{bmatrix}-1.5 & 1\end{bmatrix},\mathbf{B}_P^{(1)}=\begin{bmatrix}1 & 0.5\end{bmatrix},$$
$$\mathbf{y}^1=\begin{bmatrix}y_P(k) & y_P(k+1)\end{bmatrix}^{\mathrm{T}},\mathbf{u}_P^1=\begin{bmatrix}u_P(k) & u_P(k+1)\end{bmatrix}^{\mathrm{T}}. \tag{13}$$

The plant is controlled by a first order PDS controller:

$$\begin{gathered}\mathbf{A}_C^{(2)}\mathbf{u}^2=\mathbf{B}_C^{(2)}\boldsymbol{\varepsilon}^2, T_1=1, T=0.01\,\text{sec}.\\ \mathbf{A}_C^{(2)}=\begin{bmatrix}\frac{1-0.01}{0.01} & \frac{T-2T_1}{T} & \frac{T_1}{T}\end{bmatrix}=\begin{bmatrix}99 & -199 & 100\end{bmatrix},\\ \mathbf{B}_C^{(1)}=\begin{bmatrix}\frac{K_D}{T}+K_ST-K & K-\frac{2K_D}{T} & \frac{K_D}{T}\end{bmatrix}=\\ =\begin{bmatrix}100K_D+0.01K_S-K & K-200K_D & 100K_D\end{bmatrix}.\end{gathered} \tag{14}$$

The overall closed loop system is described in the compact form by:

$$\mathbf{A}^{(3)}(\alpha,\beta,\gamma)\mathbf{y}^3=\mathbf{B}^{(3)}(\alpha,\beta,\gamma)\mathbf{i}^3, \forall k\in\mathbb{N}_0, \alpha=K, \beta=K_D, \gamma=K_S,$$

$$\mathbf{A}^{(3)}(\alpha,\beta,\gamma)=\begin{bmatrix}a_0(\alpha,\beta,\gamma) & a_1(\alpha,\beta,\gamma) & a_2(\alpha,\beta,\gamma) & a_3(\alpha,\beta,\gamma)\end{bmatrix}=$$
$$=\begin{bmatrix}\begin{matrix}-0.01\alpha+\beta+\\ +0.0001\gamma-\\ -1.485\end{matrix} & \begin{matrix}0.005\alpha-1.5\beta+\\ +0.00005\gamma+\\ +3.975\end{matrix} & 0.005\alpha-3.49 & 1+0.5\beta\end{bmatrix},$$

$$\mathbf{B}^{(3)}(\alpha,\beta,\gamma)=\begin{bmatrix}\mathbf{B}_0^{\mathrm{T}}(\alpha,\beta,\gamma)\\ \mathbf{B}_1^{\mathrm{T}}(\alpha,\beta,\gamma)\\ \mathbf{B}_2^{\mathrm{T}}(\alpha,\beta,\gamma)\\ \mathbf{B}_3^{\mathrm{T}}(\alpha,\beta,\gamma)\end{bmatrix}^{\mathrm{T}}=\begin{bmatrix}\begin{bmatrix}-0.01\alpha+\beta+10^{-4}\gamma & 0.99\end{bmatrix}^{\mathrm{T}}\\ \begin{bmatrix}0.005\alpha-1.5\beta+0.00005\gamma & -1.495\end{bmatrix}^{\mathrm{T}}\\ \begin{bmatrix}0.005\alpha & 0.005\end{bmatrix}^{\mathrm{T}}\\ \begin{bmatrix}0.5\beta & 0.5\end{bmatrix}^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}},$$

$$\mathbf{y}^3=\begin{bmatrix}y(k) & y(k+1) & y(k+2) & y(k+3)\end{bmatrix}^{\mathrm{T}},$$
$$\mathbf{i}^3=\begin{bmatrix}\mathbf{i}^{\mathrm{T}}(k) & \mathbf{i}^{\mathrm{T}}(k+1) & \mathbf{i}^{\mathrm{T}}(k+2) & \mathbf{i}^{\mathrm{T}}(k+3)\end{bmatrix}^{\mathrm{T}}, \mathbf{i}=\begin{bmatrix}r & d\end{bmatrix}^{\mathrm{T}}.$$

4.1 Full transfer function matrix

$$\mathbf{F}(z)=\mathbf{F}_D^{-1}(z)\mathbf{F}_N(z)=\left[\mathbf{A}^{(3)}(\alpha,\beta,\gamma)\mathbf{S}_1^{(3)}(z)\right]^{-1}\cdot$$
$$\cdot\left[\mathbf{B}^{(3)}(\alpha,\beta,\gamma)\mathbf{S}_2^{(3)}(z) \quad -\mathbf{B}^{(3)}(\alpha,\beta,\gamma)\mathbf{Z}_2^{(3)}(z) \quad \mathbf{A}^{(3)}(\alpha,\beta,\gamma)\mathbf{Z}_1^{(3)}(z)\right],$$

$$\mathbf{F}(z)=\frac{1}{\begin{pmatrix}(1+0.5\beta)z^3+\\ +\\ +(0.005\alpha-3.49)z^2\end{pmatrix}+\begin{pmatrix}(0.005\alpha-1.5\beta+\\ +0.00005\gamma+\\ +3.975)z\end{pmatrix}+\begin{pmatrix}-0.01\alpha+\beta+\\ +0.0001\gamma-\\ -1.485\end{pmatrix}}\cdot$$

$$\cdot\begin{bmatrix} 0.5\beta z^3+0.005\alpha z^2+\left(0.005\alpha-1.5\beta+0.00005\gamma\right)z+ \\ +\left(-0.01\alpha+\beta+10^{-4}\gamma\right) \\ 0.5z^3+0.005z^2-1.495z+0.99 \\ -0.5\beta z^3-0.005\alpha z^2-\left(0.005\alpha-1.5\beta+0.00005\gamma\right)z \\ -0.5z^3-0.005z^2+1.495z \\ -0.5\beta z^2-0.005\alpha z \\ -0.5z^2-0.005z \\ -0.5\beta z \\ -0.5z \\ \left(1+0.5\beta\right)z^3+\left(0.005\alpha-3.49\right)z^2+\left(0.005\alpha-1.5\beta+0.00005\gamma+3.975\right)z \\ \left(1+0.5\beta\right)z^2+\left(0.005\alpha-3.49\right)z \\ \left(1+0.5\beta\right)z \end{bmatrix}^T.$$

4.2 Relative stability

The full transfer function matrix is row nondegenerate, which implies that the system characteristic equation reads as follows:

$$\begin{gathered}\left(0.5\beta+1\right)z^3+\left(0.005\alpha-3.49\right)z^2+\left(0.005\alpha-1.5\beta+0.00005\gamma+3.975\right)z+ \\ +\left(-0.01\alpha+\beta+0.0001\gamma-1.485\right)= \\ =\overline{a}_3\left(\alpha,\beta,\gamma\right)z^3+\overline{a}_2\left(\alpha,\beta,\gamma\right)z^2+\overline{a}_1\left(\alpha,\beta,\gamma\right)z+\overline{a}_0\left(\alpha,\beta,\gamma\right)=0, \\ \overline{a}_j\left(\alpha,\beta,\gamma\right)=\overline{b}_j\alpha+\overline{c}_j\beta+\overline{d}_j\gamma+\overline{e}_j,\ j=0,1,2,3.\end{gathered}$$

In order to get loci in the $\alpha\beta\gamma$ parameter space of the constant damping coefficient, we choose the constant value of ζ, $0\leq\zeta=const.\leq 1$, change the values of the natural frequency ω_n with a numerical step, $\omega_n>0$, and use expressions (9) where:

$$\begin{gathered}\overline{B}_1\left(\omega_n,\zeta\right)=-0.01+0.005e^{-0.01\omega_n\zeta}\cos\left(0.01\omega_n\sqrt{1-\zeta^2}\right)+ \\ +0.005e^{-0.02\omega_n\zeta}\cos\left(0.02\omega_n\sqrt{1-\zeta^2}\right), \\ \overline{B}_2\left(\omega_n,\zeta\right)=0.005e^{-0.01\omega_n\zeta}\sin\left(0.01\omega_n\sqrt{1-\zeta^2}\right)+0.005e^{-0.02\omega_n\zeta}\sin\left(0.02\omega_n\sqrt{1-\zeta^2}\right) \\ \overline{C}_1\left(\omega_n,\zeta\right)=1-1.5e^{-0.01\omega_n\zeta}\cos\left(0.01\omega_n\sqrt{1-\zeta^2}\right)+0.5e^{-0.03\omega_n\zeta}\cos\left(0.03\omega_n\sqrt{1-\zeta^2}\right) \qquad (15a) \\ \overline{C}_2\left(\omega_n,\zeta\right)=-1.5e^{-0.01\omega_n\zeta}\sin\left(0.01\omega_n\sqrt{1-\zeta^2}\right)+0.5e^{-0.03\omega_n\zeta}\sin\left(0.03\omega_n\sqrt{1-\zeta^2}\right), \\ \overline{D}_1\left(\omega_n,\zeta\right)=0.0001+0.00005e^{-0.01\omega_n\zeta}\cos\left(0.01\omega_n\sqrt{1-\zeta^2}\right), \\ \overline{D}_2\left(\omega_n,\zeta\right)=0.00005e^{-0.01\omega_n\zeta}\sin\left(0.01\omega_n\sqrt{1-\zeta^2}\right),\end{gathered}$$

$$\begin{aligned}\bar{E}_1(\omega_n,\zeta) &= -1.485+3.975e^{-0.01\omega_n\zeta}\cos\left(0.01\omega_n\sqrt{1-\zeta^2}\right)- \\ &-3.49e^{-0.02\omega_n\zeta}\cos\left(0.02\omega_n\sqrt{1-\zeta^2}\right)+e^{-0.03\omega_n\zeta}\cos\left(0.03\omega_n\sqrt{1-\zeta^2}\right), \\ \bar{E}_2(\omega_n,\zeta) &= 3.975e^{-0.01\omega_n\zeta}\sin\left(0.01\omega_n\sqrt{1-\zeta^2}\right)-3.49e^{-0.02\omega_n\zeta}\sin\left(0.02\omega_n\sqrt{1-\zeta^2}\right)+ \\ &+e^{-0.03\omega_n\zeta}\sin\left(0.03\omega_n\sqrt{1-\zeta^2}\right).\end{aligned} \tag{15b}$$

4.3 Performance index

Plant Equation (13) leads to:

$$Y(z)=\begin{bmatrix}\frac{(0.5z+1)}{(z-1.5)} & \frac{-0.5z}{(z-1.5)} & \frac{z}{(z-1.5)}\end{bmatrix}\begin{bmatrix}U_P(z)\\ u_P(0)\\ y(0)\end{bmatrix}. \tag{16}$$

Controller Equation (14) yields:

$$U(z)=\begin{bmatrix}\frac{0.01(z-1)\alpha+(z-1)^2\beta+10^{-4}\gamma}{\left(z^2-1.99z+0.99\right)}\\ \frac{-0.01z\alpha-(z-2)z\beta}{z^2-1.99z+0.99}\\ \frac{-\beta z}{z^2-1.99z+0.99}\\ \frac{(z-1.99)z}{z^2-1.99z+0.99}\\ \frac{z}{z^2-1.99z+0.99}\end{bmatrix}^{\mathrm{T}}\cdot\begin{bmatrix}E(z)\\ \varepsilon(0)\\ \varepsilon(1)\\ u(0)\\ u(1)\end{bmatrix}. \tag{17}$$

Using Equation (12) and Expressions (16), (17) we obtain:

$$E(z)=\begin{bmatrix}\frac{\left(z^2-1.99z+0.99\right)(z-1.5)}{\left(z^2-1.99z+0.99\right)(z-1.5)+(0.5z+1)\left[0.01(z-1)\alpha+(z-1)^2\beta+10^{-4}\gamma\right]}\\ -\frac{(0.5z+1)\left(z^2-1.99z+0.99\right)}{\left(z^2-1.99z+0.99\right)(z-1.5)+(0.5z+1)\left[0.01(z-1)\alpha+(z-1)^2\beta+10^{-4}\gamma\right]}\\ \frac{\left[0.01z\alpha+(z-2)z\beta\right](0.5z+1)}{\left(z^2-1.99z+0.99\right)(z-1.5)+(0.5z+1)\left[0.01(z-1)\alpha+(z-1)^2\beta+10^{-4}\gamma\right]}\\ \frac{z(0.5z+1)\beta}{\left(z^2-1.99z+0.99\right)(z-1.5)+(0.5z+1)\left[0.01(z-1)\alpha+(z-1)^2\beta+10^{-4}\gamma\right]}\\ \frac{-(0.5z+1)(z-1.99)z}{\left(z^2-1.99z+0.99\right)(z-1.5)+(0.5z+1)\left[0.01(z-1)\alpha+(z-1)^2\beta+10^{-4}\gamma\right]}\\ \frac{-z(0.5z+1)}{\left(z^2-1.99z+0.99\right)(z-1.5)+(0.5z+1)\left[0.01(z-1)\alpha+(z-1)^2\beta+10^{-4}\gamma\right]}\\ \frac{0.5z\left(z^2-1.99z+0.99\right)}{\left(z^2-1.99z+0.99\right)(z-1.5)+(0.5z+1)\left[0.01(z-1)\alpha+(z-1)^2\beta+10^{-4}\gamma\right]}\\ \frac{-z\left(z^2-1.99z+0.99\right)}{\left(z^2-1.99z+0.99\right)(z-1.5)+(0.5z+1)\left[0.01(z-1)\alpha+(z-1)^2\beta+10^{-4}\gamma\right]}\end{bmatrix}^{\mathrm{T}}\cdot\begin{bmatrix}R(z)\\ D(z)\\ r(0)-y(0)\\ r(1)-y(1)\\ u(0)\\ u(1)\\ u(0)+d(0)\\ y(0)\end{bmatrix}. \tag{18}$$

By means of Expressions (10) and (18) the performance index values were calculated for 29,700 points, using the initial conditions: $u(0)=0$, $u(1)=0.2$, $y(0)=2$, $y(1)=1.9$, $r(0)=1$, $r(1)=1$, $d(0)=0$, but only the minimal values were applied to the constant damping coefficient surface, as shown in Fig. 3.

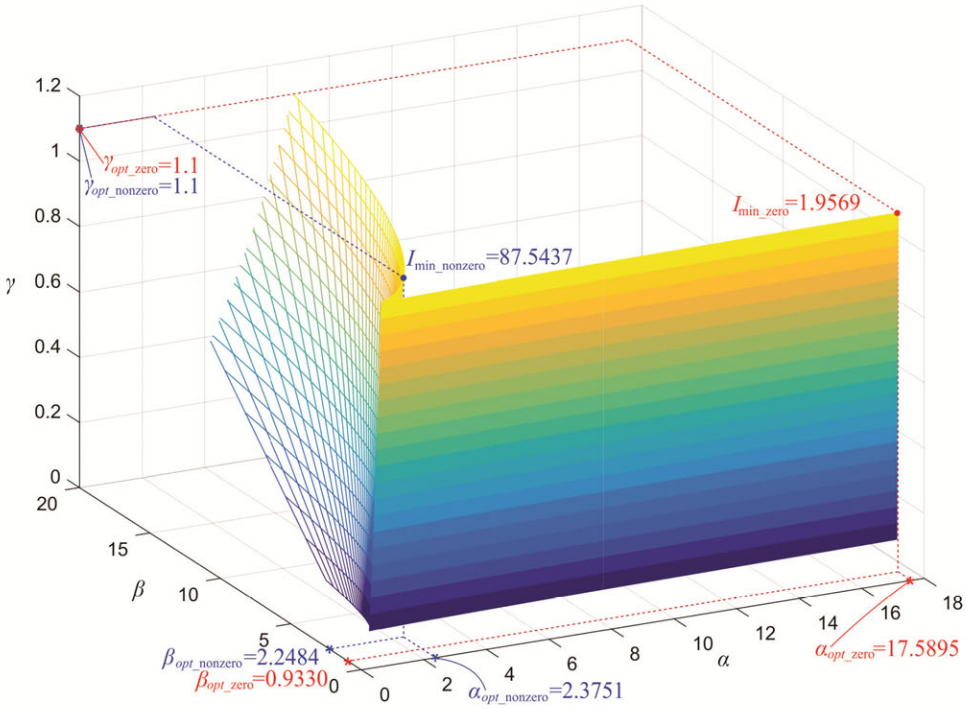

Fig. 3 Constant damping coefficient surface with the minimal performance index values applied to it for both cases, zero and non-zero initial conditions

The set of the optimal values of parameters α, β, γ, which gives the optimal minimal value of the performance index $I_{\min\_nonzero}=87.5437$ is as follows: $\alpha_{\text{opt}\_nonzero}=2.3751$, $\beta_{opt\_\text{nonzero}}=2.2484$, $\gamma_{opt\_\text{nonzero}}=1.1$. In order to indicate a difference between the novel approach and the classical one, the performance index values were also calculated using all zero initial conditions and for 29,700 points.

4.4 Discussion of the simulation results

The question is what kind of comparison between the classical conditional optimisation theory and the developed and proposed new optimisation theory would make sense. The developed and proposed new conditional optimisation theory implies non-zero initial conditions. Optimal values of adjustable unknown parameters are obtained assuming the initial conditions are non-zero and they are dependent on initial conditions. The system unit step response marked blue in Fig. 4 is obtained by the new theory optimal parameters which are valid only for the concrete initial conditions used. The classical conditional optimisation theory implies zero initial conditions. Optimal values of adjustable unknown parameters are obtained assuming the initial conditions are zero and they are treated as universally optimal for any working regime and any initial conditions. However, if the system unit step response obtained by the classical theory optimal parameters (marked green (*c*) in Fig. 4) starts from zero initial conditions, the response is quite correct because these are initial conditions in relation to which the optimal parameters are designed.

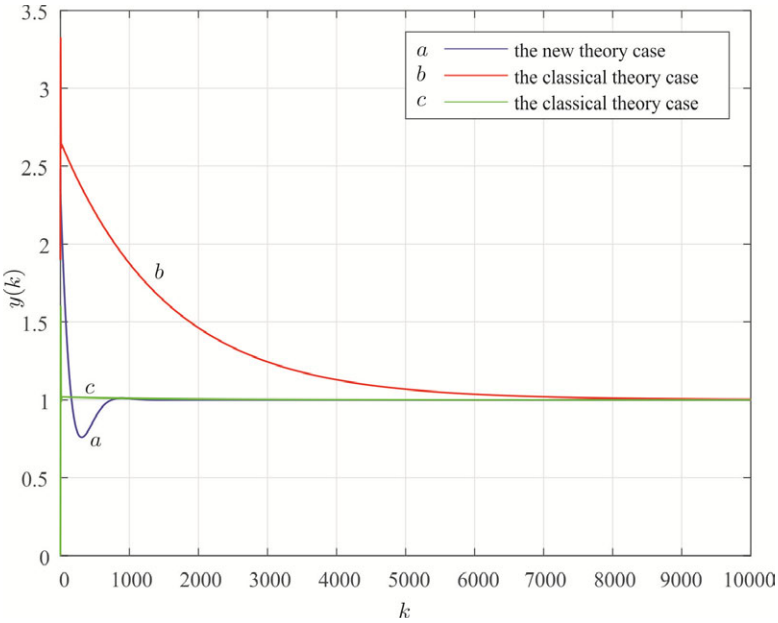

Fig. 4 System unit step responses for cases of non-zero (new theory) and zero (classical theory) initial conditions

An enlarged segment of Fig. 4 for $0 \le k \le 100$ is shown in Fig. 5.

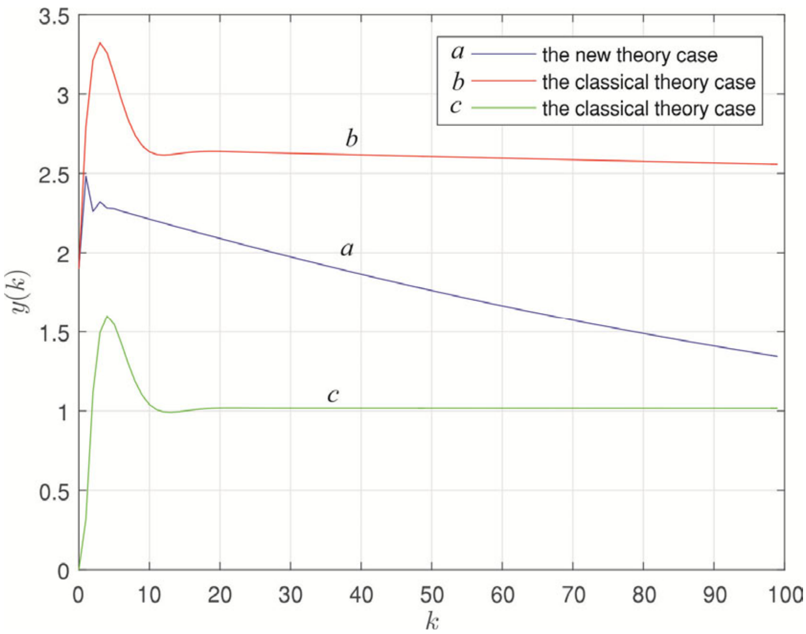

Fig. 5 Augmented part of Fig. 4

5. Experimental example

The proposed design procedure has been applied and tested experimentally on a DC servo motor with a gearbox and load. For the case that the system can be accurately modelled without considering the major nonlinear effects (speed dependent friction, dead zone and backlash), a linear model of the DC motor can be given as:

$$J\ddot{\theta}(t) + B\dot{\theta}(t) = \frac{\eta_g \eta_m k_t K_g}{R_m} U(t). \tag{19}$$

For the nominal plant parameters J=0.0021 kgm^2, B=0.084 Nms/rad, R_m=2.6 Ω, k_t=0.0077 Nm/A, η_m=0.69, η_g=0.9 and K_g=70, choosing $y=\theta$ and $u_P=U$, a discrete-time IO model of the system for the sampling time T=0.01 sec is:

$$\mathbf{A}_P^{(2)}\mathbf{y}^2=\mathbf{B}_P^{(1)}\mathbf{u}_P^1,\mathbf{A}_P^{(2)}=\begin{bmatrix}0.6746 & -1.6746 & 1\end{bmatrix},\mathbf{B}_P^{(1)}=\begin{bmatrix}0.00232852 & 0.002653\end{bmatrix},$$
$$\mathbf{y}^2=\begin{bmatrix}y(k) & y(k+1) & y(k+2)\end{bmatrix}^{\mathrm{T}},\mathbf{u}_P^1=\begin{bmatrix}u_P(k) & u_P(k+1)\end{bmatrix}^{\mathrm{T}}. \quad (20)$$

Let the plant (20) be controlled by a zero order PS controller:

$$\mathbf{A}_C^{(1)}\mathbf{u}^1=\mathbf{B}_C^{(1)}\boldsymbol{\varepsilon}^1,$$
$$\mathbf{A}_C^{(1)}=\begin{bmatrix}-1 & 1\end{bmatrix},\mathbf{B}_C^{(1)}=\begin{bmatrix}K_ST-K & K\end{bmatrix}=\begin{bmatrix}0.01K_S-K & K\end{bmatrix},T=0.01\text{sec}. \quad (21)$$

The overall closed loop system is described in the compact form by:

$$\mathbf{A}^{(3)}(\alpha,\beta)\mathbf{y}^3(k)=\mathbf{B}^{(2)}(\alpha,\beta)\mathbf{i}^2(k),\forall k\in\mathbb{N}_0,\alpha=K,\beta=K_S,$$
$$\mathbf{A}^{(3)}(\alpha,\beta)=\begin{bmatrix}a_0(\alpha,\beta) & a_1(\alpha,\beta) & a_2(\alpha,\beta) & a_3(\alpha,\beta)\end{bmatrix}=$$
$$=\begin{bmatrix}\begin{matrix}-0.00232852\alpha+\\+0.00232852\cdot10^{-2}\beta-\\-0.6746\end{matrix} & \begin{matrix}-3.2448\cdot10^{-4}\alpha+\\+2.653\cdot10^{-5}\beta+\\+2.3492\end{matrix} & \begin{matrix}0.002653\alpha-\\-2.6746\end{matrix} & 1\end{bmatrix},$$
$$\mathbf{B}^{(2)}(\alpha,\beta)=\begin{bmatrix}\mathbf{B}_0(\alpha,\beta) & \mathbf{B}_1(\alpha,\beta) & \mathbf{B}_2(\alpha,\beta)\end{bmatrix}=$$
$$=\begin{bmatrix}\begin{bmatrix}\begin{matrix}-2.3285\cdot10^{-3}\alpha+\\+2.3285\cdot10^{-5}\beta\end{matrix} & -0.00232852\end{bmatrix}^{\mathrm{T}}\\ \begin{bmatrix}\begin{matrix}-3.2448\cdot10^{-4}\alpha+\\+2.653\cdot10^{-5}\beta\end{matrix} & -3.2448\cdot10^{-4}\end{bmatrix}^{\mathrm{T}}\\ \begin{bmatrix}0.002653\alpha & 0.002653\end{bmatrix}^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}},$$
$$\mathbf{y}^3(k)=\begin{bmatrix}y(k) & y(k+1) & y(k+2) & y(k+3)\end{bmatrix}^{\mathrm{T}},$$
$$\mathbf{i}^2(k)=\begin{bmatrix}\mathbf{i}^{\mathrm{T}}(k) & \mathbf{i}^{\mathrm{T}}(k+1) & \mathbf{i}^{\mathrm{T}}(k+2)\end{bmatrix}^{\mathrm{T}},\mathbf{i}=\begin{bmatrix}r & d\end{bmatrix}^{\mathrm{T}}.$$

The full transfer function matrix is obtained as in the simulation example (section 4.1) and the system characteristic polynomial is:

$$z^3+(0.002653\alpha-2.6746)z^2+\left(-3.2448\cdot10^{-4}\alpha+2.653\cdot10^{-5}\beta+2.3492\right)z+$$
$$+\left(-0.00232852\alpha+0.00232852\cdot10^{-2}\beta-0.6746\right).$$

In order to obtain the loci in the $\alpha\beta$ parameter plane of the constant damping coefficient, we repeat the same procedure as in the simulation example (section 4.2).

5.1 Performance index

As steady state error ε_s of the system is equal to zero, $\varepsilon_s=0$, for the control algorithm that contains the S (sum) action, we use expression (10) to determine the performance index. Plant Equation (20) leads to:

$$Y(z)=\begin{bmatrix}\frac{0.002653z+0.00232852}{z^2-1.6746z+0.6746} & \frac{-0.002653z}{z^2-1.6746z+0.6746} & \frac{z^2-1.6746z}{z^2-1.6746z+0.6746} & \frac{z}{z^2-1.6746z+0.6746}\end{bmatrix}\cdot \begin{bmatrix}U_P(z)\\ u_P(0)\\ y(0)\\ y(1)\end{bmatrix}. \tag{22}$$

Controller Equation (21) yields:

$$U(z)=\begin{bmatrix}\frac{\alpha(z-1)+0.01\beta}{z-1} & \frac{-\alpha z}{z-1} & \frac{z}{z-1}\end{bmatrix}\begin{bmatrix}E(z)\\ \varepsilon(0)\\ u(0)\end{bmatrix}. \tag{23}$$

Using Equation (12) and Expressions (22), (23), we obtain Expression for $E(z)$ in the same way as in the simulation example. Using the non-zero initial conditions: $u(0)=0.1$, $y(0)=0.2$, $y(1)=0.226$, $r(0)=0$, the performance index values were calculated for 2,033 points, but only some of them were applied to the constant damping coefficient curve, as shown in Fig. 6.

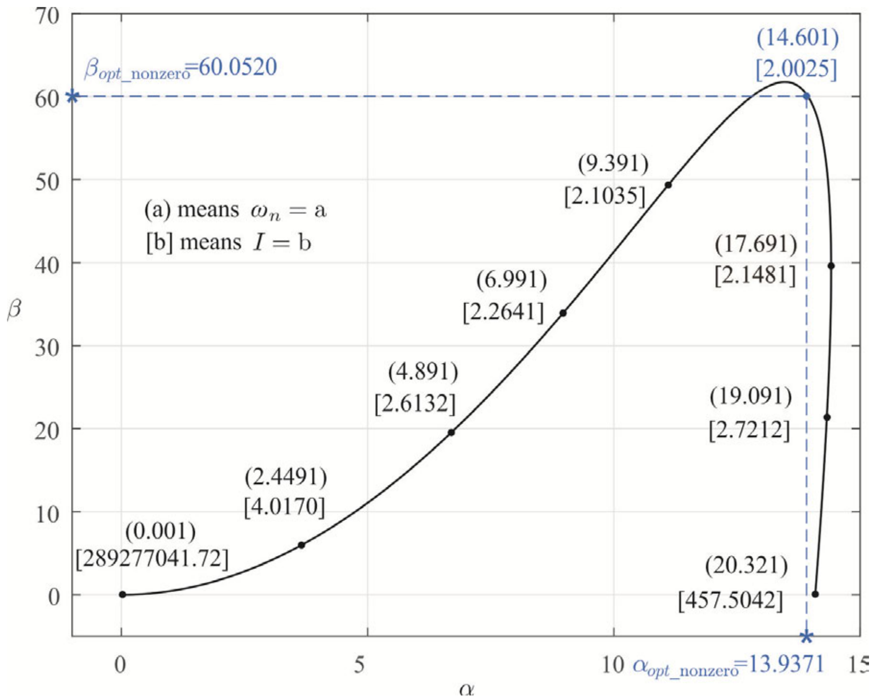

Fig. 6 Constant damping coefficient curve with some performance index values applied to it for non-zero initial conditions (new theory case)

The sample of the optimal values of parameters α, β, which gives the optimal minimal value of the performance index $I_{min\_nonzero}=2.0025$, is as follows: $\alpha_{opt\_nonzero}=13.9371$, $\beta_{opt\_nonzero}=60.0520$. To highlight the difference between the new and the classical approach, the performance index values were also calculated using the zero initial conditions and for 2,033 points as shown in Fig. 7.

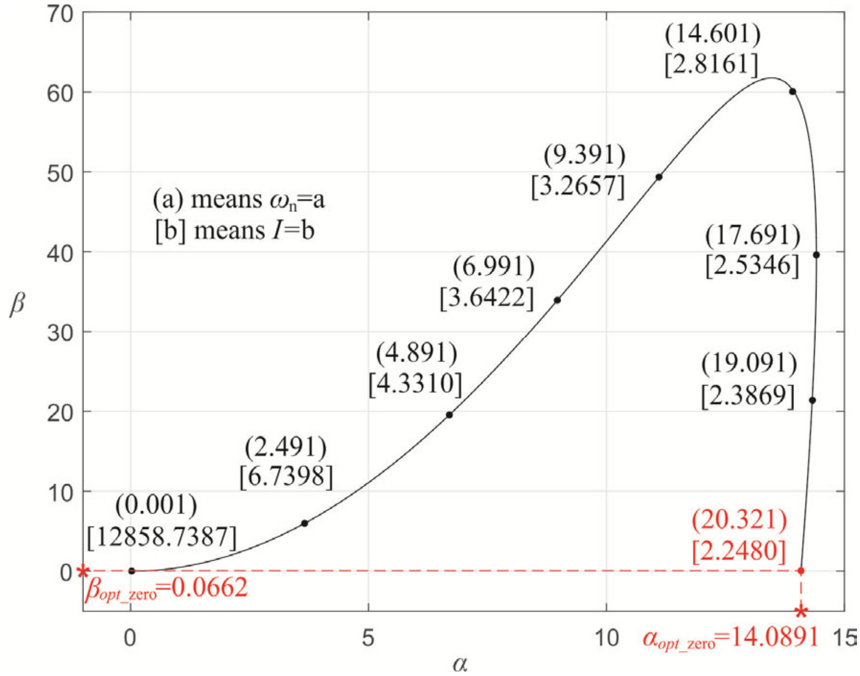

Fig. 7 Constant damping coefficient curve with some performance index values applied to it for zero initial conditions (classical theory case)

5.2 Discussion of the simulation and experimental results

The system simulated 0.6 step responses for both cases (non-zero and zero initial conditions, that is, for the new theory and for the classical theory, respectively) are shown in Fig. 8.

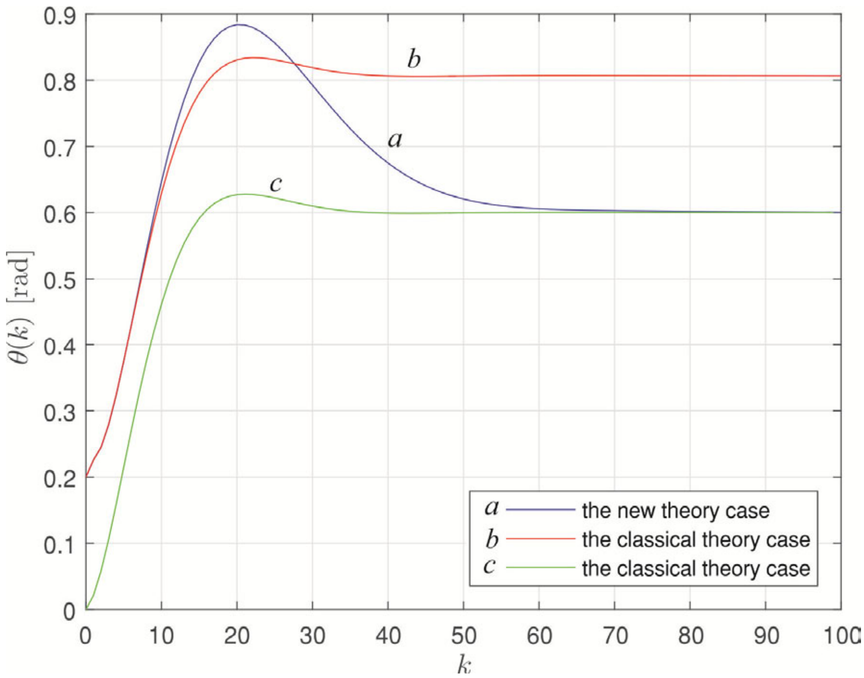

Fig. 8 Simulated 0.6 step responses with new theory optimal parameters (blue - starting from non-zero initial conditions) and classical theory optimal parameters (red - starting from non-zero initial conditions and green - starting from zero initial conditions)

The system experimental 0.6 step responses for the cases of non zero (new theory) and zero (classical theory) initial conditions are shown in Fig. 9.

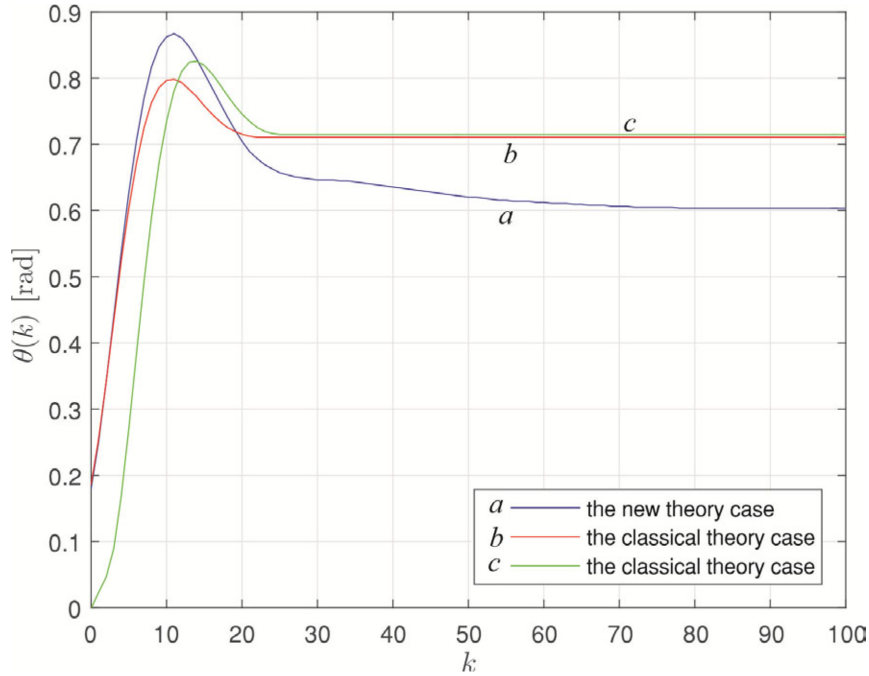

Fig. 9 Experimental 0.6 step responses with new theory optimal parameters (blue - starting from non-zero initial conditions) and classical theory optimal parameters (red - starting from non-zero initial conditions and green - starting from zero initial conditions)

The "green" response is similar to the "red" one, which is not good enough, however, this is not due to concepts of the conditional optimisation (new and classical). It is the consequence of the following: the simulations were carried out on the basis of the linear model, while the experiment was carried out, of course, on the system itself, which is non-linear in nature; the nonlinear effects (static + Coulomb friction) are mostly present when the system starts from the zero initial conditions, and the energy of control is still small and it takes time for this energy to accumulate on the basis of the S control algorithm action.

6. Conclusion

In the paper, the new conditional optimisation procedure for linear discrete-time systems is developed and introduced, which is the only adequate and appropriate procedure compared to the classical one. The optimisation is made in the parameter space of three unknown and adjustable parameters, unlike the classical optimisation which was done in the parameter plane of two parameters. The procedure is based on the solution [27, 29] of the major controversies between the classical transfer function matrix and the system stability investigation using this matrix. This controversy has been recently solved by developing and introducing the full transfer function matrix [27, 29] so that during the system conditional optimisation the characteristic polynomial of the full transfer function matrix is used and not of the classical transfer function. More precisely, it is the characteristic polynomial of the so-called row nondegenerate full transfer function matrix, the same one as used for the only adequate and appropriate testing of the system stability. Also, a new, compact calculus [27, 29] is used, without which determining the full transfer function matrix is impossible.

Fully compatible with the use of the full transfer function matrix, a new form of the performance index is introduced, the sum of squared errors occurred in the most general and realistic situation under all actions performed on the system at the same time, the non-zero external inputs and non-zero initial conditions.

Illustrative examples are given to show the difference in the system behaviour, when it is designed in the classical and the proposed novel way. The differences are detected by carrying out simulations and a practical experiment. Each time when the system is designed in the classical way and starts from non-zero initial conditions, its response is much worse than when the system is designed in a new way and starts from non-zero initial conditions too, or when the system is designed in the classical way and starts from zero initial conditions.

The answer to the question posed at the beginning is the following: the set of the controller parameters optimal for the control system behaviour under all zero initial conditions is not optimal for its operation under non-zero initial conditions. Figures 3, 6 and 7 illustrate this concluding statement.

Considering the solution to the problem just for discrete-time systems makes place for the application of a high quality microprocessor compensator in the systems.

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