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Approximate method for stress intensity factors determination in case of multiple site damage

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ABSTRACT

A simple and easy to use approximate procedure, for calculating stress intensity factors, was proposed. The procedure was developed based on existing solution for stress intensity factor in the case of two unequal cracks in an infinite plate subjected to remote uniform stress. The solution for this configuration was used for obtaining interaction effect coefficients which take into consideration the increase of stress intensity factor of analyzed crack tip due to interaction with existing adjacent crack. Accuracy and application of suggested procedure were verified through two different computer programs which are based on two different computational methods: finite element method (FEM) with singularity elements and extended finite element method (X-FEM). The analysis of the results has shown that a very good agreement between solutions was achieved, and that this method can provide stress intensity factors with acceptable accuracy.

1. Introduction

The continuous integrity of supporting structures of aging aircrafts is of great concern to the aviation community. The long service life of aging aircrafts increases the possibility of diminishing, or even total loss of structural integrity due to multiple fatigue cracks. Multiple site damage (MSD) represents the simultaneous development of fatigue cracks at multiple sites in the same structural element. Those cracks are close enough to influence each other and to affect the overall structural integrity. MSD often occurs in longitudinal and circumferential riveted lap joints in wings and fuselages. It can be very serious, because of possible link up of adjacent cracks creating one large crack that can cause catastrophic failure, due to reduction of residual strength of structural element.

The prediction of crack growth rate and residual strength of cracked structure demands accurate calculation of stress intensity factors (SIFs). In order to predict those factors, several analytical and theoretical studies, as well as numerical models and have been presented over the years.

Analytical procedure had been presented in which the stress function was assumed to be the sum of stress functions with singularities on the crack faces and at the infinity and the stress intensity factors for various configurations of two-dimensional interacting cracks in an infinite body subjected to a remote tension were given as a power series formula [1]. Theoretical analysis followed which enabled formation of an approximate expansion polynomial expression for the stress intensity factor [2].

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The analytical method introduced for solving stress intensity factor problems on multiple holes by Zhao et al. [3]. This modified analytical method is easier to apply than some traditional analytical methods.

The Schwartz–Neumann alternating method, together with the boundary element method was used to determine the mixed mode stress intensity factors and weight functions for cracks in finite bodies [4], while an alternating indirect boundary element (AIBE) technique was used to calculate stress intensity factors for multiple interacting cracks in two-dimensional cracked structure by Dawicke and Newman [5].

A two-dimensional plane stress elastic fracture mechanics analysis of a clap joint fastened by rigid pins was performed, where two types of MSD are considered: MSD with equal length cracks and MSD with variable crack length and the plate treated as being of infinite thickness. The mode I stress intensity factors and changes in the compliance due to the existence of MSD were determined in paper presented by Beuth and Hutchinson [6].

Hybrid finite element method was used, together with complex variable theory of elasticity to calculate the stress intensity factor at the crack tips, stress concentration factors in the stiffeners, and rivet loads for a stiffened structure with multiple cracks [7].

Regardless of the mentioned research, there is still a lack of available solutions in case of more complex configurations with more than few cracks. The solutions for these configurations, now more than ever, imply the usage of numerical methods, as technology and computer sciences became more available. Nevertheless, this kind of analysis can be very complicated especially because of the mutual influence of the adjacent cracks. This is the main reason for introducing approximation methods and procedures which will enable faster and simpler determination of stress intensity factors of supporting aircraft structures with multiple cracks, but they were very occasional the topic of the researcher's studies. One of the rare methods of this kind was a compounding method for determining approximate stress intensity factors which is performed by adding individual boundary effects, presented by Cartwright and Rooke [8]. But, the evaluation of the interaction between boundaries effect was very difficult, since it increases with the increase of boundary number and with the crack tip approaching to the boundary. This significantly influenced the accuracy of the method. This method was used for the assessment of influence of the adjacent cracks for calculating the stress intensity factor in the case of multiple elliptical and through cracks that develop from adjacent rivet holes of a thin plate by Pastrama and De Castro [9]. A simple method of stress analysis in elastic solids with many cracks was proposed by Kachanov [10]. It was based on the superposition technique and the ideas of self-consistency applied to the average tractions on individual cracks. This method was specialized to arbitrary length collinear cracks with arbitrary spacing under far field tension by Millwater [11].

However, the assessment of mutual influence of the adjacent cracks remains one of the major problems for approximate methods.

In this paper a versatile and easy to use approximate procedure for stress intensity factor determination in case of multiple cracks is presented. This procedure takes into consideration the effect of the interaction between multiple crack tips as corresponding coefficients, which represent the influences of adjacent cracks on stress intensity factor of analyzed crack. The accuracy of the procedure was verified by comparison with the solutions obtained with finite element method and extended finite element method.

2. Approximate method for stress intensity factor determination in case of MSD

The procedure was developed based on existing solutions for stress intensity factors in the case of two unequal cracks in an infinite plate subjected to remote uniform stress [12].

For this model, normalized SIFs for crack tips A, B, C i D are calculated with following expressions:

$$\beta_A = \frac{K_{IA}}{K_{01}} = \sqrt{\frac{2b}{a_1}} \cdot \frac{x_A^2 - C_1 x_A + C_2}{\sqrt{x_A(x_A + x_C)(x_A + x_D)}}, \tag{1}$$

$$\beta_B = \frac{K_{IB}}{K_{01}} = \sqrt{\frac{2b}{a_1}} \cdot \frac{C_2}{\sqrt{x_A \cdot x_C \cdot x_D}}, \tag{2}$$

$$\beta_C = \frac{K_{IC}}{K_{02}} = \sqrt{\frac{2b}{a_2}} \cdot \frac{x_C^2 + C_1 x_C + C_2}{\sqrt{x_C(x_A + x_C)(x_D - x_C)}}, \tag{3}$$

$$\beta_D = \frac{K_{ID}}{K_{02}} = \sqrt{\frac{2b}{a_2}} \cdot \frac{x_D^2 + C_1 x_D + C_2}{\sqrt{x_D(x_A + x_D)(x_D - x_C)}}, \tag{4}$$

where:

$K_{01} = \sigma\sqrt{\pi a_1}$, for crack tips A i B, and $K_{02} = \sigma\sqrt{\pi a_2}$ for crack tips C i D.

Those normalized SIFs for opening mode are given as a function of dimensionless parameters x_A, x_C and x_D, which can be expressed as:

$$x_A = \frac{2a_1}{b}; \quad x_C = 1 - \frac{a_1}{b}; \quad x_D = 1 - \frac{a_1}{b} + 2\frac{a_2}{b}.$$

Coefficients C_1 and C_2 are determined as:

$$C_1 = \frac{(x_A - x_D)\cdot K(k) - 2x_A \cdot \Pi(n,k) + 2x_D \cdot \Pi(m,k) + (x_A + x_D)\cdot [J(n,k) - J(m,k)]}{K(k) - \Pi(n,k) - \Pi(m,k)}, \tag{5}$$

$$C_2 = \frac{C_1}{K(k)}\cdot [x_A \cdot K(k) - (x_A + x_D)\cdot \Pi(n,k)] - \frac{1}{K(k)}\cdot [x_A^2 \cdot K(k) - 2x_A \cdot (x_A + x_D)\cdot \Pi(n,k) + (x_A + x_D)^2 \cdot J(n,k)], \tag{6}$$

where $K(k)$ is complete elliptic integral of the first kind, while $\Pi(n,k)$ and $\Pi(m,k)$ are complete elliptic integrals of the third kind, which are given as:

$$K(k) = \int_0^{\frac{\pi}{2}} \frac{d\phi}{\sqrt{1 - k^2 \sin^2\phi}}, \tag{7}$$

$$\Pi(t,k) = \int_0^{\frac{\pi}{2}} \frac{d\phi}{(1 + t\cdot \sin^2\phi)\cdot \sqrt{1 - k^2 \sin^2\phi}}, \tag{8}$$

where $t = n$ or $t = m$, and function J is defined as:

$$J(t,k) = \int_0^{\frac{\pi}{2}} \frac{d\phi}{(1 + t\cdot \sin^2\phi)^2 \cdot \sqrt{1 - k^2 \sin^2\phi}}. \tag{9}$$

Parameters n, m and k are defined as:

$$n = \frac{x_D - x_C}{x_A + x_C}; \quad m = \frac{x_A}{x_D}; \quad k^2 = m\cdot n. \tag{10}$$

On the other hand, in this case (Fig. 1) stress intensity factor (for mode I) for the first crack (for tip B which is closer to the adjacent influential crack) will be increased due to the existence of the adjacent crack, and, as such, can be expressed as:

$$K_{I1B} = K_{I1} + c_{2b}\cdot K_{I2}, \tag{11}$$

where:

K_{I1B} – total stress intensity factor for tip B of the first crack in presence of the adjacent one;

$K_{I1} = \sigma\sqrt{a_1\pi}$ – individual stress intensity factor for the first crack [12,13] (only crack in the configuration);

$K_{I2} = \sigma\sqrt{a_2\pi}$ – individual stress intensity factor for the second crack [12,13] (only crack in the configuration);

c_{2b} – coefficient that takes into consideration the increase of stress intensity factor of first crack tip B, due to presence of the adjacent crack. Index **2** refers to influence of the **second** crack, and index **b** refers to the influence of the second crack on the stress intensity factor of the **closer** tip of analyzed crack i.e. in this case tip B.

If the previous equation is written as a function of geometry factors, i.e. normalized stress intensity factors ($\beta_i = \frac{K_{Ii}}{\sigma\sqrt{\pi\cdot a_i}}$), and then divided by $\sigma\sqrt{a_1\pi}$, the following equation is obtained:

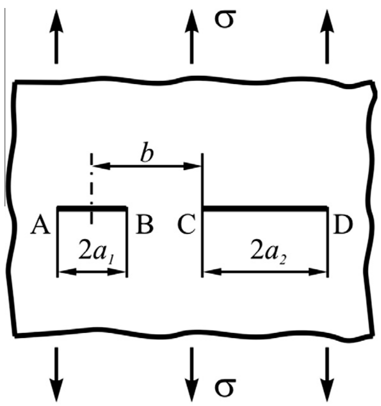

Fig. 1. Two unequal cracks in an infinite plate subjected to remote uniform stress.

$$\beta_{1B} = \beta_1 + c_{2b} \cdot \beta_2 \sqrt{\frac{a_2}{a_1}}, \tag{12}$$

where a_1 is one half of the length of the first (analyzed) crack and a_2 is one half of the length of the second (influential) crack.

The coefficient c_{2b} that takes into consideration the increase of stress intensity factor of crack tip B, due to the presence of the adjacent crack, can be expressed as (for shown configuration β_1 and β_2 are 1):

$$c_{2b} = (\beta_{1B} - 1) \cdot \sqrt{\frac{a_1}{a_2}}. \tag{13}$$

Eq. (11) written for K_{I1A}, can be expressed as:

$$K_{I1A} = K_{I1} + c_{2d} \cdot K_{I2}. \tag{14}$$

The coefficient that takes into consideration the increase of stress intensity factor of left further crack tip A, due to the presence of the adjacent crack, now nominated as c_{2d} (index **d** refers to the influence of adjacent crack on the **further** crack tip of the analyzed crack which is now tip A) can be similarly expressed as:

$$c_{2d} = (\beta_{1A} - 1) \cdot \sqrt{\frac{a_1}{a_2}}. \tag{15}$$

Again, using Eq. (11), now for the second crack tips C and D, the corresponding influential coefficients for second crack tips C and D are obtained in the same manner

$$c_{1b} = (\beta_{2C} - 1) \cdot \sqrt{\frac{a_2}{a_1}}, \tag{16}$$

$$c_{1d} = (\beta_{2D} - 1) \cdot \sqrt{\frac{a_2}{a_1}}. \tag{17}$$

It should be noticed that c_{1b} represents the coefficient that takes into consideration the increase of stress intensity factor of crack tip C, due to the presence of the adjacent crack, whereby index **1 now** refers to influence of the **first** crack, and index **b** refers to the influence of the first crack on the stress intensity factor of the **closer** tip of analyzed, second crack i.e. in this case tip C. Also, c_{1d} represents the coefficient that takes into consideration the increase of stress intensity factor of crack tip D, due to the presence of the adjacent crack, whereby index **1 now** refers to influence of the **first** crack, and index **d** refers to the influence of the first crack on the stress intensity factor of the **further** tip of analyzed, second crack i.e. in this case tip D.

The geometry factors β_{1A}, β_{1B}, β_{2C} and β_{2D} are known and can be calculated with Eqs. (1)–(4), presented by Rooke and Cartwright [12].

So, according to Eq. (11), the stress intensity factor for opening mode of analyzed crack in any given configuration with n cracks can be estimated as:

$$K_{IjA,B} = c_{1b,d} \cdot K_{I1} + \cdots + c_{jb,d} \cdot K_{Ij} + \cdots + c_{nb,d} \cdot K_{In} = \sum_{i=1}^{n} c_{ib,d} \cdot K_{Ii}, \tag{18}$$

where:

$K_{IjA,B}$ – represents stress intensity factor for tip A, or B of analyzed crack in presence of all other cracks in configuration;

K_{Ii} – individual stress intensity factor of all cracks in configuration, i.e., stress intensity factors of auxiliary configurations;

$c_{ib,d}$ – the coefficient that takes into consideration influence of i-th crack on stress intensity factor of analyzed crack (if the tip of the analyzed crack is closer to the influential crack, i.e. if it's on the same side, the coefficient is c_{ib}, and if it is further, i.e., if it's on the opposite side, the coefficient is c_{id}), and that the influential coefficient of the analyzed crack on itself is $c_{jb,d} = 1$.

In this manner the analyzed complex configuration is represented as a combination of several simpler (auxiliary) configurations. The number of those configurations is equal to the number of cracks, such that every auxiliary configuration contains only one crack. The determination of stress intensity factor of analyzed crack is reduced to determination of the influence that every crack in the initial configuration has on the analyzed one, as for the many configurations with one crack the solutions for stress intensity factors are available. This influence here, as it is shown, is represented with corresponding coefficients. For their determination Eqs. (3) and (4) can be used, but the distances between the cracks and the position of the influential crack must be especially taken care for. Those equations, in general can be written as:

$$c_{ib} = (\beta_{jB} - 1) \cdot \sqrt{\frac{a_j}{a_i}}, \tag{19}$$

$$c_{id} = (\beta_{jA} - 1) \cdot \sqrt{\frac{a_j}{a_i}}, \tag{20}$$

where: β_{jB} – geometry factor of analyzed crack for tip B, in the case when only analyzed crack and influence crack are present in the configuration (determined with Eq. (1)); β_{jA} – geometry factor of analyzed crack for tip A, in the case when only analyzed crack and influence crack are present in the configuration (determined with Eq. (2));

a_j – half length of the analyzed crack;

a_i – half length of the influential crack.

So, Eqs. (13) and (15)–(17), i.e. (19) and (20), with appropriate crack lengths and distance between them, can be used for obtaining the approximate influential coefficients for any given configuration, as long as the tips of the cracks are far enough from neighboring boundaries. Those coefficients, for vast number of crack lengths and distances between them, i.e. for their combinations, are computed by usage of MathCAD computer program by Kastratović [14].

2.1. Numerical example

In this paper, the SIFs were determined for a typical aero structural configuration. It is a thin plate with three circular holes subjected to uniform uniaxial tensile stress. Material of the plate is aluminum alloy Al-2024 T3 [15]. Middle hole has two radial cracks and other two holes have one radial crack. This configuration is shown in Fig. 2.

It should be noticed that for this type of aircraft structural element, the dominant fraction of loading originates from fuselage pressurisation, and thus, tension in the direction perpendicular to the middle line of the plate prevails. Hence, the determination of opening mode SIFs is sufficient enough.

Implementing Eq. (18) on this model, total stress intensity factor for crack tip B is:

$$K_{IB} = c_{1b,d} \cdot K_{I1} + c_{2b} \cdot K_{I2} + c_{3d} \cdot K_{I3} \tag{21}$$

Normalized total stress intensity factor for crack tip B, considering the fact that the analyzed configuration is symmetrical (Fig. 3(a)), $K_{I2} = K_{I3},\ a_2 = a_3$, and that the influential coefficient of the analyzed crack on itself is $c_{1b,d} = 1$, is:

$$\beta_B = \beta_1 + (c_{2b} + c_{3d}) \cdot \beta_2 \sqrt{\frac{a_2}{a_1}}, \tag{22}$$

where

c_{2b} – coefficient that takes into consideration the increase of stress intensity factor of crack with tip B, due to presence of the crack with tip D (on the same side of tip B),

c_{3d} – coefficient that takes into consideration the increase of stress intensity factor of crack with tip B, due to presence of the crack with tip C (on the opposite side of tip B).

Similarly, based on Eq. (18), total stress intensity factor for crack tip D is:

$$K_{ID} = c_{1b} \cdot K_{I1} + c_{2b} \cdot K_{I2} + c_{3b,d} \cdot K_{I3} \tag{23}$$

and normalized total stress intensity factor for crack tip D, (the influential coefficient of the analyzed crack on itself is $c_{3b,d} = 1$) is:

$$\beta_D = (1 + c_{2b}) \cdot \beta_2 + c_{1b} \cdot \beta_1 \cdot \sqrt{\frac{a_1}{a_2}}, \tag{24}$$

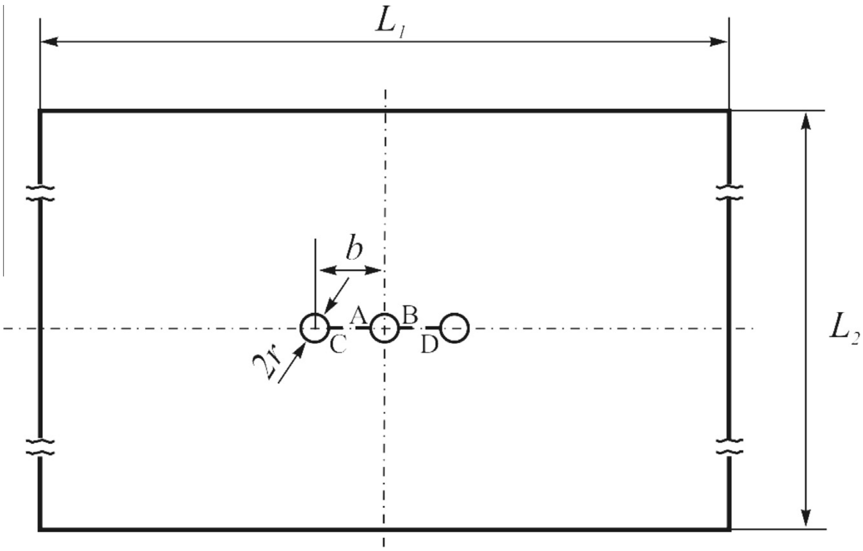

Fig. 2. Analyzed configuration with multiple cracks.

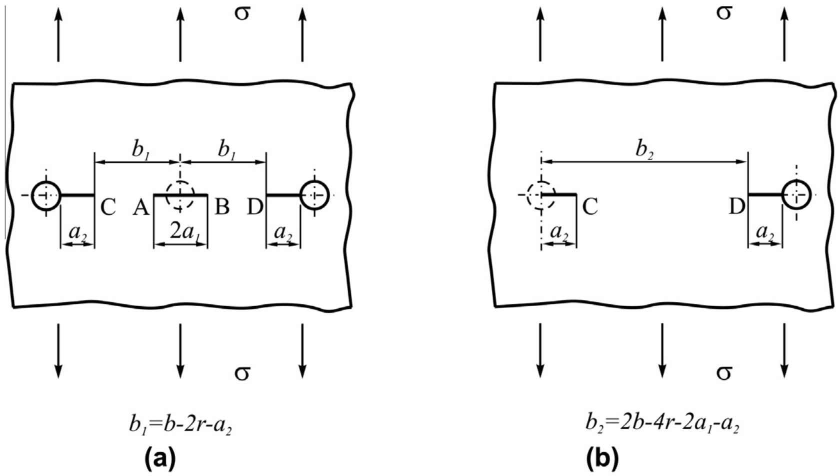

Fig. 3. The distances between the cracks and the position of the analyzed and influential crack.

where

c_{1b} – coefficient that takes into consideration the increase of stress intensity factor of crack with tip D, due to presence of the crack with tip B (on the same side of tip D),

c_{2b} – coefficient that takes into consideration the increase of stress intensity factor of crack with tip D, due to presence of the crack with tip C (on the same side of tip D).

Special attention was addressed to determination of distances between cracks, which is very important for calculation of influential coefficients, as already mentioned. Those distances are shown in Fig. 3, and they are determined in the same manner as in [8]. When calculating the influential coefficients we consider only two crack at the same time: the analyzed crack and the influential crack. Thus, initial configuration presented on Fig. 1 is emulated.

So, first the auxiliary configurations were established, whereby every auxiliary configuration had only one crack. Then, the distances between the cracks were determined, as already described. Then with those distances, using Eqs. (1) and (2) β_{jB} and β_{jA} (j = 1,2,3) were calculated, and they were than used for calculating $c_{1b},\ c_{2b},\ c_{3d}$ using Eqs. (19) and (20). Those coefficients were computed by usage of MathCAD computer program, for a range of different crack sizes.

The auxiliary configurations used for numerical example were a thin plate with central circular hole with one radial crack subjected to uniform uniaxial tensile stress (side holes, geometry factor β_2), and a thin plate with central circular hole with two radial cracks subjected to uniform uniaxial tensile stress (central hole, geometry factor β_1).

The solutions for β_1 and β_2, also known as Bowies solutions, were obtained from following Eqs. (13) and (15):

$$\beta_1 = 0.5 \cdot (3 - s) \cdot \lfloor 1 + 1.243 \cdot (1 - s)^3 \rfloor \quad \text{where } s = \frac{a_1}{r + a_1}, \tag{25}$$

$$\beta_2 = \lfloor 1 + 0.2 \cdot (1 - s) + 0.3 \cdot (1 - s)^6 \rfloor \cdot [2.243 - 2.64 \cdot s + 1.352 \cdot s^2 - 0.248 \cdot s^3] \quad \text{where } s = \frac{a_2}{r + a_2}. \tag{26}$$

3. Verification of stress intensity factors solutions

Further, the stress intensity factors solutions were obtained and verified by using two different computational methods: finite element method (FEM) with singularity elements and extended finite element method (XFEM).

3.1. Finite element method

The most popular numerical method nowadays is Finite element method (FEM), which is used for SIF determination for various structural configurations [14,16–18]. There are basically two different groups of methods for estimation of SIFs when the FEM is concerned. They are the field (displacement and stress) extrapolation techniques (local approach) and those based on energy (global approach). Ansys v14, the FEM software used here, uses displacement extrapolation technique, for 2D models. To be more specific, the analysis uses a fit of the nodal displacements in the vicinity of the crack [19].

This kind of analysis demands solutions for models with cracks. The stress and deformation fields around the crack tip generally have high gradients. To capture the rapidly varying stress and deformation fields, a very refined mesh must be used in the region around the crack tip. This can be accomplished with so called singular elements. So, in this case, the 2D finite element model of analyzed plate was created.

Because Ansys v14 cannot simulate the crack growth, it was necessary to calculate SIF for vast number of cracked configuration, with different crack sizes.

In order to create required finite element model, the computer code was written within the used computer program, which enabled automatic execution of iteration procedure for obtaining required results. This code enabled geometrical modeling, specification of material properties, application of loads, generation of finite element mesh, as well as solving and obtaining corresponding output data. In this case the output data were stress intensity factors for given crack sizes.

The whole calculating procedure can be schematically shown in the form of an algorithm that is presented in Fig. 4.

Analysis file is an input file which containing a complete analysis sequence: preprocessing, solution, and post processing. The file must contain a parametrically defined model using parameters to represent all inputs and outputs to be used as random input variables (RVs), which are in this case cracks length, and random output parameters (RPs), which are in this case stress intensity factors.

Then this analysis file is run thru **Probabilistic design database**, which represents the current probabilistic design environment, which includes:

- Random input variables (RVs).
- Random output parameters (RPs).
- Settings for probabilistic methods.
- Which probabilistic analyses have been performed and in which files the results are stored.
- Which output parameters of which probabilistic analyses have been used for a response surface fit, the regression model that has been used for the fitting procedure, and the results of that fitting procedure.

The database can be saved as **Probabilistic design based database file** or resumed at any time.

The probabilistic design loop file is created automatically by via the **Analysis file** and samples. Samples represent unique set of parameter values that represents a particular model configuration. A sample is characterized by random input variable values.

In each loop, the PDS uses the values of the RVs from one sample and executes the user-specified analysis (**Model database**). The PDS collects the values for the RPs following each loop. The PDS uses the **Loop file** to perform analysis loops. Each model database can be also be saved as **Ansys database file**, or resumed at any time.

The calculation is finally finished when models for all prescribed samples are solved, and stress intensity factor all the cracks, with different crack lengths are obtained.

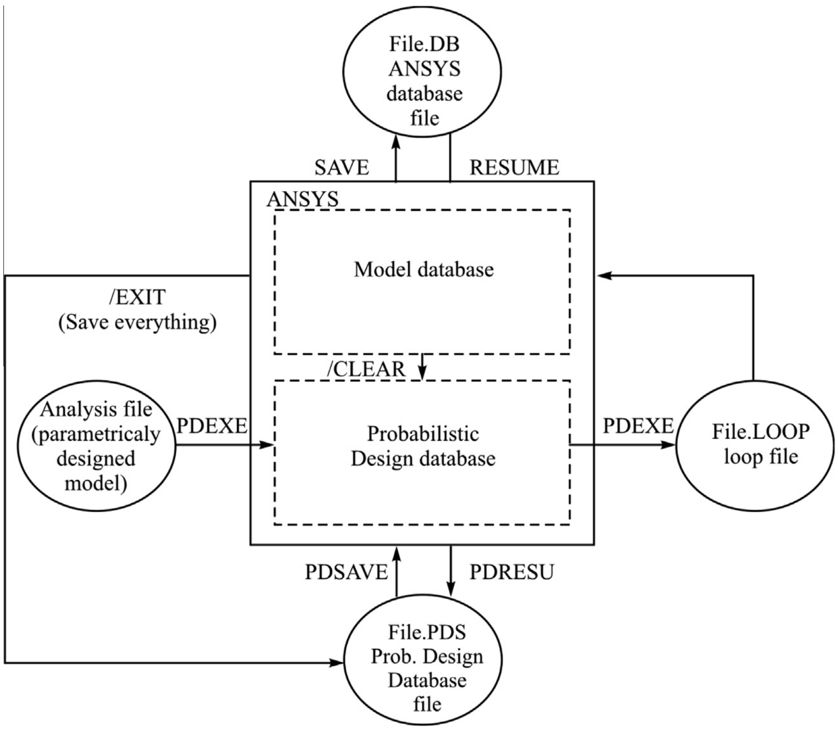

Fig. 4. Calculation algorithm (from Ansys User manual [19]).

3.2. Extended finite element method (XFEM) for stress intensity factor in case of MSD

FEM has been used for decades for solving different engineering problems, but it has some restrictions in crack propagation simulations mainly because the finite element mesh needs to be updated after each propagation step in order to track the crack path. Extended Finite Element Method (XFEM) suppresses the need to mesh and remesh the crack surfaces and is used for modeling different discontinuities in 1D, 2D and 3D domains. XFEM allows for discontinuities to be represented independently of the FE mesh by exploiting the Partition of unity finite element method (PUFEM) [20]. In this method additional functions (commonly referred to as enrichment functions) can be added to the displacement approximation as long as the partition of unity is satisfied. The XFEM uses these enrichment functions as a tool to represent a non-smooth behavior of field variables.

There are many enrichment functions for a variety of problems in areas including cracks, dislocations, grain boundaries and phase interfaces. Recently, XFEM and its coupling with level set method were intensively studied. The level set method allows for treatment of internal boundaries and interfaces without any explicit treatment of the interface geometry.

Due to the relatively short history of the XFEM, commercial codes which have implemented the method are not prevalent. There are however, many attempts to incorporate the modeling of discontinuities independent of the FE mesh by either a plug-in or native support. Cenaero [21] has developed a crack growth prediction add-in *Morfeo/Crack for Abaqus* which relies on the implementation of the XFEM method available in Abaqus software (the functionality of Abaqus is however limited to the calculation of stationary cracks). Problems involving static cracks in structures, evolving cracks, cracks emanating from voids etc., were numerically studied and the results were compared against the analytical and experimental results to demonstrate the robustness of the XFEM and precision of *Morfeo/Crack for Abaqus* [22].

Relying on the assumptions that plate with 3 holes (shown in Fig. 3) remains in the elastic regime everywhere, and that small-strain yielding conditions prevail in the vicinity of the crack front, the material is considered isotropic linear elastic and the simulation is carried out under the assumptions of the linear elastic fracture mechanics (LEFM). An implicit representation of the cracks is adopted in the spirit of the level set method.

The cracks are represented with the help of two signed distance functions that are discretized on the same mesh as the displacement field with first-order shape functions. Method for representing the cracks in this application is exactly the same as described in [23]. After each step of the propagation simulation, the SIFs are computed from the numerical solution at several points along the crack fronts. Interaction integrals are used to extract the mixed-mode SIFs with the help of auxiliary fields.

Technique used here relies on a sub structuring approach that decomposes the computation domain into several subdomains of two kinds: one or several safe subdomains, handled by the FEA code, and one or several cracked subdomains, handled by the XFEM code. The latter contain elements in the vicinity of the initial cracks and in the region where they are approximately expected to propagate.

It should be mentioned that used finite element meshes in both FEM and X-FEM, were fine enough, with adequate element number to eliminate any doubt regarding correctness and accuracy of conducted SIFs calculations.

4. Analysis of the results

The results obtained by described approximate procedure were compared against results obtained thru calculation with mentioned finite element softwares.

The SIFs are calculated for different models with different crack sizes for all the cracks in the configuration, but with same crack increment for all the cracks, because the service data shows that in MSD all cracks are roughly the same length ("catch-up" phenomenon) [6].

The results are presented through normalized stress intensity factors (geometry factors β) for all cracks in analyzed configuration denoted as in Fig. 1. The length of the cracks B and A is marked as a1, and the length of the cracks C and D is marked as a2 (the model is symmetrical). The results are shown in following Tables 1 and 2 and Figs. 5 and 6:

As it can be seen from the graphs, there are excellent agreements between results obtained by used computer programs and approximate procedure which uses influential coefficients, especially for cracks C and D. All differences are within 6%, which is really remarkable considering the different methods used for calculations.

For cracks B and A, the agreements between results are also very good. The differences are within 8%, except for the initial cracks sizes where the difference is 10%, between the Ansys and approximate procedure on the one side and Morfeo on the other. This can be explained by the fact that the initial crack size was less than the thickness of the plate which in case of Morfeo was modeled as 3D solid. This could be improved by better meshing of the region between the cracks and even crack numeration.

Nevertheless, the obtained solutions for SIFs are absolutely acceptable from an engineering point of view.

Also, it should be mentioned that time for SIFs calculation in case of approximate procedure was by far less than time that was used by Ansys and especially by Morfeo. But, only Morfeo simulate crack growth.

Table 1
Comparison of the SIFs solutions for crack tips A and B.

a_1 (mm)	r (mm)	b (mm)	a_2 (mm)	β_B-ANS	β_B-method	β_B-xfem	β_A-xfem	β_1-Bowie	β_2-Bowie	C_{2b}	C_{3d}
1.00	2.4	25	1.00	1.9736	1.9490	1.8046	1.7691	1.944	1.858	0.00129	0.00117
1.60	2.4	25	1.53	1.6969	1.6584	1.6658	1.6787	1.649	1.599	0.00322	0.00275
2.00	2.4	25	1.86	1.5951	1.5424	1.6267	1.5823	1.529	1.490	0.00496	0.00406
2.60	2.4	25	2.28	1.4849	1.4289	1.5206	1.5072	1.410	1.384	0.00803	0.00618
3.00	2.4	25	2.53	1.4285	1.3778	1.4614	1.4378	1.356	1.333	0.01041	0.00769
3.60	2.4	25	2.87	1.3701	1.3234	1.4152	1.3954	1.295	1.276	0.01450	0.01005
4.00	2.4	25	3.00	1.3467	1.2960	1.3729	1.3522	1.265	1.256	0.01691	0.01124
5.00	2.4	25	3.56	1.2974	1.2546	1.3510	1.3221	1.211	1.185	0.02703	0.01608
6.00	2.4	25	4.00	1.2728	1.2323	1.3065	1.3062	1.176	1.141	0.03951	0.02094
7.00	2.4	25	4.43	1.2548	1.2242	1.3092	1.2766	1.151	1.105	0.05674	0.02658
8.00	2.4	25	4.80	1.2545	1.2262	1.2640	1.2546	1.132	1.078	0.07966	0.03268

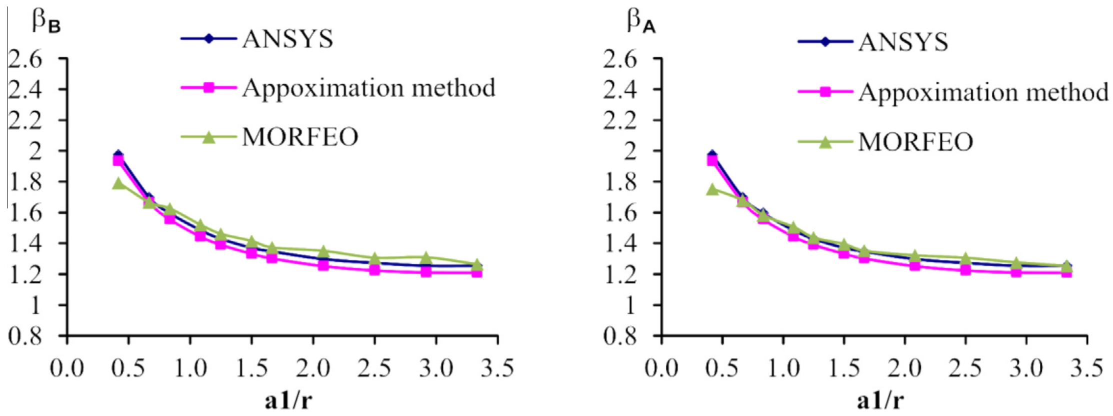

Fig. 5. Normalized stress intensity factors for crack tips B and A.

Table 2
Comparison of the SIFs solutions for crack tips C and D.

a_1 (mm)	r (mm)	b (mm)	a_2 (mm)	β_D-ANS	β_D-method	β_C-xfem	β_D-xfem	β_1-Bowie	β_2-Bowie	C_{1b}	C_{2b}
1.00	2.4	25	1.00	1.9736	1.8615	1.8091	1.7786	1.944	1.858	0.00129	0.00035
1.60	2.4	25	1.53	1.6969	1.6061	1.6146	1.6351	1.649	1.599	0.00336	0.00088
2.00	2.4	25	1.86	1.5951	1.5002	1.5470	1.5384	1.529	1.490	0.00529	0.00138
2.60	2.4	25	2.28	1.4849	1.4006	1.4195	1.4361	1.410	1.384	0.00899	0.00226
3.00	2.4	25	2.53	1.4285	1.3551	1.3676	1.3820	1.356	1.333	0.01200	0.00295
3.60	2.4	25	2.87	1.3701	1.3060	1.3414	1.3288	1.295	1.276	0.01738	0.00415
4.00	2.4	25	3.00	1.3467	1.2930	1.3402	1.3274	1.265	1.256	0.02117	0.00480
5.00	2.4	25	3.56	1.2974	1.2442	1.2900	1.2571	1.211	1.185	0.03448	0.00797
6.00	2.4	25	4.00	1.2728	1.2288	1.2357	1.2418	1.176	1.141	0.05148	0.01200
7.00	2.4	25	4.43	1.2548	1.2318	1.2521	1.2305	1.151	1.105	0.07408	0.01790
8.00	2.4	25	4.80	1.2545	1.2573	1.2431	1.2471	1.132	1.078	0.10328	0.02610

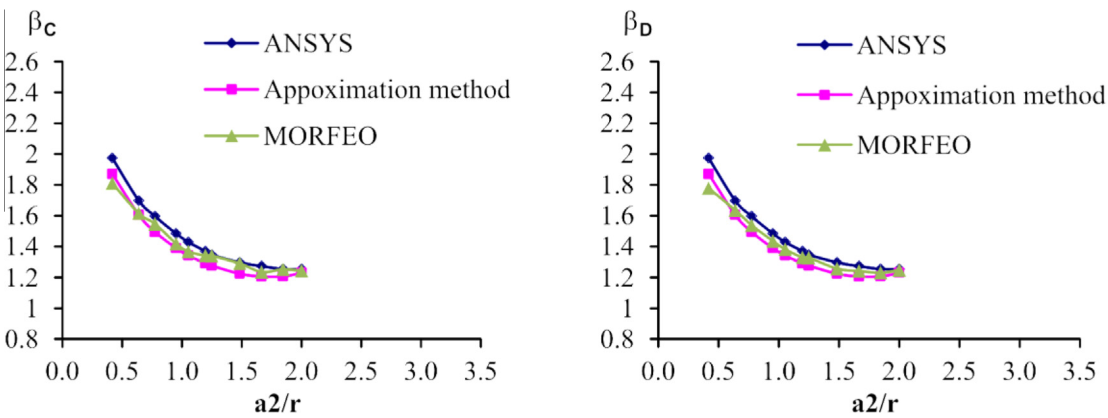

Fig. 6. Normalized stress intensity factors for crack tips D and C.

5. Conclusion

The prediction of crack growth rate and residual strength of cracked structure demands accurate calculation of stress intensity factors. There are some of solutions available, but only for simple geometry configurations with few cracks. On the other hand, there is a lack of available solutions in case of more complex configurations with more than few cracks. The solutions for these configurations require usage of finite element method, which can be very complicated. The mutual influence of the adjacent cracks additionally increases the complexity of stress intensity factors determination. So, a simple and easy to use approximate procedure, for calculating stress intensity factors, was proposed in this paper. The procedure was developed based on existing solution for stress intensity factor in the case of two unequal cracks in an infinite plate subjected to remote uniform stress. The solution for this configuration was used for obtaining interaction effect coefficients which take into consideration the increase of stress intensity factor of analyzed crack tip due to interaction with existing adjacent crack. The calculations of those coefficients were executed thru MathCAD computer program. Special attention was addressed to determination of distances between cracks, which was very important for these calculations.

To demonstrate the capability of the proposed method, it was used for determination of stress intensity factors for a thin plate with three circular holes subjected to uniform uniaxial tensile stress.

The accuracy of the procedure was verified by comparison with the solutions for the same model, obtained by finite element method, where analysis demanded solutions for 2D models with cracks, and extended finite element method where analysis allowed simulation of crack growth in 3D model.

The analysis of the results had shown that the solutions obtained by the proposed approximate procedure are in very good agreement with the results calculated with *Ansys v14* and *Morfeo/Crack for Abaqus* software, and that these solutions can provide stress intensity factors of the analyzed configuration with acceptable accuracy.

It should be mentioned that, the proposed approximation method could be used for determination of stress intensity factors for 3D configurations, since it uses the known solutions for auxiliary configurations with one crack, if auxiliary configurations are 3D, especially for thin configurations. But in case of thicker configurations, the influential coefficients should be determined for 3D cracks and that is the topic of this authors' future work.

References

[1] M. Isida, Analysis of stress intensity factors for plates containing random array of cracks, Bull. JSME 13 (1970) 635–642.
[2] M. Isida, H. Igawa, Doubly-periodic array and zig-zag array of cracks in solids under uniaxial tension, Int. J. Fract. 53 (1992) 249–260.
[3] J. Zhao, L. Xie, J. Liu, Q. Zhao, A method for stress intensity factor calculation of infinite plate containing multiple hole-edge cracks, Int. J. Fatigue 35 (2012) 2–9.
[4] H. Rajiyah, S.N. Atluri, Evaluation of K-factors and weight functions for 2-d mixed-mode multiple cracks by the boundary element alternating method, Eng. Fract. Mech. 32 (1989) 911–922.
[5] D.S. Dawicke Jr., J.C. Newman, Analysis and prediction of multiple-site damage (MSD) fatigue crack growth, NASA Technical paper 3231, 1992.
[6] J.L. Beuth, J.W. Hutchinson, Fracture analysis of multi-site cracking in fuselage lap joints, Comput. Mech. 13 (1994) 315–331.
[7] P. Tong, R. Greif, L. Chen, Residual strength of aircraft panels with multiple site damage, Comput. Mech. 13 (1994) 285–294.
[8] D.J. Cartwright, D.P. Rooke, Approximate stress intensity factors compounded from known solutions, Eng. Fract. Mech. 6 (1974) 563–571.
[9] S.D. Pastrama, P.M.S.T. De Castro, Approximate stress intensity factor for multiple elliptical and through cracks developing from holes, in: The Ninth Portuguese Conference on Fracture, 2004.
[10] M. Kachanov, Elastic solids with many cracks: a simple method of analysis, Int. J. Solids Struct. 23 (1987) 23–43.
[11] H.R. Millwater, A simple and accurate method for computing stress intensity factors of collinear interacting cracks, Aerosp. Sci. Technol. 14 (2010) 542–550.
[12] D.P. Rooke, D.J. Cartwright, Compendium of Stress Intensity Factors, HMSO, London, 1976.
[13] H. Tada, P.C. Paris, G.R. Irwin, The Stress Analysis of Cracks Handbook, Del Research Corporation, 1973.
[14] G. Kastratović, Determination of stress intensity factor of supporting aero structures with multiple site damage (Ph.D. thesis) (in Serbian), Faculty of Mechanical Engineering, University of Belgrade, Serbia, 2006.
[15] B. Rašuo, Aircraft Production Technology (in Serbian), Faculty of Mechanical Engineering, University of Belgrade, Belgrade, 1995.
[16] A. Grbovic, B. Rašuo, FEM based fatigue crack growth predictions for spar of light aircraft under variable amplitude loading, Eng. Fail. Anal. 26 (2012) 50–64.
[17] A. Grbović, B. Rašuo, N. Vidanović, M. Perić, Simulation of crack propagation in titanium mini dental implants (MDI), FME Trans. 39 (2011) 165–170.
[18] A. Grbović, N. Vidanović, G. Kastratović, The use of finite element method (FEM) for simulation crack growth in mini dental implants (MDI), in: Proceedings of The Third International Congress of Serbian Society of Mechanics, Vlasina Lake, 2011, pp. 490–501.
[19] Ansys Inc., in: Mecanical APDL Theory reference, 2011.
[20] I. Babuska, J.M. Melenk, The partition of unity method, Int. J. Numer. Methods Eng. 40 (1998) 727–758.
[21] Cenaero, Morfeo. Available at <http://www.cenaero.be/Page_Generale.asp?DocID=21686&la=1&langue=EN>, last accessed on 20th september 2013.
[22] A. Grbović, Investigation of fatigue life in superalloys structural components (Ph.D. thesis) (in Serbian), Faculty of Mechanical Engineering, University of Belgrade, Serbia, (2012).
[23] N. Moës, A. Gravouil, T. Belytschko, Non-planar 3D crack growth by the extended finite element and level sets – Part I: Mechanical model, Int. J. Numer. Methods Eng. 53 (2002) 2549–2568.