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Free Planar Vibration of Structures Composed of Rigid Bodies and Elastic Beam Segments

This article presents free vibration analysis of structures composed of rigid bodies connected with elastic beam segments. It is assumed that the mass centers of rigid bodies are not located on the neutral axes of undeformed elastic beam segments as well as rigid bodies perform planar motion in the same plane and their mass centers are located in that plane. For determination of natural frequencies of the system, modification of the conventional continuous-mass transfer matrix method has been performed. The elastic beam segments are treated as Euler-Bernoulli beams. Numerical example is presented.

Keywords: *free vibration analysis, rigid bodies, elastic beams, conventional continous-mass transfer matrix method, Euler-Bernoule beams.*

1. INTRODUCTION

Many engineering structures can be modeled with a system of rigid bodies connected with elastic am segments, hence, free vibration analysis of these models of structure are of crucial importance. Many papers deal with vibration analysis of the system composed of a single rigid body and two elastic beam segments [1-3] as well as of the system of cantilever beam with a rigid body attached to its free end [4-6]. In [7] two dimensional structures composed of two-part elastic beam-rigid body elements are analyzed by using transfer matrix and direct approach. Vibration of hybrid elastic beam carrying several elastic-supported rigid bodies is analyzed in [8]. All above references consider that the mass centers of the rigid bodies are located on the neutral axis of elastic beams.

This paper presents the extension of the existing results of free vibration of structures of rigid bodies connected with elastic beam segments, but unlike existing results, in this paper mass centers of rigid bodies are not located on the neutral axes of elastic beam segments. Also, all elastic beam segments are in the same plane and during oscillations, rigid bodies perform planar motion. For determination of natural frequencies of the system, modification of the conventional continuous-mass transfer matrix method (CTMM) [9] has been performed. Performed modification of CTMM gives the coefficients of lower-order determinant as compared to the determinant obtained in [9], which has importance in numerical analysis of the systems with a large number of elastic beam segments and rigid bodies. Theoretical apporach of this paper is based on paper [10]. In this paper, the case when the left side of structure is clamped and the right side of structure is simply supported, is applied. But the beam is cantilevered and obtained results can be applied easily on any type of constraints on these places.

2. SYSTEM MODELING AND EQUATIONS OF MOTION

A system of rigid bodies (α_i) connected by homogenous elastic beam segments (BS_i) is shown in Fig. 1 [10]. C_i represents the mass center of body V_i, α_i is the angle between the longitudinal axes of undeformed ad0jacent segments (BS_i) and (BS_{i+1}). O_i is the point of body (V_i) which represents the intersection point of the longitudinal axes of undeformed adjacent segments (BS_i) and BS_{i+1}. Rigid bodies perform planar motions in the plane where elastic segments are positioned. $w_i(z_i,t)$ presents the transverse displacement in the y_i direction and $u_i(z_i,t)$ presents the axial displacement in the z_i direction, where z_i axe coincide with the neutral axis of segment (BS_i).

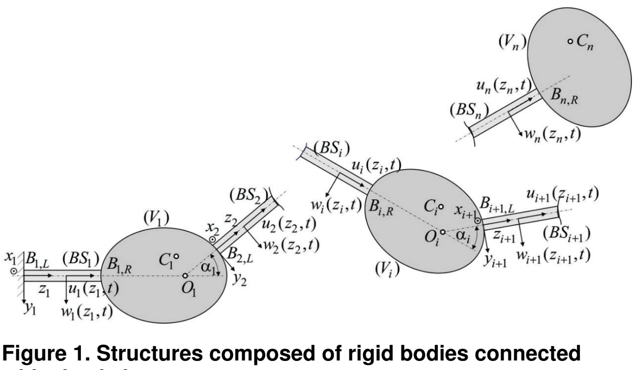

Figure 1. Structures composed of rigid bodies connected with elastic beams

The partial differential equations for bending and axial vibrations of the beam segments (BS_i) is [11]:

$$E_i I_{x(i)} w''''(z_i,t) + \rho_i A_i \ddot{w}_i(z_i,t) = 0, \quad i = 1,\ldots,n, \quad (1)$$

$$\rho_i A_i \ddot{u}_i(z_i,t) - E_i A_i u''(z_i,t) = 0, \quad i = 1,\ldots,n, \quad (2)$$

where E_i presents modulus of elasticity, $I_{x(i)}$ is the cross-sectional area moment of inertia about axis x_i

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which passes trough center of the cross section, A_i is the cross-sectional area, ρ_i is the mass density. The beam segments are modeled as the Euler-Bernoulli beams (rotary and shear effects are ignored) [11]. Deformations $u_i(z_i,t)$ and $w_i(z_i,t)$ as well as rotations $w'_i(z_i,t)$ are small.

Using the separation of variables method, the displacements $w_i(z_i,t)$ and $u_i(z_i,t)$ can be written as

$$w_i(z_i,t)=W_i(z_i)T(t), \tag{3}$$

$$u_i(z_i,t)=U_i(z_i)T(t), \tag{4}$$

where $W_i(z_i)$ and $U_i(z_i)$ (i=1,...n) are the normal modes in bending and axial vibrations, respectively. According to (3) and (4), (1) and (2) can be rewritten as the following system of $2n$+1 ordinary differential equations:

$$W_i^{''''}(z_i)-k_i^4 W_i(z_i)=0,\ \ i=1,\dots,n, \tag{5}$$

$$U_i^{''}(z_i)+p_i^2 U_i(z_i)=0,\ \ i=1,\dots,n, \tag{6}$$

$$\ddot{T}(t)+\omega^2 T(t)=0, \tag{7}$$

where ω is the natural frequency of vibration of the entire system and

$$k_i^4=\frac{\rho_i A_i}{E_i I_{x(i)}}\omega^2,\ \ p_i^2=\frac{\rho_i}{E_i}\omega^2,\ \ i=1,\dots,n. \tag{8}$$

The relation between quantities k_i and p_i can be seen from (8).

$$p_i=\sqrt{\frac{I_{x(i)}}{A_i}}k_i^2,\ \ i=1,\dots,n. \tag{9}$$

Taking that $k_1 = k$ and $p_1=\sqrt{I_{x(1)}A_i}k^2$, from (8) and (9) it follows

$$k_i=\sqrt[4]{\frac{E_1 I_{x(1)}\rho_i A_i}{\rho_1 A_1 E_i I_{x(i)}}}k,\ \ p_i=\sqrt{\frac{E_1 I_{x(1)}}{\rho_1 A_1}}k^2 \tag{10}$$

and

$$\omega=\sqrt{\frac{E_1 I_{x(1)}}{\rho_1 A_1}}k^2 \tag{11}$$

The general solutions of (5) and (6) can be expressed as [11]

$$\begin{aligned}W_i(z_i)&=C_{1(i)}\cos(k_i z_i)+C_{2(i)}\sin(k_i z_i)+\\&+C_{3(i)}\cosh(k_i z_i)+C_{4(i)}\sinh(k_i z_i),\\ i&=1,\dots,n,\end{aligned} \tag{12}$$

$$\begin{aligned}U_i(z_i)&=C_{5(i)}\cos(p_i z_i)+C_{6(i)}\sin(k_i z_i),\\ i&=1,\dots,n,\end{aligned} \tag{13}$$

3. BOUNDARY CONDITIONS

3.1 Boundary conditions at the left end beam segment

Let the segment (BS_1) be clamped at the left end $B_{1,L}$. Based on this, following boundary conditions hold:

$$w_1(0,t)=0,\ \ w_1'(0,t)=0,\ \ u_1(0,t)=0, \tag{14}$$

which, taking into account (3), (4), (12), (13), (14) can be written in the developed form as follows:

$$C_{1(1)}+C_{3(1)}=0, \tag{15}$$

$$k_1 C_{2(1)}+k_1 C_{4(1)}=0, \tag{16}$$

$$C_{5(1)}=0. \tag{17}$$

The following matrix relation can be formed:

$$[C_1]=[T_0][C_0] \tag{18}$$

where

$$[C_1]=\begin{bmatrix}C_{1(1)} & C_{2(1)} & C_{3(1)} & C_{4(1)} & C_{5(1)} & C_{6(1)}\end{bmatrix}^{\mathrm{T}}, \tag{19}$$

$$[C_0]=\begin{bmatrix}C_{1(1)} & C_{2(1)} & C_{6(1)}\end{bmatrix}^{\mathrm{T}}, \tag{20}$$

$$[T_0]=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ -1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}. \tag{21}$$

3.2 Boundary conditions of the rigid body (V_i)

The rigid body (V_i) is presented in Fig. 2 [10]. C_i^* and C_i^{**} represents the perpendicular projections of the mass center C_i to the directions $B_{i,R}O_i$ and $B_{i+1,L}O_i$, respectively.

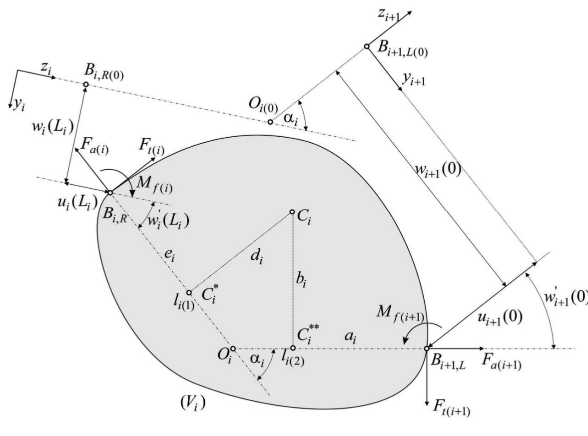


Figure 2. Free-body diagram of the body (V_i)

In further considerations the following quantities will be used to describe the material and geometric characteristic of the rigid bodies (V_i): body mass m_i, mass moment of inertia about centroidal axis J_i, $\overline{B_{i,R}C_i^*}=e_i$, $\overline{C_i^{**}B_{i+1,L}}=a_i$, $\overline{C_iC_i^*}=d_i$, $\overline{C_iC_i^{**}}=b_i$, $\overline{O_iB_{i,R}}=l_{i(1)}$, $\overline{O_iB_{i+1,L}}=l_{i(2)}$. The slopes of the displacements at the ends $B_{i,R}$ and $B_{i+1,L}$ of the segments (BS_i) and (BS_{i+1}) equal the angle of rotation of the body (V_i):

$$w_i'(L_i,t) = w_{i+1}'(0,t) \tag{22}$$

or, in developed form:

$$\begin{aligned} & k_i\left(-C_{1(i)}\sin k_iL_i + C_{2(i)}\cos k_iL_i + \right.\\ & \left. + C_{3(i)}\sinh k_iL_i + C_{4(i)}\cosh k_iL_i\right) = \\ & = k_{i+1}\left(C_{2(i+1)} + C_{4(i+1)}\right). \end{aligned} \tag{23}$$

Further, according to the assumption on small elastic deformations of the beam segments, the displacement vector of point O_i determined based on the displacement of point $B_{i,R}$ and the slope $w'_i(L_i,t)$ reads

$$\begin{aligned} & \overrightarrow{(O_i)_0 O_i} = \\ & = \left(w_i(L_i,t) + \overline{B_{i,R}O_i}\, w_i'(L_i,t)\right)\vec{j}_i + u_i(L_i,t)\vec{k}_i. \end{aligned} \tag{24}$$

Also, the displacement vector of point O_i can be expressed through the displacement of point $B_{i+1,L}$ and deflection $w'_{i+1}(L_i,t)$ as follows:

$$\begin{aligned} & \overrightarrow{(O_i)_0 O_i} = \left(w_{i+1}(0,t) - \overline{B_{i+1,R}O_i}\, w_{i+1}'(0,t)\right)\vec{j}_{i+1} + \\ & + u_{i+1}(0,t)\vec{k}_{i+1}. \end{aligned} \tag{25}$$

Equating (24) and (25) and taking dot product of such obtained expression by the $\vec{j}_i$ and $\vec{k}_i$, respectively, yields

$$\begin{aligned} & u_i(L_i,t) = u_{i+1}(0,t)\cos\alpha_i + \\ & + \left[w_{i+1}(0,t) - \ell_{i(2)}w_{i+1}'(0,t)\right]\sin\alpha_i, \end{aligned} \tag{26}$$

$$\begin{aligned} & w_i(L_i,t) + \ell_{i(1)}w_i'(L_i,t) = -u_{i+1}(0,t)\sin\alpha_i + \\ & + \left[w_{i+1}(0,t) - \ell_{i(2)}w_{i+1}'(0,t)\right]\cos\alpha_i, \end{aligned} \tag{27}$$

or in the developed form

$$\begin{aligned} & C_{5(i)}\cos p_iL_i + C_{6(i)}\sin p_iL_i = C_{5(i+1)}\cos\alpha_i + \\ & + \left[C_{1(i+1)} + C_{3(i+1)} + \right.\\ & \left. - \ell_{i(2)}k_{i+1}(C_{2(i+1)} + C_{4(i+1)})\right]\sin\alpha_i, \end{aligned} \tag{28}$$

$$\begin{aligned} & C_{1(i)}\cos k_iL_i + C_{2(i)}\sin k_iL_i + C_{3(i)}\cosh k_iL_i + \\ & + C_{4(i)}\sinh k_iL_i + \ell_{i(1)}k_i\left(-C_{1(i)}\sin k_iL_i + \right.\\ & + C_{2(i)}\cos k_iL_i + C_{3(i)}\sinh k_iL_i + \\ & \left. + C_{4(i)}\cosh k_iL_i\right) = -C_{5(i+1)}\sin\alpha_i + \left[C_{1(i+1)} + \right.\\ & \left. + C_{3(i+1)} - \ell_{i(2)}k_{i+1}\left(C_{2(i+1)} + C_{4(i+1)}\right)\right]\cos\alpha_i. \end{aligned} \tag{29}$$

The angular acceleration and the acceleration of the mass center C_i of the body (V_i), respectively, is

$$\varepsilon_i = \ddot{w}_{i+1}'(0,t) = -\omega^2 k_{i+1}\left(C_{2(i+1)} + C_{4(i+1)}\right)T(t), \tag{30}$$

$$\vec{a}_{Ci} = \vec{a}_{Bi+1,L} + \vec{\varepsilon}_i \times \overrightarrow{B_{i+1,L}C_i}, \tag{31}$$

where $\vec{a}_{Bi+1,L}$ is the acceleration of point $B_{i+1,L}$ and $\vec{\varepsilon}_i = \varepsilon_i\vec{i}_{i+1}$. In (31) on account of assumption about small deformations of the segments, the term $\vec{\omega}_i \times \vec{\omega}_i \times B_{i+1,L}C_i$ which represents normal acceleration of the mass center C_i is ignored. In that case, $\vec{\omega}_i = \dot{w}_{i+1}'\vec{i}_{i+1}$ is the vector of angular velocity of body (V_i). Now, Newton-Euler differential equations of motion of the body is

$$\begin{aligned} & J_i\varepsilon_i = M_{f(i)} - M_{f(i+1)} + F_{t(i)}e_i + \\ & + F_{a(i)}d_i + F_{t(i+1)}a_i - F_{a(i+1)}b_i, \end{aligned} \tag{32}$$

$$\begin{aligned} & m_i\left(\ddot{u}_{i+1}(0,t) + b_i\varepsilon_i\right) = \\ & = F_{a(i+1)} - F_{a(i)}\cos\alpha_i + F_{t(i)}\sin\alpha_i, \end{aligned} \tag{33}$$

$$m_i\left(\ddot{w}_{i+1}(0,t) - a_i\varepsilon_i\right) = F_{t(i+1)} - F_{a(i)}\sin\alpha_i - F_{t(i)}\cos\alpha_i, \tag{34}$$

where $F_{t(i)}$ and $F_{t(i+1)}$ are the shear forces of beam segments (BS_i) and (BS_{i+1}), respectively, defined as:

$$F_{t(i)} = -E_iI_{x(i)}w_i'''(L_i,t), \tag{35}$$

$$F_{t(i+1)} = -E_{i+1}I_{x(i+1)}w_{i+1}'''(0,t). \tag{36}$$

$F_{a(i)}$ and $F_{a(i+1)}$ are the axial forces of beam segments (BS_i) and (BS_{i+1}), respectively, defined as:

$$F_{a(i)} = E_iA_iu_i'(L_i,t), \tag{37}$$

$$F_{a(i+1)} = E_{i+1}A_{i+1}u_{i+1}'(0,t), \tag{38}$$

and, finaly, $M_{f(i+1)}$ and $M_{f(i)}$ are the bending moments of beam segments (BS_i) and (BS_{i+1}), respectively, defined as:

$$M_{f(i)} = -E_iI_{x(i)}w_i''(L_i,t), \tag{39}$$

$$M_{f(i+1)} = -E_{i+1}I_{x(i+1)}w_{i+1}''(0,t). \tag{40}$$

Based on above relations, (32)-(34) can be written in a developed form as follows:

$$\begin{aligned} & -\omega^2 J_ik_{i+1}\left(C_{2(i+1)} + C_{4(i+1)}\right) = \\ & = -E_iI_{x(i)}k_i^2\left[-C_{1(i)}\cos k_iL_i - C_{2(i)}\sin k_iL_i + \right.\\ & \left. + C_{3(i)}\cosh k_iL_i + C_{4(i)}\sinh k_iL_i\right] + \\ & + E_{i+1}I_{x(i+1)}k_{i+1}^2\left(-C_{1(i+1)} + C_{3(i+1)}\right) + \\ & - E_iI_{x(i)}e_ik_i^3\left(C_{1(i)}\sin k_iL_i - C_{2(i)}\cos k_iL_i + \right.\\ & \left. + C_{3(i)}\sinh k_iL_i + C_{4(i)}\cosh k_iL_i\right) + \\ & + E_iA_id_ip_i\left(-C_{5(i)}\sin p_iL_i + C_{6(i)}\cos p_iL_i\right) + \\ & - E_{i+1}I_{x(i+1)}a_ik_{i+1}^3\left(-C_{2(i+1)} + C_{4(i+1)}\right) + \\ & - E_{i+1}A_{i+1}b_ip_{i+1}C_{6(i+1)}, \end{aligned} \tag{41}$$

$$\begin{aligned} & -m_i\omega^2\left[C_{5(i+1)} + b_ik_{i+1}\left(C_{2(+1)} + C_{4(i+1)}\right)\right] = \\ & = E_{i+1}A_{i+1}p_{i+1}C_{6(i+1)} - E_iA_ip_i \cdot \\ & \left[-C_{5(i)}\sin p_iL_i + C_{6(i)}\cos p_iL_i\right]\cos\alpha_i + \\ & - E_iI_{x(i)}k_i^3\left[C_{1(i)}\sin k_iL_i - C_{2(i)}\cos k_iL_i + \right.\\ & \left. + C_{3(i)}\sinh k_iL_i + C_{4(i)}\cosh k_iL_i\right]\sin\alpha_i, \end{aligned} \tag{42}$$

$$\begin{aligned} & m_i\omega^2\left[-C_{1(i+1)} - C_{3(i+1)} + \right.\\ & \left. + a_ik_{i+1}\left(C_{2(i+1)} + C_{4(i+1)}\right)\right] = E_{i+1}I_{x(i+1)}k_{i+1}^3 \cdot \\ & \left(C_{2(i+1)} - C_{4(i+1)}\right) + E_iA_ip_i \cdot \\ & \left(C_{5(i)}\sin p_iL_i - C_{6(i)}\cos p_iL_i\right)\sin\alpha_i + \\ & + E_iI_{x(i)}k_i^3\left[C_{1(i)}\sin k_iL_i - C_{2(i)}\cos k_iL_i + \right.\\ & \left. + C_{3(i)}\sinh k_iL_i + C_{4(i)}\cosh k_iL_i\right]\cos\alpha_i. \end{aligned} \tag{43}$$

Equations (23), (29), (30), (41), (42), and (43) can be written in the matrix form as follows:

$$\mathbf{T}_{iL}\mathbf{C}_i = \mathbf{T}_{iR}\mathbf{C}_{i+1} \tag{44}$$

where $C_i = [C_{1(i)}\ C_{2(i)}\ C_{3(i)}\ C_{4(i)}\ C_{5(i)}\ C_{6(i)}]^T$, $C_{i+1} = [C_{1(i+1)}\ C_{2(i+1)}\ C_{3(i+1)}\ C_{4(i+1)}\ C_{5(i+1)}\ C_{6(i+1)}]^T$. Finally, based on equations (44), the following recurrence relation can be written as

$$\mathbf{C}_{i+1} = \mathbf{T}_i\,\mathbf{C}_i, \quad i = 1,\ldots,n-1, \tag{45}$$

where $T_i \in R^{6x6}$ is transfer matrix between the integration constants for beam segments (BS_i) and (BS_{i+1}) determined as

$$\mathbf{T}_i = \mathbf{T}_{iR}^{-1}\mathbf{T}_{iL}, \quad i = 1,\ldots,n-1, \tag{46}$$

After n-1 successive application of the reccurence relation (45), it can be obtained:

$$\mathbf{C}_n = \mathbf{T}_{n-1}\mathbf{T}_{n-2}\cdots\mathbf{T}_1\mathbf{T}_0\mathbf{C}_0. \tag{47}$$

3.3 Boundary conditions at the right end beam segment

Let the segment (BS_n) be simply supported at the right end $B_{n,R}$. Based on this, following boundary conditions hold:

$$w_n(L_n,t) = 0, \quad w_n''(L_n,t) = 0, \tag{48}$$

which, taking into account equations (3), (4), (12), (13), (48) can be written in the developed form as follows:

$$\begin{aligned} &C_{1(n)}\cos(k_nL_n) + C_{2(n)}\sin(k_nL_n) + \\ &+ C_{3(n)}\cosh(k_nL_n) + C_{4(n)}\sinh(k_nL_n) = 0, \end{aligned} \tag{49}$$

$$\begin{aligned} &-k_n^2C_{1(n)}\cos(k_nL_n) - k_n^2C_{2(n)}\sin(k_nL_n) + \\ &+ k_n^2C_{3(n)}\cosh(k_nL_n) + \\ &+ k_n^2C_{4(n)}\sinh(k_nL_n) = 0. \end{aligned} \tag{50}$$

The following matrix relation can be formed:

$$\mathbf{T}_n\mathbf{C}_n = \mathbf{O}_{3\times1}, \tag{51}$$

where

$$\begin{aligned} &T_{11(n)} = T_{12(n)} = T_{13(n)} = \\ &= T_{14(n)} = T_{15(n)} = T_{11(n)} = 0, \end{aligned} \tag{52}$$

$$\begin{aligned} &T_{21(n)} = \cos(k_nL_n), T_{22(n)} = \sin(k_nL_n), \\ &T_{23(n)} = \cosh(k_nL_n), T_{24(n)} = \sinh(k_nL_n), \\ &T_{25(n)} = T_{26(n)} = 0, \end{aligned} \tag{53}$$

$$\begin{aligned} &T_{31(n)} = -k_n^2\cos(k_nL_n), \\ &T_{22(n)} = -k_n^2\sin(k_nL_n), \\ &T_{23(n)} = k_n^2\cosh(k_nL_n), \\ &T_{24(n)} = k_n^2\sinh(k_nL_n), \\ &T_{25(n)} = T_{26(n)} = 0. \end{aligned} \tag{54}$$

3.4 Frequency equation and mode shapes

Taking into account (47), it follows from (51) that

$$\mathbf{TC}_0 = \mathbf{O}_{3\times1} \tag{55}$$

where $\mathbf{T} \in R^{3\times3}$ represents overall transfer matrix determined by the following expression:

$$\mathbf{T} = \mathbf{T}_n\mathbf{T}_{n-1}\cdots\mathbf{T}_1\mathbf{T}_0. \tag{56}$$

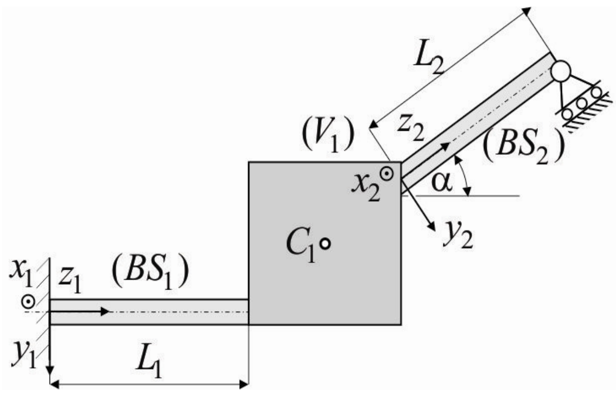
Eq. (53) represents a matrix form of the homogeneous system of equations for unknown components of the matrix $\mathbf{C}_0$. In order that this system can have non-trivial solutions, it is needed to hold that

$$\det\mathbf{T} = 0. \tag{57}$$

4. NUMERICAL EXAMPLE

In this example, rigid body with two elastic beam segments is considered (Fig. 3). The beam segments have circular cross section and the rigid body has square cross section. The following values of the system are used: Young's modulus $E_1 = E_2 = 2.069\times10^{11} N/m^2$, mass destiny $\rho_1 = \rho_2 = 7500\ kg/m^3$, diameters of the beam segments $D_1 = D_2 = 0.05m$, length of the beam segments $L_1 = L_2 = 1m$, mass of the rigid body $m = 50kg$, dimension of the rigid body $a = 0.3m$.

The first four mode shapes are presented in Figures 4, 5, 6, 7. Figure 8 shows the effect of angle α on the first four coefficients k. The characteristic equation for angle $\alpha = \pi/4$ is presented in Figures 9, 10, 11, 12. Determined coefficients k from these figures, as well as first four lowest natural frequencies ω are presented in table 1.



k_r

Figure 3. Rigid body with two elastic segments

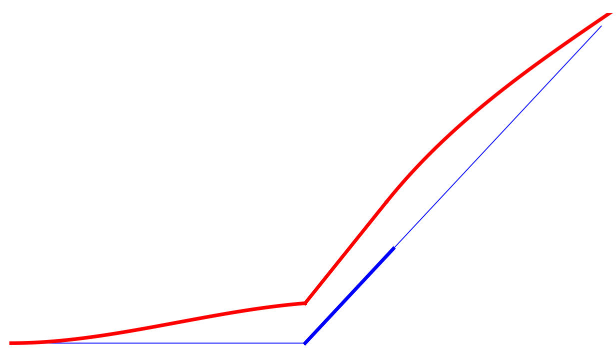


Figure 4. The first mode shape

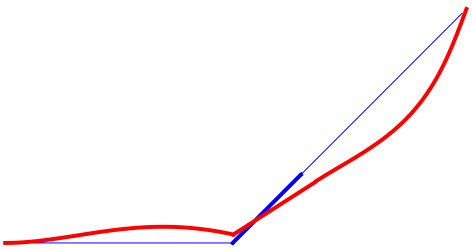


Figure 5. The second mode shape

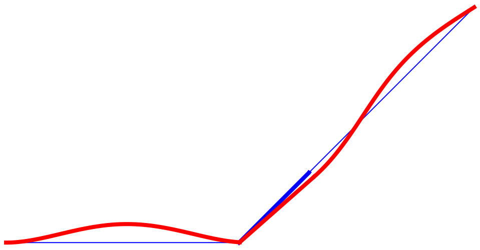


Figure 6. The third mode shape

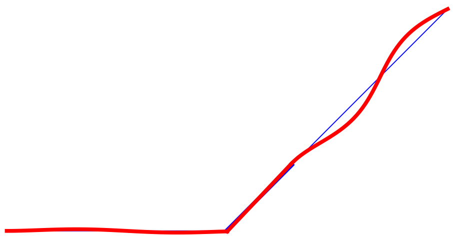


Figure 7. The fourth mode shape

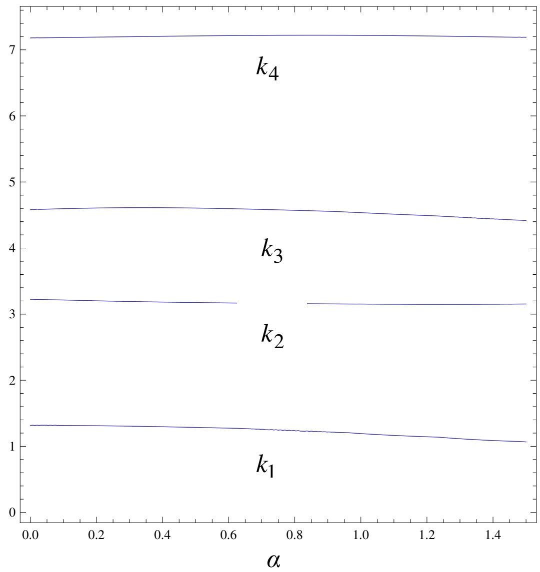


Figure 8. The effect of angle α on the coefficients k

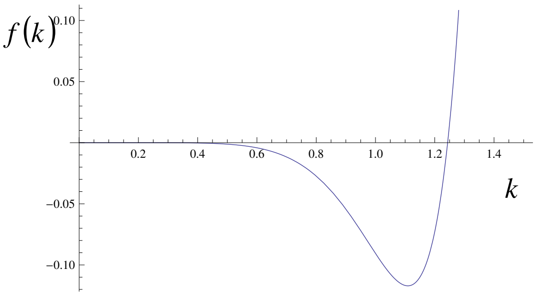


Figure 9. Characteristic equation (determination of k_1)

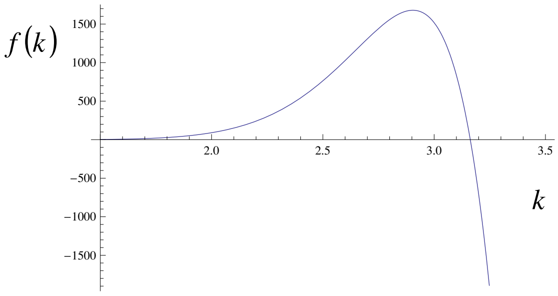


Figure 10. Characteristic equation (determination of k_2)

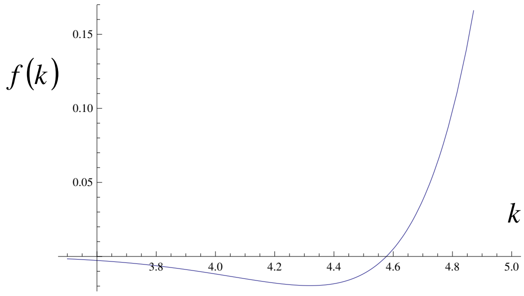


Figure 11. Characteristic equation (determination of k_3)

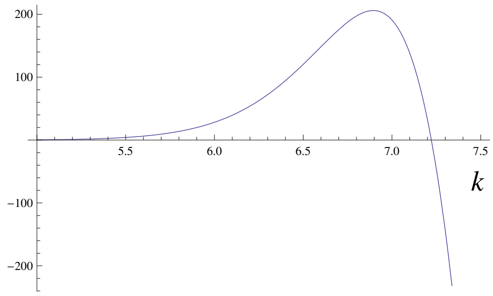


Figure 12. Characteristic equation (determination of k_4)

Table 1. The first four natural frequency of the system

Mode	k	ω [rad/s]
1	1.24	89.46
2	3.16	580.96
3	4.57	1215.08
4	7.22	3032.83

5. CONCLUSION

Free vibrations of structures composed of rigid bodies connected with elastic beam segments are presented in this paper. It is assumed that mass centres of rigid bodies are not located on the neutral axes of elastic beam segments. For determination of natural frequencies of the system, modification of the conventional continuous-mass transfer matrix method (CTMM) [9] has been performed. The matrix $\mathbf{T}$ can be formed by using software tools like MatLab and Mathematica. Also, using the procedure developed in this paper, with the help of software tools, it can be found easily the solution of equation $\det\mathbf{T}=0$. in the analytical form. This provides possibility to analyze dependence on frequencies of any parameter of a given system. Numerical example is provided in order to represent possibilities of the developed procedure.

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АНАЛИЗА СЛОБОДНИХ РАВАНСКИХ ОСЦИЛАЦИЈА СТРУКТУРА САСТАВЉЕНИХ ОД КРУТИХ ТЕЛА И ЕЛАСТИЧНИХ ГРЕДНИХ СЕГМЕНАТА

Н. Радовановић, Н. Зорић, Н. Тришовић, А. Томовић

Овај рад представља анализу слободних вибрација структура састављених од крутих тела међусобно спојених са еластичним гредама. Претпоставља се да се центри маса крутих тела не налазе на неутралној оси недеформисаног еластичног гредног сегмента као и да крута тела врше равно кретање у истој равни и да се њихови центри маса налазе у тој истој равни. За одређивање фреквенција система, модификација класичне "CTMM" методе је употребљена. Еластични гредни сегменти се третирају као Ојлер-Бернулијеве греде. Приказан је нумерички пример.