

EFFICIENCY ANALYSIS OF PLANETARY GEARS

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Abstract: By kinematic combinations of toothed pairs with external and internal contacts, we can obtain planetary gears with a considerably improved performance than the corresponding ones with fixed axes, as well as planetary gears with notably poor performance regarding the efficiency. In regard to that, the reference literature and papers almost regularly emphasize that planetary gears, under the same technical conditions, have a smaller mass and a higher degree of efficiency than the ones with fixed axes. The main aim of this paper is to examine the above statement and to determine the scope of the gear ratios in which the planetary gears are more suitable than the fixed axes gears.

Key words: planetary gear, efficiency, gear ratio, central gear, satellite

1. INTRODUCTION

Gear trains in operation are characterized by power losses which can be attributed to the gear-mesh and bearing contact frictional effects as a consequence of friction between the contact surfaces of the meshing teeth and the friction in the bearings. The gear power loss, expressed by means of the efficiency, is inevitable during power transformation, and depend upon the type of gear train, the bearing, manufacturing precision, loading, lubrication, etc. Nowadays it is essential to effectively predict power losses during the design stage of the gear box, which makes it possible to make right design improvements prior to gearbox manufacturing and testing [6].

Planetary gearboxes are widely used in various mechanisms and machines such as industrial drives, rotorcraft, automobiles, wind turbines, etc., where they can offer higher gear ratios and higher power densities with less noise and higher torque-to-weight ratios, especially compared to ones with fixed axes [4]. However, the construction of such gearboxes involves multiple planet branches which also reduces efficiency, where the multiple gear mesh and bearing contact regions in planetary gear sets dictate the overall efficiency.

Great number of papers focuses on the gear efficiency and power loss which are related to considerations of efficiency of parallel axis gear systems [8, 1], where the majority of studies focuses on the mechanical power loss. Analysis of the gear power loss in parallel axis gear systems is typically studied separately from the bearing losses [2]. Papers considering the power losses of planetary systems are sparse and mainly focused on power loss in planetary gear sets which have generally been experimental. Typically, such models study efficiency through gear train kinematics, employing speed and torque equations to analyse planetary gearbox efficiency [5]. However, these studies fail to consider the power loss due to lubrication interactions, under applied non-uniform gear normal loads.

The primary objective of this paper is to investigate mechanical power loss in certain variant constructions of

single stage and two-stage planetary gear systems, for which efficiencies, as well as the range of practical applications in which application of planetary gears is more suitable than gears with fixed axes is determined.

2. DETERMINATION OF INSTANTANEOUS EFFICIENCY

In most of the papers dealing with the efficiency of geared system, some kind of average coefficient of friction representing the frictional phenomena during the gear engagement was used. In this paper, however, the coefficient of friction is treated as a variable factor during the engagement cycle. It was also assumed that the frictional losses in gearing are not only due to the relative sliding velocity of the two surfaces, but also to the combination of rolling and sliding motion of tooth surfaces. The instantaneous efficiency is determined according to the expression:

$$\eta_i = \frac{T_2}{T_1} \cdot \frac{1}{i} \quad (1)$$

where T_1 is external torque acting on the driving gear, Nm, T_2 is external torque acting on the driven gear, Nm, and i is the gear ratio.

The overall efficiency for gearing under consideration is determined according to:

$$\eta_a = \frac{1}{l} \int_{A1}^{E1} \eta_i dx \quad (2)$$

where l represents active length.

An iterative procedure for determination of the instantaneous efficiency of a gear pair for both external and internal gearings is given in detail in [7]. Based upon the developed models, computer programs for instantaneous efficiency determination were devised. The computer numerical results for determination of the instantaneous efficiency of a gear pair with internal gearing are shown in figure 1.

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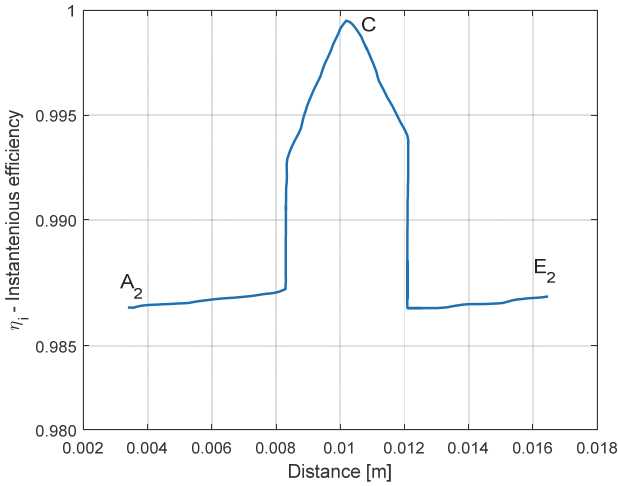


Fig.1. Variations of instantaneous of the efficiencies for the couple under consideration during the contact period

The intensity of friction is obtained from the following equation:

$$F_{\mu(x)} = \mu(x)F_n \quad (3)$$

where the coefficient of friction is determined according to:

$$\mu(x) = 0.0127 \cdot \log \left(\frac{29.66}{b} \frac{F_n}{\eta \cdot v_s \cdot v_R^2} \right) \quad (4)$$

and v_s is a sliding velocity, m/s, v_R is rolling velocity, m/s, and η is fluid dynamic viscosity, Ns/m².

The intensity of the rolling friction is determined according to:

$$F_R(x) = C \cdot h(x) \cdot b \quad (5)$$

where b is a width of gear, m, the equation for minimum film thickness is due to Dowson and Higginson [3] is

$$h(x) = 1.6 \cdot \alpha^{0.6} \cdot (\eta \cdot V_R)^{0.7} \cdot E^{0.003} \frac{R^{0.43}}{F_n^{0.13}} \quad (6)$$

and α is viscosity-pressure coefficient of lubricant, m²/N, R is the effective radius of curvature, m, and E is Young modulus of gear material, N/m². On the basis of the models developed for a gear pair with external and internal gearing, the efficiency of a planetary gear train can be determined.

3. THE SINGLE STAGE PLANETARY GEAR TRAIN

Before approaching the determination of the planetary gear train efficiency, it is, first of all, necessary to identify the driving and the driven members of the planetary gear. Specifically, for the given gear, the central gear is the driving, and the satellite is the driven member. The total power through the planetary gear train is carried partly by

conjugating the gear sets, and partly by coupling. The conjugating power, i.e., the relative power, is represented by the product of the torque and the relative angular velocity of the member under consideration:

$$P_{Ha} = T_a \cdot \omega_{ra} = T_a (\omega_a - \omega_H) \quad (7)$$

where T_a is torque acting on the pinion, Nm, ω_a is absolute angular velocity of the central gear, rad/s, ω_{ra} is relative angular velocity, rad/s, and ω_H is angular velocity of the satellite carrier, rad/s.

The coupling power can be defined as the product of the torque and the angular velocity of the transmission, i.e., the angular velocity of the satellite carrier:

$$P_{Sa} = T_a \cdot \omega_H \quad (8)$$

Therefore, the absolute power of the central gear of the planetary gear train equals to the sum of the conjugating power and the coupling power. For an analysis of the efficiency of a planetary gear train, it is necessary to know the ratio between the relative power and the absolute power of the central gear, which may be written in the following form:

$$\varphi_a = \frac{P_{Ha}}{P_a} = 1 - \frac{\omega_H}{\omega_a} = 1 - \frac{1}{U_{aH}^b} \quad (9)$$

Figure 2 represents the value of φ_a as function of the absolute gear ratio, which has the form of a hyperbole with the horizontal asymptote $\varphi_a = 1$.

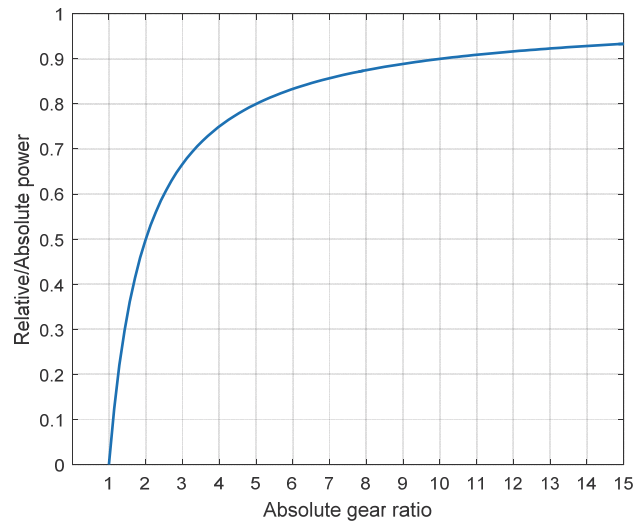


Fig.2. The ratio of the coupling and the absolute power in the functions of the absolute gear ratio

Based upon the results from figure 2, it can be concluded that value φ_a increases with the increase of the absolute gear ratio and asymptotically approaches the $\varphi_a = 1$. For the absolute gear ratios higher than 10, value φ_a ranges between 0.9 and 1, which means that the power is carried through the gear mainly by the conjugation of the gear sets. Thus, the practical values of the absolute gear ratios for the single stage planetary gears can be found in the range of $\varphi_a \in (0.5, 0.9)$.

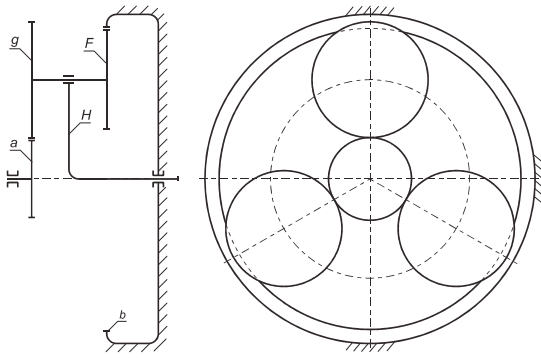


Fig.3. A kinematic sketch of a two-stage planetary gear train

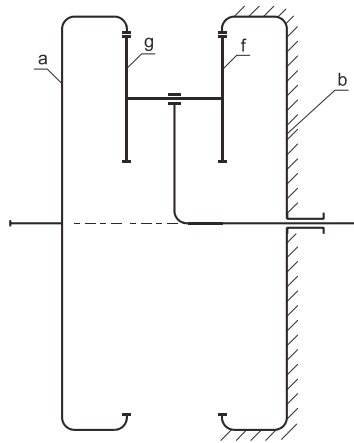


Fig.4. A kinematic sketch of a two-stage planetary gear train

4. THE TWO-STAGE PLANETARY GEAR TRAIN

To further analyze efficiency of planetary gearboxes, the kinematic representation of a two-stage planetary gearbox with one external and one internal contact is illustrated in figure 3. The absolute efficiency for this type of gear is determined according to the following expression:

$$\eta_{aH}^b = \frac{1 - u_{ab}^H \cdot \eta_{ab}^H}{1 - u_{ab}^H} \quad (10)$$

where the relative gear ratio is determined as:

$$u_{ab}^H = u_{ag}^H u_{fb}^H - \frac{z_g z_b}{z_a z_f} \quad (11)$$

and u_{ag}^H is relative first stage gear ratio (with fixed carrier), u_{fb}^H is relative second stage gear ratio, z_g, z_f are number of the corresponding teeth and $\eta_{ab}^H = \eta_{ag}^H \eta_{fb}^H$ is the relative efficiency.

Starting with the expression establishing the relationship between the absolute and the relative gear ratio:

$$u_{ab}^H = 1 - u_{aH}^b \quad (12)$$

and after substitution of the u_{ab}^H in the above equation, the expression for the absolute gear ratio is obtained:

$$u_{aH}^b = \frac{z_a \cdot z_f + z_g \cdot z_b}{z_a \cdot z_f} \quad (13)$$

The functional interdependence of the absolute efficiency upon the absolute gear ratio is shown in figure 5. In addition to that, the same graph shows the relative efficiency which is constant, and for the given system, it amounts to 0.9604. Based upon figure 5, it can be concluded that the absolute efficiency decreases with the increase of the gear ratio, as well as that it is higher than the efficiency of a two-stage gear with fixed axes. Keeping in mind the fact that the efficiency of the two-stage planetary gear decreases with the increase of the absolute gear ratio, it is necessary to consider the product of the efficiency and the gear ratio, which represents the relationship of the torques at the inlet and the outlet shafts of the planetary gear train.

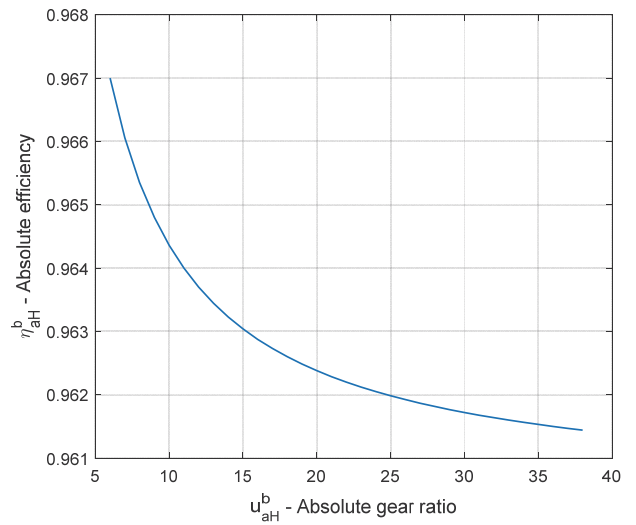


Fig.5. The graph showing the absolute efficiency

The next construction of the two-stage planetary gear, shown in figure 4, has been performed with two gears with the internal toothing. The satellite carrier, connected to the inlet shaft, the first central gear with the internal toothing is connected to the outlet shaft, while the second central gear with the internal contact is stationary.

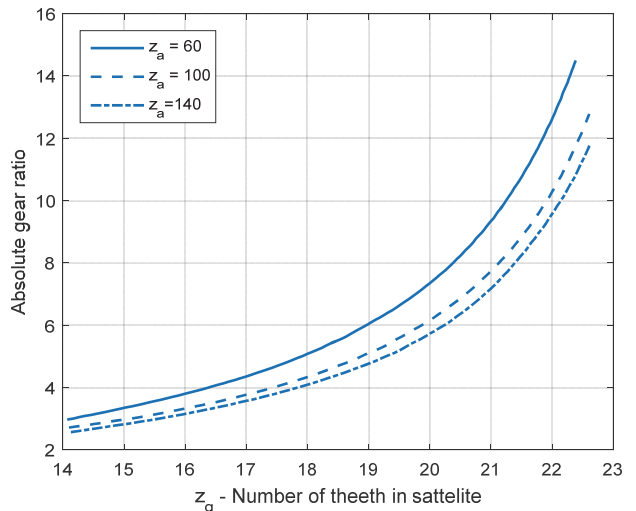


Fig.6. The absolute gear ratio as a function of the number of teeth in the satellite

Figure 6 and figure 7 show the absolute gear ratio and the efficiency of the planetary gear train as a function of the number of teeth of the satellite for definite values of the number of teeth of the central gear respectively.

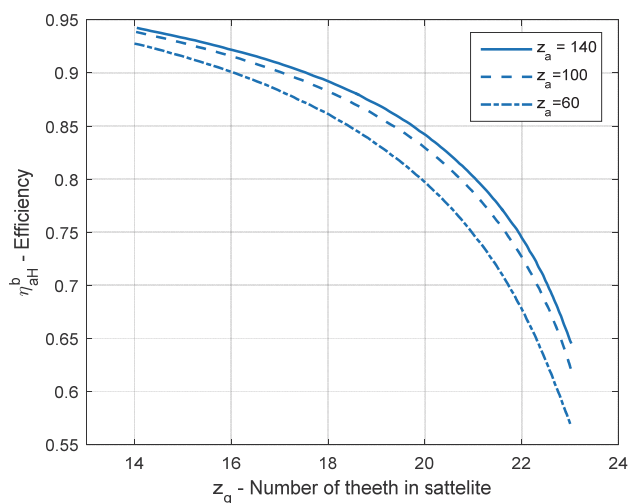


Fig. 7. The efficiency in the function of the number of teeth in the satellite

By this construction of the planetary gear relatively low values of the gear ratio, as well as of the efficiency of the gear are obtained. It should particularly be pointed out that the efficiency and the gear ratio are mutually opposite, which means that this gear also gives low values of the ratios of the corresponding torques. Therefore, it makes sense to use this type of the planetary gear only for the purpose of speed transformation.

5. CONCLUSION

In this paper mechanical power loss model, considering the power loss due to lubrication interactions, has been developed and applied to several common planetary gear constructions to examine the area and possibility of application of such planetary gear sets. The analysis presented in this paper focused on the relationship of the gear ratio with overall efficiency of certain variant constructions. In accordance with the analyses described above, it can be concluded that only those types of planetary gear trains should be used that give better performance as compared to the fixed axis gear trains. According to the results, the planetary gear trains with

one series of satellites are able to obtain satisfactory relation of efficiency and performance, especially when compared with fixed axis parallel-axis gear pairs with same performance. Such single stage planetary gear sets are able to achieve satisfactory efficiency for a large domain of gear ratio values. Furthermore, it has been shown that the two-stage planetary gear trains with one external and one internal toothing, when the central gear with the external toothing is mounted upon the inlet shaft, and the satellite carried connected to the outlet shaft at the stationary central internal gear are able to provide good efficiency.

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