

# Basic Mathematical Model and Simplified Computer Simulation of Swarming Tactics for Unmanned Ground Combat Platforms

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This paper presents a methodological attempt to simulate tactics of simultaneous attacks using robotized (UGV- Unmanned Ground Vehicle) platforms. The numerical method is used to determine the time of departure and the location of platforms in order to achieve their simultaneous arrival to the zone of effective fire onto moving armored targets. The method of proportional navigation, with given initial conditions of velocity and iteratively changing conditions of the initial position, is used. Time delay departure of robotized platforms has been established and calculated in relation to the farthest platform with respect to the target. The simulation results are presented for two hypotheses of target motion (rectilinear and curvilinear) in the lateral oblique attack.

*Key words:* combat system, robotized vehicle, unmanned vehicle, AT rocket, simulation of combat operations, proportional navigation, guidance law, mathematical model.

## Nomenclature

$K$	– Command post
$T$	– Target
$P$	– Platform
$P_i$	– $i$ -th platform
$\vec{r}_{P_i}$	– Position vector of the $i$ -th platform
$\vec{r}_{T_j}$	– Position vector of the $j$ -th target
$\vec{r}_{P_i T_j}$	– Vector of the line of sight of the $i$ -th platform and the $j$ -th target
$\dot{\vec{r}}_{P_i T_j}$	– Relative velocity vector of the $i$ -th platform and the $j$ -th target
$\vec{v}_{P_i}$	– Velocity vector of the $i$ -th platform
$\vec{v}_{T_j}$	– Velocity vector of the $j$ -th target
$\vec{a}_{P_i}$	– Acceleration vector of the $i$ -th platform
$\vec{a}_{T_j}$	– Acceleration vector of the $j$ -th target
$\varphi$	– Angle of the position vector
$\varphi_{P_i}$	– Angle of the position vector of the $i$ -th platform and the $x$ axis [rad]
$\varphi_{T_j}$	– Angle of the position vector of the $j$ -th target and the $x$ -axis [rad]
$\varphi_{P_i T_j}$	– Angle between the $i$ -th platform, the $j$ -th target and the $x$ -axis [rad]
$\dot{\varphi}_{P_i T_j}$	– Angular velocity of the line of sight between the $i$ -th platform, the $j$ -th target and the $x$ axis
$\dot{\varphi}_r$	– Angular velocity of the line of sight
$\varphi_r$	– Angle of the line of sight [rad]

$\theta_P, \theta_T$	– Platforms and target angular components [rad]
$D_{rP}$	– Relative distance between each unmanned combat platform
$\Delta t_{P_i}, \forall i \in (1, n)$	– Delay time for each platform from 1 to $n$
$m_P$	– Mass of platform [kg],
$a_{p_n}$	– Intensity of platform acceleration perpendicular to the longitudinal axis [ $m/s^2$ ],
$g$	– Acceleration due to gravity [ $m/s^2$ ],
$h_{TP}$	– Platform center of gravity height [m] and
$b/2$	– Half the distance (width) of the platform support

## Indices

$T$	– Target
$P$	– Platform
$r$	– Relative position of the platform to the target
$c$	– Command

## Introduction

MODERN combat operations are characterized by their action from a distance with reduced involvement of men in direct clashes. This causes a reduced human interference on the outcome of the action. The focus of the paper will be on the use of combat platforms armed with rocket systems for antitank close combat. The classic use of unguided rockets for antitank close combat requires an operator-gunner at the effective range from 200 to 500 m. The research in this paper considers the use of unmanned combat platforms armed with appropriate antitank systems

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to replace the gunner by a distant operator in the firing loop. This reduces the number of engaged soldiers, as a single gunner in attack tactics, replacing a few of them by one operator, is able to guide a couple of combat platforms by remote guidance and control techniques. This requires an appropriate solution of unmanned combat platform navigation for directing platforms to the firing pod where they can have an impact on tracked targets applying the appropriate tactics. In order to navigate unmanned guided vehicles (UGVs) as combat platforms, the proportional guidance law with varying navigation constants is used in this paper, [1-3]. The navigation of combat platforms, UGVs, armed with suitable systems for close antitank combat against known targets, is organised in the form of so-called swarming tactics [4]. A condition for the successful employment of swarming tactics is to collect a certain number of platforms to an effective rocket range at the same time in order to attack onto the selected target. This means that antitank systems have to be presented in a limited number of platforms required to shoot and destroy the target with a given total probability.

Due to the target superiority and manoeuvrability compared to UGV combat platforms, the paper proposes the use of swarming of unmanned platforms for the efficient impact on targets. The problem of target destruction efficiency and losses of combat platforms during the action was not discussed in this paper, but is covered in detail in [5].

In the simulation program for the „swarming” tactics which use UGV combat platforms, short range antitank rocket systems are used as appropriate platform weapons. The target did not take counterattack action and the platform attacks onto the target had a full sudden and prompt ambush effect.

The typical paths of UGV movements are simulated utilizing the proportional method of guidance with various navigation constants. The simulation of the swarming tactics of a couple of UGVs against the known target did not encounter unguided rockets trajectory simulation, so the total probability of shoots, for every single firing, was the same for all of the engaged platforms. It is accepted that platforms operate simultaneously when access to the target from different directions at the same firing range (OTOT-on time on target model of attack).

### System description and definition of the problem

For this paper, we examine the performance of proportional navigation for the two-dimensional intercept geometry of the point mass target and the unmanned combat platform. The target and the unmanned combat platform are assumed to be point mass models in a plane, moving with the velocities  $V_T$  and  $V_P$ , respectively.

The initial position of the unmanned combat platform is assumed to be the reference point of the relative coordinate system with its initial velocity vector pointing at the initial target position.

A modular unmanned combat platform armed with a close range antitank system (Fig. 1) consists of:

- Unmanned medium-size tracked vehicle (weighing up to 250 kg) [6-7],
- remote controlled system,
- system for navigation and orientation, and
- modular missile system for the near combat with two 90 mm antitank launchers or one 120 mm launcher [6-7]. Platforms are wireless remote controlled with the

movement rate fixed at a ( $V_P$ ) depending on the terrain type and accepted to be about 1.38 m/s. The command control maximum distance ( $(D_{\kappa})_{\max}$ ) depends on the used communication equipment and terrain forms, ranging up to 1000 m. The effective range of the missile system ( $(D_{ef})$ ) depends on the type of built-in armament, ranging below 500 m.

The target is an armored vehicle traveling in the horizontal plane at a speed  $V_T$  of 4.16 m/s. The target movement was arbitrarily limited to a horizontal surface of 4000 m x 2000 m on the perimeter.



**Figure 1.** Unmanned combat platform armed with a short range antitank system

The simulations of engagement of the minimum number of platforms, from their initial positions to the same effective range of their armament ( $(D_{ef})$ ), according to the swarming tactics, could be achieved by the platforms movement as follows:

1. Change in the velocity vector of each UGV platform ( $\vec{v}_{Pi}$ ,  $\forall i \in (1, n)$ ) from its initial position regardless of the initial conditions of each platform,
2. Introducing delay times ( $\Delta t_{Pi}$ ,  $\forall i \in (1, n)$ ) in terms of arbitrary initial position and velocity of movement of constant intensity in the defined guided direction, and
3. Selection of initial firing position vectors ( $\vec{r}_{Pi}$ ) of UGV platforms for the simultaneous departure while moving at a constant speed and in a controlled direction.

In this paper it is assumed that the speed of the platforms and the target are constant ( $V_P = \text{const}$  and  $V_T = \text{const}$ ), and the mentioned cases 2 and 3 are discussed. This requires the introduction of time delays ( $\Delta t_{Pi}$ ) for the starting of each platform (as an initial condition). Case 3 causes adopting the initial position vector for each UGV platforms in order to avoid initial time delays.

The platforms guidance effective range is determined by the method of proportional navigation with a constant coefficient, [1], as it will be discussed below.

### The basic spatial parameters of the swarming tactics

The command post ( $K$ ) is the position that has the ability to track and transmit information about targets and unmanned combat platforms and to send data to unmanned combat platforms. The maximum range of action from the command post is  $D_{T_{\max}}$ , and the acting range on the platforms is  $(D_P)_{\max}$ . The space  $A$  in which the simulation occurs is defined by the coordinates ( $X_{\min}$ ,  $Y_{\min}$ ) and ( $X_{\max}$ ,  $Y_{\max}$ ), respectively,  $A(0.0; 4000, 200)$ .

### The parameters of the navigation method

In this paper, the performance of proportional navigation

is studied. Proportional Navigation Geometry is presented in Figs. 2 and 2a. The proportional navigation guidance law is given by commanded acceleration (1),

$$a_c = NV_r \dot{\varphi}_r, \quad (1)$$

There are two cases:

a) True Proportional Navigation (TPN), where  $a_c$  is applied normal to the line of sight (LOS) [3], where commanded acceleration for the unmanned combat platform is:

$$a_p = \frac{a_c}{\cos(\theta_p - \varphi_r)}. \quad (2)$$

b) Pure Proportional Navigation (PPN), where  $a_c$  is applied normal to the unmanned combat platform velocity vector,  $V_p$  [3].

Here, the commanded acceleration to the unmanned combat platform is  $a_p = a_c$ .

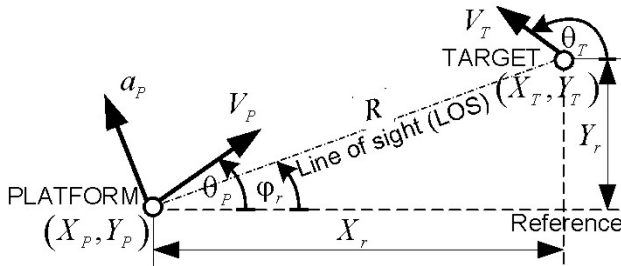


Figure 2. Proportional Navigation Geometry

The basic geometric relations of one unmanned combat platform and the target are given in Fig.2. The letter P refers to the  $i$ -th, the platform, while the T letter refers to the  $j$ -th, the target.

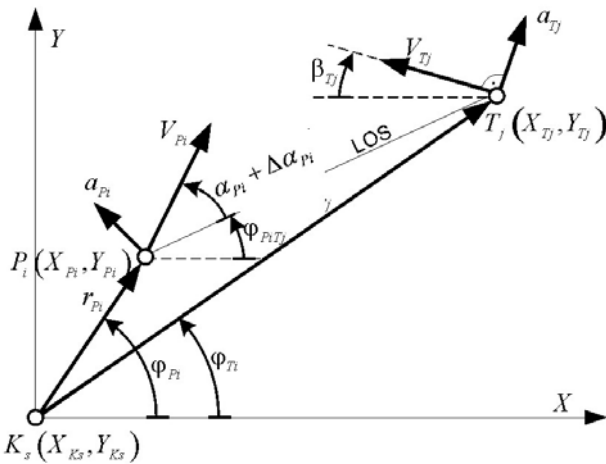


Figure 2a. The geometry relation of the  $i$ -th platform interception of the  $j$ -th target in the horizontal plane

The relation is represented in the inertial coordinate frame Oxy bounded to the surface of earth where:

- $(X_p, Y_p)$  are the coordinates of the platform P, and
- $(X_T, Y_T)$  are the coordinates of the target T, and their initial values at the beginning of the motion action  $t_0 = 0$  s are  $(X_p(0), Y_p(0))$  and  $(X_T(0), Y_T(0))$ .

In accordance with Fig.2, the following refers to:

$\overline{PT}$  - the line of sight,

$R_r = |\vec{r}_{PT}|$  - relative distance between the platform and the target in accordance with expression (3) is,

$$R_r = \sqrt{X_r^2 + Y_r^2} = \sqrt{(X_T - X_P)^2 + (Y_T - Y_P)^2}, \quad (3)$$

where  $X_r$  and  $Y_r$  are the components of the platform and the target relative distance vector to the  $x$  and the  $y$  axis. Their expressions are as follows:

$$X_r = X_T - X_P \quad (4a)$$

$$Y_r = Y_T - Y_P. \quad (4b)$$

The angle of the target sighting line with respect to the  $x$  axis, according to Figs. 2 and 2a ( $\varphi_r = \varphi_{PTi}$ ), will be in the form (5),

$$\varphi_r = \arctg\left(\frac{Y_T - Y_P}{X_T - X_P}\right) = \arctg\left(\frac{Y_r}{X_r}\right). \quad (5)$$

The components of the angles of inclination of platforms and target velocities, Fig.2, are:

$$\theta_p = \arctg\left(\frac{\dot{Y}_p}{\dot{X}_p}\right), \quad (6a)$$

$$\theta_T = \arctg\left(\frac{\dot{Y}_T}{\dot{X}_T}\right). \quad (6b)$$

The target and the platforms move with a constant speed. The target velocity vector ( $\vec{V}_T$ ) forms the angle  $\beta$  with the  $x$  axis. In the general case, the target T can make a manoeuvre with the normal acceleration,  $a_T$  of constant intensity whose vector direction is perpendicular to the target velocity vector making the variable angle  $\beta$ .

In general, the platform velocity vector  $\vec{V}_{Pi}$  makes an overpass angle with the line of sight  $\alpha_p = \alpha + \Delta\alpha$  (Fig.2a). A special case is when  $\Delta\alpha = 0$ . Then, the angle is equal to the initial angle of the overpass,  $\alpha_p = \alpha$ , and provides the platform to meet the target. If the target motion is with a constant speed in a straight line, this results in the trajectory of the UGV combat platform, also a straight line. In this case, no turning commands for the UGV combat platform are required. From Fig.2a, it is obvious that  $\alpha = \alpha_{Pi}$ , so this angle can be determined in the form (7),

$$\alpha = \arcsin \frac{V_T \sin(\beta + \varphi_r)}{V_T}. \quad (7)$$

The unmanned combat platform is not precisely directed to the target at the overpass starting point, since this direction is not known in advance. It depends on the further target motion. To calculate the point where the platform meets the target, it is necessary to have continuous prediction of the future target position. This requires data restoration for each time interval of the iterative procedure in accordance with the recognized and accepted hypothesis of target motion. According to [4], the following hypotheses are adopted:

1. Hypothesis of the rectilinear motion of the target established at a constant speed and a straight-line course, and
2. Hypothesis of the target constant normal acceleration vector.

The hypotheses above correspond to the mathematical models of target motion as follows: the target motion with a constant speed in a straight line and curvilinear. Unknown parameters of motion with arbitrary initial conditions require adjustment to the initial angle  $\alpha$  of overtaking for the value  $\Delta\alpha$ . Then the projections of the initial velocity values of the platform are given by:

$$V_{p,x}(0) = V_p \cos(\varphi + \alpha + \Delta\alpha) \quad (8)$$

$$V_{p,y}(0) = V_p \sin(\varphi + \alpha + \Delta\alpha).$$

A relative distance between unmanned combat platforms ( $Dr_p$ ), is limited and is determined from the condition of nonintersecting trajectories along the current line of target sighting. Further limitations of the method are given in the following chapter.

### Variation of the navigation constant in the proportional navigation law

Theoretically, the proportional guidance law [1] can be represented in different forms as well as the ability to manage the velocity vector steering using the command of the normal acceleration  $a_p = a_r$ . The desired command, expressed by the normal acceleration of the platform [1] is the input to the automatic control loop of each UGV platform. The automatic control block is not considered in this paper, and neither is the essence of the principle of the proportional navigation guidance method. A simplified treatment is accepted on the basis of the centralized control loops in accordance with [1-2].

According to Fig.2, [1] the command parameter can be written in the form:

$$a_r = N v_r \dot{\varphi}_r, \quad (9)$$

Respectively,

$$a_r = N \dot{R}_r \dot{\varphi}_r \quad (10)$$

where:

$N$  - navigation constant (dimensionless ratio). This value was varied from 3 to 5 [1-2] in order to assess its influence on the platform trajectory, during the attack by swarming manoeuvre tactics, as well as its influence on the time of the target interception.

The intensity of the platform acceleration  $a_r$  vector used as the command parameter and its scalar coordinates on the  $x$  and  $y$  axes of the frame are:

$$a_{xr} = -a_r \sin \varphi_r, \quad (11a)$$

$$a_{yr} = a_r \cos \varphi_r. \quad (11b)$$

The intensity of the relative speed of the line of sight  $v_r$ , (relative approaching speed of the platform and the target), according to Fig.2 and [1] is given by the expression:

$$v_r = -\dot{R}_r = -\frac{(X_r \dot{X}_r + Y_r \dot{Y}_r)}{\sqrt{X_r^2 + Y_r^2}} \quad (12)$$

where:

-  $X_r$  and  $Y_r$  are the relative distance vector projections,

-  $\dot{X}_r$  and  $\dot{Y}_r$  are the relative velocity vector projections of the approaching platforms

$$\dot{X}_r = \dot{X}_T - \dot{X}_P \quad (13a)$$

$$\dot{Y}_r = \dot{Y}_T - \dot{Y}_P. \quad (13b)$$

The sighting line angular velocity, according to Fig. 2 and [1], is determined as:

$$\dot{\varphi}_r = \frac{X_r \dot{Y}_r - Y_r \dot{X}_r}{X_r^2 + Y_r^2}. \quad (14)$$

Since the speed of UGV combat platform  $V_p$ , the speed of the target  $V_T$ , and  $\varphi + \alpha_p$  and  $\beta_T$  (angles of inclination of the platform position vector and the target with the  $x$  axis) are known, all variables defined in expressions (9) and (10) can determine the platform acceleration command (10), and its components on the  $x$  and  $y$  frame axis, which would lead to the target interception.

The maximum intensity of the command acceleration of the UGV platform is determined from the condition of the platform stability (15) Fig.3 by the expression,

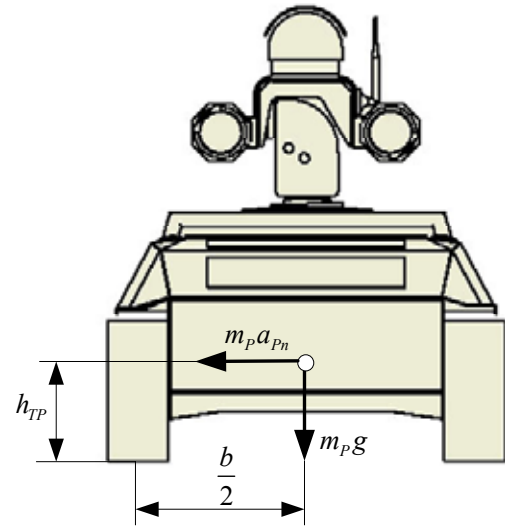


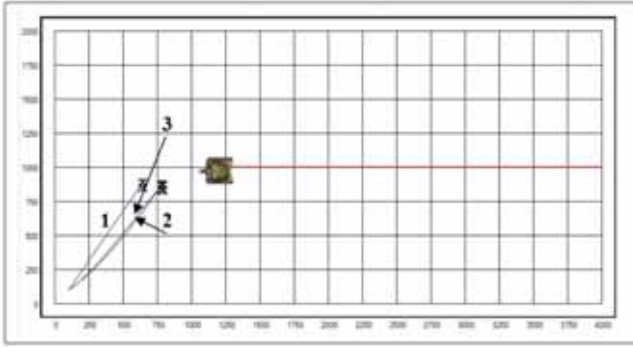
Figure 3. The overturn effect of the command acceleration forces on the UCV combat platform

$$m_p a_{pn} h_{TP} \leq m_p g \left( \frac{b}{2} \right) \quad (15)$$

or, respectively,

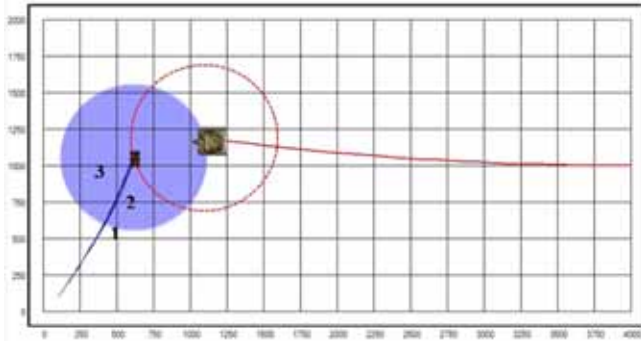
$$a_{pn} \leq g \left( \frac{b}{2h_{TP}} \right) \quad (16)$$

In Fig.4, the simulation trajectories of a single UGV combat platform are given with curve 1 representing the trajectory that corresponds to the theoretical value of the initial angle of overtaking ( $\alpha$ ), given by expression (9). Curve 2 in Fig.4 is given for the deviation (error) of the initial angle of overtaking  $\Delta\alpha = -20^\circ$  of the UGV platform and for the navigation constant  $N=3$ . Curve 3 in Fig.4 is given for the initial angle of the overtaking deviation  $\Delta\alpha = -20^\circ$  and for the navigation constant  $N=4$ .



**Figure 4.** Simulation of the platform and target movements with a variation of the navigation constant  $N$ . ( $V_p=1.38$  m/s=const,  $V_T=4.16$  m/s=const,  $\beta=0^\circ$ , Curve 1-  $\Delta\alpha=0^\circ$ ,  $N=3$ , Curve 2-  $\Delta\alpha=-20^\circ$ ,  $N=3$ , Curve 3-  $\Delta\alpha=-20^\circ$ ,  $N=4$ )

Fig.5 presents the trajectory of the platform that moves at  $V_p=1.38$  m/s = const, with an initial angle of the overtaking deviation  $\Delta\alpha=-20^\circ$ , and the navigating constant  $N=3$ ,  $N=4$  and  $N=5$ , and a moving target speed of  $V_T=4.16$  m/s, with the normal acceleration  $a_T=0.01$  m/s<sup>2</sup>, and the initial angle  $\beta=0^\circ$ .



**Figure 5.** Simulation of the platform and target movements with a variation of the navigation constant  $N$  and the existence of the lateral target acceleration  $a_T$ . ( $V_p=1.38$  m/s=const,  $\Delta\alpha=-20^\circ$ =const,  $V_T=4.16$  m/s=const,  $a_T=0.01$  m/s<sup>2</sup>,  $\beta=0^\circ$ ; Curve 1- $N=3$ ; Curve 2- $N=4$ ; Curve 3- $N=5$ )

All of variations mentioned above are the examples of a single platform motion towards a single target, utilizing the programmed navigation trajectory.

### Centralized navigation of the platforms onto the targets

Fig.2a gives the basic geometric relationship of the  $i$ -th unmanned combat platform in a swarming impact guidance model to the  $j$ -th target.

The command post  $K_s$  has the capability of determining distances to  $T_j$  targets corresponding to the intensity of the  $j$ -th target vector sensors measurements ( $|\vec{r}_{Tj}|$ ,  $\forall j \in (1, m)$ ), as well as the target angle of sighting ( $\varphi_{Tj}$ ,  $\forall j \in (1, m)$ ) measured relative to the  $x$  axis. The command post also determines the positions of the platforms in the polar coordinates ( $(|\vec{r}_{Pi}|$ ,  $\forall i \in (1, n)$ ,  $\varphi_{Pi}$ ,  $\forall i \in (1, n)$ ), measured relative to the  $x$  axis. Also, the command post sends commands to each platform corresponding to the scalar components of the acceleration vector of each platform on the  $x$  and  $y$  axes, by the expression,

$$\vec{a}_{PiTj} = N \left| \dot{\vec{r}}_{PiTj} \right| \dot{\varphi}_{PiTj}, \forall i \in (1, n), \forall j \in (1, m) \quad (17)$$

Since the velocity of the combat platform  $V_{Pi}$ , the target velocity  $V_{Tj}$ , and the angles  $\alpha_{Pi}$  and  $\beta_{Tj}$  are known, for the solution of equation (17),  $\left| \dot{\vec{r}}_{PiTj} \right|$  and  $\dot{\varphi}_{PiTj}$  must be determined.

Based on the geometric relations in Fig.2a and [1] and [2], it follows that:

$$\dot{r}_{PiTj} = \frac{(x_{PiTj}\dot{r}_{x, PiTj} + y_{PiTj}\dot{r}_{y, PiTj})}{r_{PiTj}}, \text{ for } \forall i \in (1, n), \forall j \in (1, m) \quad (18)$$

$$\dot{\varphi}_{PiTj} = \frac{x_{PiTj}\dot{r}_{y, PiTj} - y_{PiTj}\dot{r}_{x, PiTj}}{r_{PiTj}^2}, \text{ for } \forall i \in (1, n), \forall j \in (1, m) \quad (19)$$

where:

- position vector of the  $i$ -th UGV combat platform and its projections on the  $x$  and  $y$  axes are,

$$\vec{r}_{Pi} = (x_{Pi}, y_{Pi}), \text{ for } \forall i \in (1, n), \forall j \in (1, m) \quad (20a)$$

$$x_{Pi} = |\vec{r}_{Pi}| \cos(\varphi_{Pi}), \text{ for } \forall i \in (1, n) \quad (20b)$$

$$y_{Pi} = |\vec{r}_{Pi}| \sin(\varphi_{Pi}), \text{ for } \forall i \in (1, n) \quad (20c)$$

- position vector of the  $j$ -th target and its projections on the  $x$  and  $y$  axes are

$$\vec{r}_{Tj} = (x_{Tj}, y_{Tj}), \text{ for } \forall i \in (1, n), \forall j \in (1, m) \quad (21a)$$

$$x_{Tj} = |\vec{r}_{Tj}| \cos(\varphi_{Tj}), \text{ for } \forall j \in (1, m) \quad (21b)$$

$$y_{Tj} = |\vec{r}_{Tj}| \sin(\varphi_{Tj}), \text{ for } \forall j \in (1, m) \quad (21c)$$

- vector of the relative distance between the  $i$ -th combat platform and the  $j$ -th target with its projections on the  $x$  and  $y$  axes are

$$\vec{r}_{PiTj} = (x_{PiTj}, y_{PiTj}), \text{ for } \forall i \in (1, n), \forall j \in (1, m) \quad (22a)$$

$$x_{PiTj} = |\vec{r}_{PiTj}| \cos(\varphi_{PiTj}), \text{ for } \forall i \in (1, n), \forall j \in (1, m) \quad (22b)$$

$$y_{PiTj} = |\vec{r}_{PiTj}| \sin(\varphi_{PiTj}), \text{ for } \forall i \in (1, n), \forall j \in (1, m) \quad (22c)$$

- relative distance of the  $i$ -th unmanned combat platforms and  $j$ -th target with their components on the  $x$  and  $y$  axes,

$$r_{PiTj} = \sqrt{x_{PiTj}^2 + y_{PiTj}^2}, \text{ for } \forall i \in (1, n), \forall j \in (1, m) \quad (23a)$$

$$x_{PiTj} = x_{Tj} - x_{Pi}, \text{ for } \forall i \in (1, n), \forall j \in (1, m) \quad (23b)$$

$$y_{PiTj} = y_{Tj} - y_{Pi}, \text{ for } \forall i \in (1, n), \forall j \in (1, m) \quad (23c)$$

- relative approaching velocity vector of the  $i$ -th unmanned combat platforms and the  $j$ -th target with their projections on the  $x$  and  $y$  axes are

$$\dot{\vec{r}}_{PiTj} = (\dot{r}_{x, PiTj}, \dot{r}_{y, PiTj}), \text{ for } \forall i \in (1, n), \forall j \in (1, m) \quad (24a)$$

$$\dot{r}_{x, PiTj} = \left| \dot{\vec{r}}_{PiTj} \right| \cos(\varphi_{PiTj}), \text{ for } \forall i \in (1, n), \forall j \in (1, m) \quad (24b)$$

$$\dot{r}_{y.PiTj} = \left| \dot{\vec{r}}_{PiTj} \right| \sin(\varphi_{PiTj}), \text{ for } \forall i \in (1, n), \forall j \in (1, m) \quad (24c)$$

- acceleration vector of the  $i$ -th unmanned combat platform for the  $j$ -th target with their projections on the  $x$  and  $y$  axes is:

$$\vec{a}_{PiTj} = (a_{x.PiTj}, a_{y.PiTj}), \text{ for } \forall i \in (1, n), \forall j \in (1, m). \quad (25)$$

The expression (17) can be written in the matrix form as

$$\mathbf{a}_{PT} = \mathbf{NR}_{PT}\boldsymbol{\varphi}_{PT} \quad (26)$$

where for  $\forall i \in (1, n), j = 1$  for the selected target

$$\mathbf{a}_{PT} = \begin{bmatrix} a_{P1} \\ a_{P2} \\ a_{P3} \\ \vdots \\ \vdots \\ a_{Pi} \\ \vdots \\ \vdots \\ a_{Pn} \end{bmatrix}, \mathbf{R}_{PT} = \begin{bmatrix} \dot{r}_{P1T1} \\ \dot{r}_{P2T1} \\ \dot{r}_{P3T1} \\ \vdots \\ \vdots \\ \dot{r}_{PiT1} \\ \vdots \\ \vdots \\ \dot{r}_{PnT1} \end{bmatrix}, \boldsymbol{\varphi}_{PT} = \begin{bmatrix} \dot{\varphi}_{P1T1} \\ \dot{\varphi}_{P2T1} \\ \dot{\varphi}_{P3T1} \\ \vdots \\ \vdots \\ \dot{\varphi}_{PiT1} \\ \vdots \\ \vdots \\ \dot{\varphi}_{PnT1} \end{bmatrix}. \quad (27)$$

Expression (26) determines the accelerations of every unmanned combat platform with respect to the selected target and expression (27) provides a matrix form of the central command for all UGV platforms.

### Swarming tactics simulation using the proportional navigation law

To guide UGV combat platforms successfully onto the firing pod as the position where they can attack the target together, in accordance with the swarming tactics model, it is necessary to determine the following:

1. distance to the target of each platform over the command pod distance  $K_s$  and the target range, respecting the condition that the required numbers of platforms approach to the effective range ( $D_{ef}$ ) as the obtained value;
2. possible delay of starting times for the unmanned combat platforms ( $\Delta t_i$ , for  $\forall i \in (1, n)$ ) for their simultaneous arrival to the effective range ( $D_{ef}$ ), relative to the initial start of the farthest platform;
3. safety distance between the unmanned combat platforms  $D_{rPi} \geq (D_{rPi})_{\min}$ , for the efficient use of built-in weapons;
4. step of data integration ( $t_{kz}$ ).

The condition for the determination of the limit number of unmanned combat platforms ( $k$ ) is determined according to [3], taken as  $k=4$  and  $k=3$  in this paper.

The delay time is determined in relation to the largest relative platform position vector from the target ( $\vec{r}_{PiTj}$ ), and is assigned  $t = 0$  s, while other platforms are granted the delay times:  $\Delta t_{Pi} = (t_{Pi,kr})_{\max} - t_{Pi,kr}$ , for  $\forall i \in (1, n)$ . The relative distance between the platforms ( $D_{rP}$ ) was adopted on the recommendation and is 30 m.

The distance between each of the platforms is controlled during the simulation of the range of devices for sending the data to unmanned combat platforms, and is expressed in the form  $D_{PiK} \leq (D_P)_{\max}$ , for  $\forall i \in (1, n)$ . Each step of the simulation in time is given by the interval  $H=10E-4$  seconds. The procedure is repeated until the platforms approach to the effective range ( $D_{ef}$ ).

The simulation software with the appropriate interface and characteristic symbols of the target and the platforms is shown in Fig.6. This figure graphically shows the basic vision for the swarming tactics simulation results, using the proportional guidance law for several unmanned combat platforms attacking the tracked target from the command pod.

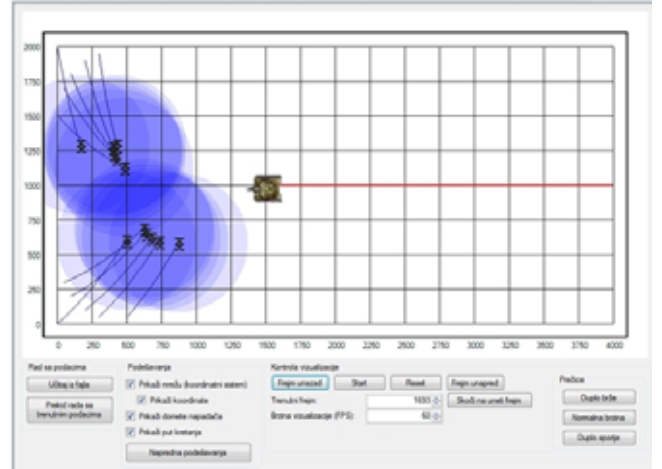


Figure 6. Visual control interface for the simulation of navigating several platforms onto one target utilizing the swarming tactics

### Input data and hypotheses

**Case A.** The target motion is with a constant speed in a straight line parallel to the  $x$  axis  $\beta=0^\circ$ , while four platforms are placed at randomly chosen arbitrary starting positions opposite its direction, Fig.7.

**Case B.** The target motion is in the curved line at a constant velocity and a constant normal acceleration [8]

$a_T = -0.02 \text{ m/s}^2$ , while three platforms are placed at randomly chosen arbitrary starting positions opposite its direction, Fig.8.

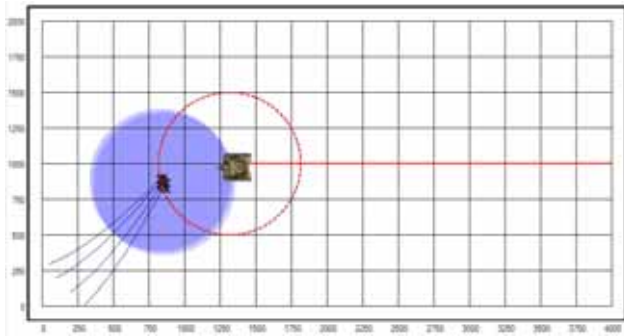
Also, some other hypotheses are adopted:

- Spatial limits of the terrain, in both cases, were  $A(0.0; 4000, 2000)$ .
- The initial positions of the platform pods were placed in the zone up to 500 m from the command position and given by the coordinates:  $P_i(X_{Pi}, Y_{Pi})$ , while the initial velocity was  $V_P=1.38 \text{ m/s}=\text{const}$ ,
- The deviation of each line of sight for every platform was given by the initial angle  $\Delta\alpha = -20^\circ$ , while the coefficient of the proportional navigation  $N=3$  was the same for all platforms.
- The effective range in which the platform was led towards the target near to the point of the impact was  $D_{ef}=300 \text{ m}$ .
- The initial position of the target was given by the coordinates  $T_i(4000, 2000)$ ,  $V_T=4.16 \text{ m/s}$ ,  $a_T \text{ m/s}^2$ .
- Command post was at the origin.

After entering these hypotheses as input data, the procedure for calculating the trajectories is started by the proportional guidance law for each platform  $P_i$  for  $i=1$  to 4 (Fig.7) and, for  $i=1$  to 3 (Fig.8).

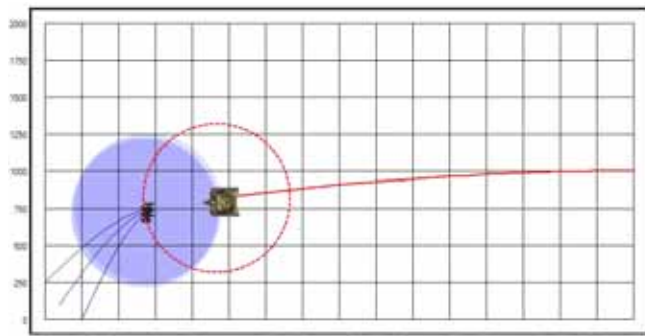
When all available platforms reach, at the same time, the distance to the target that is equally effective for the weapons range of the individual platforms ( $R_i=D_{ef}$ ) - this distance is given in the form  $r_{PiTj} = \sqrt{x_{PiTj}^2 + y_{PiTj}^2} = D_{ef}$ , for  $\forall i \in (1, 4), j = 1$  for the condition A and  $\forall i \in (1, 3), j = 1$  for the condition B - the simulation ends.

Figs. 7 and 8 present the results of the visual navigation paths as the results of the simulation of four unmanned combat platforms swarmed on to single target in swarming tactics, for the case A and the case B with appropriate conditions.



**Figure 7.** Simulation of four unmanned combat platforms and one target movement without variation of the navigation constant ( $i=1$  to 4,  $V_{pi}=1.38$  m/s=const,  $V_t=4.16$  m/s=const,  $a_t=0$ ,  $\beta=0^\circ$ ,  $\Delta\alpha_i=0^\circ$ ,  $N_i=3$ =const)

For successful swarming tactics [3], it was necessary that four platforms ( $k=4$ ) for the case A or three platforms ( $k=3$ ) for the case B arrive at the same time at the effective range ( $D_{ef}$ ) up to the target.



**Figure 8.** Simulation of three unmanned combat platforms and one target movement without variation of the navigation constant but with the target lateral acceleration ( $i=1$  to 3,  $V_{pi}=1.38$  m/s=const,  $V_t=4.16$  m/s=const,  $a_t=-0.01$  m/s<sup>2</sup>,  $\beta=0^\circ$ ,  $\Delta\alpha_i=0^\circ$ ,  $N_i=3$ =const).

### Relevance of the model

Other ways of UGV combat platform guidance may be considered from the literature, with different mathematical models of the target interception.

A simulation model applied in this paper as the part of swarming tactics has the aim to establish the minimum number of platforms to be guided at the same time to the distances where they could attack the tracked target by a simultaneous sudden attack and with the improved total probability of the target destruction by their built-in weapons (OTOT attack model). In this sense, the main task is the same time of arrival at the effective target range. This is the first required condition for the swarming tactics implementation.

Finally, it is necessary to improve the simulation model and extend it with additional 3D space requirements and calculation of realistic probability of fire and destruction for the method of guidance to be justified in combat conditions.

### Conclusion

The authors discussed the applicability of the proportional navigation guidance law for UGV combat platforms armed with antitank rockets, for close combat

attacks on single targets, with arbitrary initial positions, guided from the given command pod position.

Based on the geometric relationship between UGV combat platforms and the target, the appropriate correlations based on the model of the proportional navigation law are given as a simulation model to refer to the elements of the swarming tactics.

The paper presents the results of the simulations of the characteristic trajectories of UGV combat platforms by the appropriate selection of the navigation coefficients in their proportional guidance method used for the swarming tactics for a couple of robotized, semiautomatic and armed platforms against a single target. The necessary number of platforms depends on the type of weapons. This is given in the second simulation, where large-caliber weapons were used with a lower number of platforms. The basic preliminary analysis resulted in a reduced number of platforms from 4 to 3, sufficient for a successful attack in accordance with the tactics. The effective range was not changed.

In its further development this method provides the ability to exclude the command pod after the target motion parameters have been established and to be fully self-guided and controlled, particularly in the cases when the target trajectory is known in advance as in the case of motion along the roads the geographic coordinates of which can be known through the Geographic Information System (GIS). This allows swarming tactics with homing guidance of UGV combat platforms to be used in road ambushes in their further employment.

Future improvement of the simulation model could be related to the use of some other guidance laws for navigating robotic combat platforms. Considerations of different guidance scenarios for the navigation of robotic combat platforms onto more targets of the same or different type, with the usage of appropriate tactics, can also improve the simulation model.

The inherent simplicity of the law of proportional navigation, used in this paper, enables platforms to become self-guided devices instead of command-guided ones.

The further investigation has to deal with that problem in order to focus on all possible target trajectories and manoeuvres.

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## Osnovni matematički model i računarski pojednostavljena simulacija taktike “rojenja” kopnenih borbenih platformi bez posade

Ovaj rad predstavlja metodološki pokušaj simulacije taktike istovremenog napada robotizovanih borbenih platformi (UGV-besposadnih vozila za kretanje po zemlji). Za određivanje mesta i vremena polaska platformi u cilju istovremenog dolaska u zonu efektivne vatre na pokretni oklopni cilj koristi se numerička metoda. Korišćen je metod proporcionalne navigacije, sa datim početnim uslovima brzine i iterativno promenljivim početnim pozicijama robotizovanih borbenih platformi. Vreme kašnjenja polaska svake robotizovane borbene platforme određuje se u odnosu na platformu najudaljeniju od cilja. Rezultati simulacije su prikazani za dve hipoteze kretanja cilja u bočnom pravcu napada (pravolinijski i krivolinijski).

*Ključne reči:* borbeni sistem, robotizovano vozilo, vozilo bez posade, PO raketa, simulacija borbenih dejstava, proporcionalna navigacija, zakon vodenja, matematički model.

## Основные математические модель и упрощенное компьютерное моделирование „svorming” тактики наземных беспилотных боевых платформ

Эта статья представляет собой попытку методологии моделирования тактики одновременной атаки роботизированных боевых платформ (UGV-безэкипажные транспортные средства для передвижения по суше). Численный метод использовался для определения места и времени отправления платформы с целью одновременного прибытия в зону эффективного огня по движущейся мишени имеющей броню. Использован метод пропорциональной навигации с заданными начальными условиями скорости и многократно изменяемыми исходными позициями роботизированных боевых платформ. Время задержки отправления каждой роботизированной боевой платформы определяется ссылкой на самую отдаленную от цели платформу. Результаты моделирования представлены в течение двух гипотез движения цели в поперечном направлении атаки (прямые и изогнутые).

*Ключевые слова:* боевая система, роботизированные транспортные беспилотные средства, управляемая ракета, моделирования боевых действий, пропорциональная навигация, законы наведения, математическая модель.

## Simulation par ordinateur de la tactique Swarming chez les plates-formes de combat sans équipage

Ce papier représente un essai méthodologique de la simulation tactique d'une attaque simultanée des plates-formes de combat robotisées (UGV – des véhicules sans équipage pour les déplacements sur terrain). On utilise la méthode numérique pour la détermination du lieu et du temps de départ des plates-formes afin d'arriver en même temps dans la zone du feu effectif sur l'objectif mobile blindé. On a employé la méthode de la navigation proportionnelle avec les conditions initiales de vitesse données et avec les positions initiales variables itérativement des plates-formes de combat robotisées. Le temps de retard du départ pour chaque plate-forme de combat robotisée est déterminé par rapport à la plate-forme la plus éloignée de l'objectif. Les résultats de la simulation ont été présentés pour deux hypothèses du déplacement de l'objectif dans la direction latérale de l'attaque (rectiligne et curviligne).

*Mots clés :* système de combat, véhicule robotisé, véhicule sans équipage, missile antichar, simulation des actions de combat, navigation proportionnelle, loi de guidage, modèle mathématique.