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Original scientific paper

A NEW METHODOLOGY FOR PREDICTION OF HIGH-CYCLE CONTACT FATIGUE FOR SPUR GEARS

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Abstract: *High-cycle contact fatigue is a localized phenomenon that occurs in highly stressed grains of the material on or under the contact region. The contact zones of tooth flanks for meshed gears are subjected to contact fatigue damages that causes pitting and leads to gears failure.*
The objective of this paper is to give a new viewpoint in contact fatigue prediction in the case of high-cycle fatigue. The main aim of the presented research is to make the methodology for direct calculation of fatigue crack initiation in contact zones. This methodology is developed for spur gears and used up-to-date methods and multidisciplinary approach. Two methods are built in the new methodology: the Theory of Critical Distances (TCD) and the Finite Element Method (FEM).
In this paper the comparative analysis of standard and new methodology for prediction of fatigue crack initiation on tooth flanks is presented. The advantages of methods and procedures used in the new methodology are presented through a case study of particular gear pair. The Finite Element Analysis on 3D gear contact model is used for stress and strain calculation and prediction of the maximum stress location in contact zones along the gear facewidth. The stress gradient curves from the contact zone are made for a pinion tooth in different cross sections along gear facewidth. The Theory of Critical Distances used these stress gradients and material characteristics for fatigue crack initiation prediction. The benefits of presented methodology are shown by the detail analysis of the obtained results.

Key words: *gears, contact, fatigue, Finite Element Analysis, Theory of Critical Distances*

1. INTRODUCTION

More of hundred years, engineers and scientists investigate contact fatigue, but in recent years this phenomenon is still in focus. The prediction of high-cycle contact fatigue is of importance in engineering applications where specific contact loads appear (e.g. gears, rolling bearings and rail–wheels systems), [1]. The fatigue is a material characteristic process that depends of many factors. The most frequent causes of failure are defects such as pores or cracks introduced during manufacturing. But, an inadequate design, or approximations in calculations of stresses in contact zones as stress raiser, significantly contributes to failures. One of the key points in gears failure analysis is the ability to make accurate predictions of the fatigue load capacity in complex load conditions.
Only multidisciplinary investigations could bring solution. Material and mechanics scientists and mechanical engineers have to work together to create procedure for teeth contact fatigue prediction. Only investigations of material characteristics and material behaviour in real load cases simultaneously with calculations of real load distribution and stresses in contact zones of gears teeth, could step forward in this problem. This is the goal of the multidisciplinary investigation of contact fatigue prediction described in this paper. The improvement of pitting load capacity calculation of involute gears is the ultimate goal of developing of this new approach of crack initiation prediction in contact zones.

2. PREDICTION OF CRACK INITIATION IN GEAR CONTACT ZONE –STANDARD VS NEW METHODOLOGY

In the recent years, almost all authors have been used the Hertz analytical solutions for simple cases of contact, [2], as start point in gear contact stress calculation, [3, 4]. The Hertz's solution for two cylinders in contact is used for standard stress calculation of meshing teeth in contact zones. Some authors in their new article, [3], still used Hertz theory and only analytical approach for new standpoint in pitting load capacity calculation of spur and helical gears. But, proper computation of forces and deformations in the teeth contact zone is crucial in determining the stress distribution in teeth contact and teeth roots and couldn't be solved by Hertz's theory.
The algorithm of standard methodology for prediction of crack initiation and crack growing for gears is shown in fig.1 and includes standard analytical methods for gear load capacity calculation in combination with basic principles of linear elastic fracture mechanics (LEFM). Both of standard methods, gear load capacity calculation and LEFM for crack growing prediction, could be chanced with new methods. The result is the new methodology for fatigue crack initiation prediction in contact zones of gears, shown with algorithm in fig.2.

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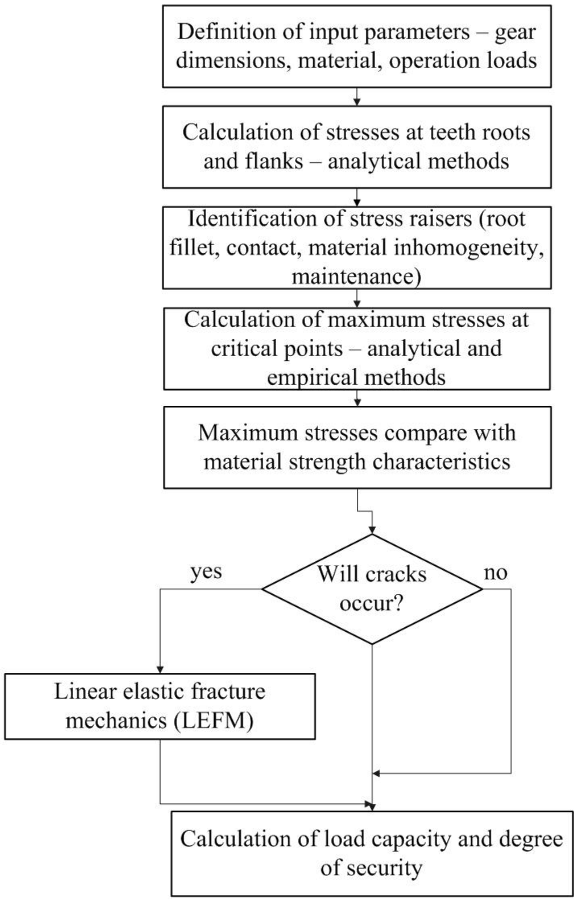


Fig.1. Standard methodology for prediction of crack initiation and crack growing for gears

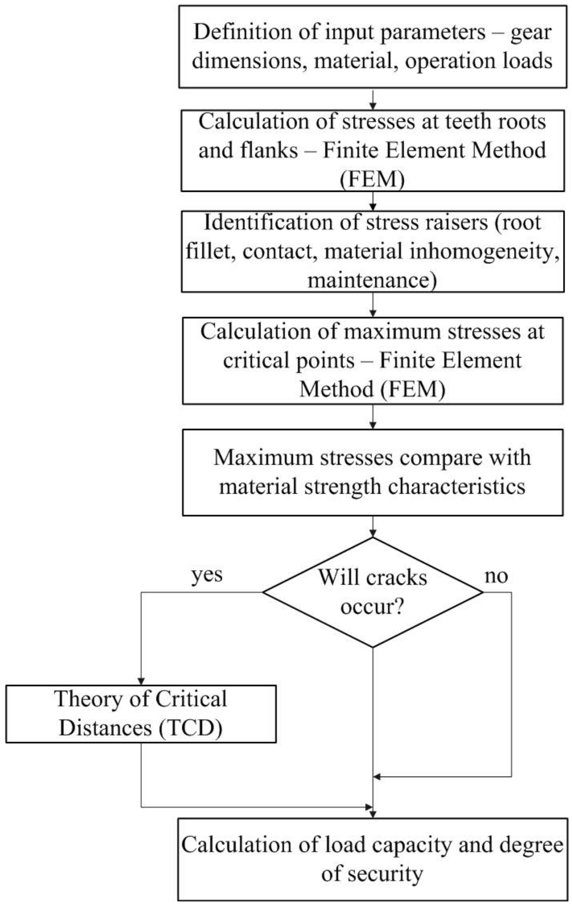


Fig.2. New methodology for crack initiation prediction for gears

Transmissions with requirements of high efficiency and reliability, calculated by standard empirical procedures, [5], lead to over dimensioning and miscalculations. Complex gears geometry and gear tooth contact with specific high-cycle load character, require accurate and reliable solving of gear teeth stresses. Only numerical iteration methods can give such solution. Finite Element Method (FEM) has proven to be the most convenient approach to investigating the gear teeth contact and the associated stress and strain characteristics under a range of loading conditions, [6–8]. The FEM possibilities on structural analysis are described in [6] through developing of 3D gear model for non-involute gear calculations. The similar FEM model for involute gear pair is developed, [9–14], and used in this paper.

The new methodology for crack initiation prediction in contact zones of spur gears described in this paper, fig.2, used nonlinear contact analysis, [10], and iteration method for calculation of contact stresses as function of real contact geometry, nominal load value and real load distribution along path of contact and line of teeth pair contact, [14].

Also, there is no commonly accepted set of standard methods for predicting the effect of contact stress raisers. The scientific community has not decided which method are most suitable, and under which circumstances. But, it seems that the problem will be solved with the appearance of basic principles of the Theory of Critical Distances (TCD), [15, 16].

The new methodology presented in this paper for crack initiation prediction in contact zones of spur gears, fig.2, joints the TCD point theory to gear fatigue capacity calculation.

3. FEA FOR SPUR GEARS CONTACT ANALYSIS

One of the key objectives in presented research has been to develop a comprehensive finite element model of meshed gears, which can be used to analyze actual contact deformations of meshed teeth, to monitor tooth flank deformations and stress, and tooth root deformations and stress at the same time. The ultimate aim of this research and the developed FEM model is to analyze and quantify the influence of the nominal load value on the gear flank and gear root load capacity.

Some theory reference, [17], discuss the capabilities and procedures of using finite elements to model contact problems, but do not give the details needed for creating and solving such a model. While real contact behaviour is modelled, numerous decisions must be made. In many cases, it may be possible to use constraints to capture some of the contact problem behaviours and treated contact problem as a linear condition. But, often it isn't sufficient and a contact must be modelled with contact finite elements and treated as nonlinear problem. During developing of gear finite element analysis authors must solved a lot of problems in order to obtain sufficiently economic and precise model in accordance with all defined targets.

Contact problems are highly nonlinear and require significant computer resources to solve. The FEM treats a contact problem as a part of the general problem of bodies' movement in space and their interaction. The problem of this type can be understood as two bodies in contact because of the action of external forces. One of these bodies is defined as a contactor body, the other as a target body. The choosing of contact body is simple when the Finite Element Method examines a rigid-to-flexible body contact. But in the case of flexible-to-flexible body contact, the FEM procedure is complex and requires excellent knowledge of the character of contact which is to be analyzed. When the bodies are in contact, contact forces appear. These forces prevent mutual penetration of bodies and provoke deformations in contact areas.

For the purpose of meshed gears modelling ANSYS 11.0 commercial software are used. This software has excellent capabilities for geometry precise modelling and nonlinear analysis when two deformable bodies are in contact. FEM supports three types of contact models: point-to-point, point-to-surface, and surface-to-surface. Each type of model is appropriate for specific types of problems. The point-to-surface contact model has been chosen for contact modelling of gear teeth in mesh. These elements are not compatible with higher-order solid elements because the mid-side nodes of the quadratic elements are not used for the contact faces. Therefore, the finite element type chosen for 3D FEM gear pair model developing is 3D isoparametric structural solid element defined by eight points. First, the areas where contact might occur during the deformation of model have been identified and then contact elements are defined. For most efficient solution smaller localized contact zones have been defined, thereby the chosen zones are adequate to capture all necessary contact.

For point-to-surface contact elements, there are two different ways to approach contact – the penalty method alone or a Lagrange multiplier added to the penalty method, [17]. The penalty method uses a contact "spring" to establish a relationship between two contact surfaces with the spring stiffness called the contact stiffness. The penalty method modifies the present stiffness matrix by adding large terms to prevent too much penetration, while the Lagrange method is an iterative series of penalty methods. The contact stresses are augmented during equilibrium iterations so that the final penetration is smaller than the allowable tolerance. Compared to the penalty method, the Lagrange method usually leads to better conditioning and is less sensitive to the magnitude of the contact stiffness. However, in some analyses, the augmented Lagrange method may require additional iterations, especially if the deformed mesh becomes too distorted.

Some experienced analysts feel that "better" contact performance and results may be found when the penalty formulation alone is used, [18]. During involute gear finite element model developing, authors choose penalty method alone for teeth contact, and have obtained enough precise and at the same time economic model, [11].

One more decision that must be made is select asymmetric or symmetric contact. Asymmetric contact has all contact elements on one surface and all target elements on the other surface. Symmetric contact is less computer time efficient than asymmetric contact, but many analyses will require its use (typically to reduce penetration). Specific situations that require symmetric contact include models where the distinction between the contact and target surfaces is not clear and both surfaces have very coarse meshes. In accordance with this, symmetric contact is chosen for simulation of contact conditions on gears teeth.

In accordance with appropriate investigations, [11], expected maximum contact stress point on path of contact is point B – point of passing from period with two tooth pairs in contact to single meshed tooth pair period. Therefore, 3D FEM tooth contact model is developed for contact in point B and for single meshed tooth pair period. In order to obtained 3D FEM gear pair model for efficiency monitoring contact stresses along the facewidth, the 2D model is sweeped (copied in normal direction along the length equal to gears facewidth), [9].

4. THEORY OF CRITICAL DISTANCES FOR CONTACT PROBLEMS

The Theory of Critical Distances (TCD) is a recent method or group of methods for the prediction of failure on engineering components and structures with various types of stress raisers.

Failure commonly occurs due to the initiation and growth of a crack, through mechanisms such as brittle fracture, fatigue and stress corrosion cracking. In these cases both the maximum stress and the stress gradient are important in determining whether failure will occur. Materials possess inherent length scales which are related (in complex ways) to their microstructure and modes of deformation and damage. The interaction between the length scale and the stress gradient determines whether crack will occur from a given feature. The major assumption is that any type of stress raiser could be considered in the same way, in particular that cracks are not a special case. This assumption is rather questionable, since different results can be obtained from a specimen containing a pre-crack induced by fatigue, and an otherwise identical specimen containing a sharp machined notch.

The local stresses around a notch, or any other stress concentration like contact, can be represented in a simple form by a diagram of the stress as a function of distance from stress concentration raiser, fig.3. We will assume that the stress analysis is an elastic one, and that the maximum normal stress is drawn. Poor accuracy can be expected if prediction of fatigue life is based on the range of stress at the stress raiser (i.e., at a distance of zero in fig.3). The use of plastic strain range instead of stress range has the disadvantage that an elastic–plastic analysis must be performed (or approximated). In any case it will be still quite inaccurate for the prediction of high-cycle fatigue in features with high K_t - stress concentration factor, [17, 19]. This means that the first step in the application of critical distance methods is to use them explicitly, generating stress–distance diagram such as fig.3 and read the necessary stress values from FEA.

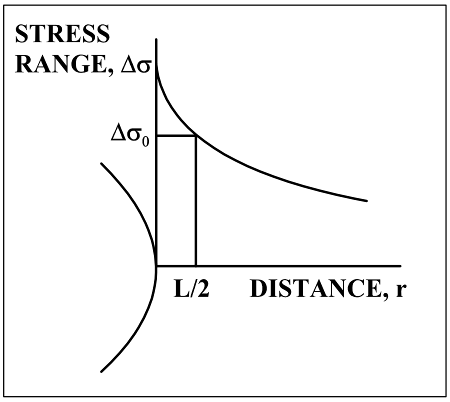


Fig.3. Schematic explanation of TCD point method

The second step is calculating the value of the critical distance. The underlying theory was suggested by and validated against experimental data by Taylor, [15, 16, 17]. Basically, it is essentially a combination of stress raiser fatigue and linear elastic fracture mechanics (LEFM) concepts. The assumption that the TCD point method and TCD line method are valid for all types of stress raisers, allows to define the stress–distance diagram at the fatigue limit for a cracked body, [18], for which the stress intensity range is equal to the threshold value for the material: $\Delta K = \Delta K_{th}$. The result is that the critical distance for the point method is $L/2$, [16, 17], where:

$$L = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta \sigma_0}\right)^2 \qquad (1)$$

In this equation $\Delta\sigma_0$ is the fatigue limit of standard, unnotched specimens of the material, and ΔK_{th} is threshold stress intensity, given in table 1 for material of investigated gear pair. The fact is that is often difficult to define the accurate fatigue limit, the stress range corresponding to a given number of cycles. The range from 1 to 10 million is generally used for determination of fatigue limit. It has been demonstrated by comparison with experimental data [19] that the use of this value of $\Delta\sigma_0$ for L gives good predictions in many different materials, [16].

The application of TCD in solving fretting fatigue was successfully so there is no reason why TCD couldn't be applicable to contact fatigue. In practice, however, contact fatigue normally involves a moving point of contact, as in rolling contact between gear teeth and bearing components. Whilst the initiation and early growth of the crack may be similar to that in standing fatigue, the moving force has a considerable effect on the subsequent crack growth and the tendency for the crack to turn back to the surface and cause spalling. Thus, the TCD is useful in modelling the early stages of this process (perhaps for predicting the limit below which only non-propagating microscopic cracks will occur). But, a crack-propagation analysis would be needed to describe the entire process, [20, 21].

5. A CASE STUDY

In this paper a particular real gear pair with high value of transmission ratio is used for developed methodology investigation. The main characteristics of the gear pair are: number of teeth z_1=20, z_2=96; standard tooth involute profile, addendum modification coefficients x_1=0.3, x_2=0.2; facewidth b=175 mm; module m_n=24; pressure angle α_n=20°; rotational wheel speed n_2=4.1596 min^{-1}; wheel torque T_2=631.7 KN·m, [11]. Gears material is steel 17CrNiMo7 with E = 206 000 N/mm^2; ν =0.3, and mechanical characteristics taken from literature, [22], and given in table 1.

Table 1. Steel 17CrNiMo7 characteristics

R_{p02} (MPa)	R_m (MPa)	$\Delta\sigma_o$ (MPa)	ΔK_{th} $(MPa\sqrt{m})$
1021	1366	612	18

After definition of input parameters for a spur gear pair and developing of 2D FEM model for gear mesh simulation, the Finite Element Analysis with iteration procedure is performed, [14]. Then, in accordance with the methodology algorithm, shown in fig.2, the contact position with maximal contact stresses are identified and appropriate 3D FEM gear model is developed. Finite Element Analysis (FEA) used this model and real load condition, while nonlinear contact solution is obtained with penalty method and 10 load sub-steps. The obtained result for normal pressure along gear facewidth is shown in fig.4. The maximum normal compressive stresses along gear facewidth for pinion tooth of modelled gear pair are used for drawing of diagram in fig.5. This diagram has shown normal load distribution along pinion tooth for single mesh period in point B – point of passing from period with two tooth pairs in contact to single meshed tooth pair period. The obtained results for stresses and load distribution along facewidth are in expected ranges, [4, 8].

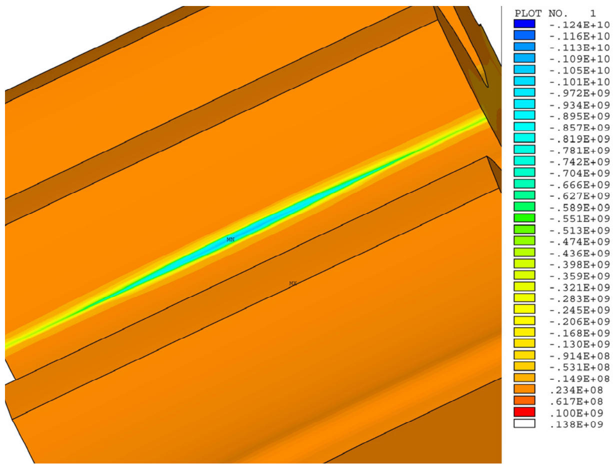


Fig.4. Normal contact stresses on the pinion tooth flank

The next step of the new methodology presented in this paper, fig.2, is the question "Will cracks occur?" The Theory of Critical Distances will give the answer. The investigation whether crack will occur or no is done for 16 sections along pinion facewidth. The 3D model of pinion has been cat with cutting planes parallel with the gears faces. The positions of cutting planes for creating of cross sections of pinion are shown in fig. 6.

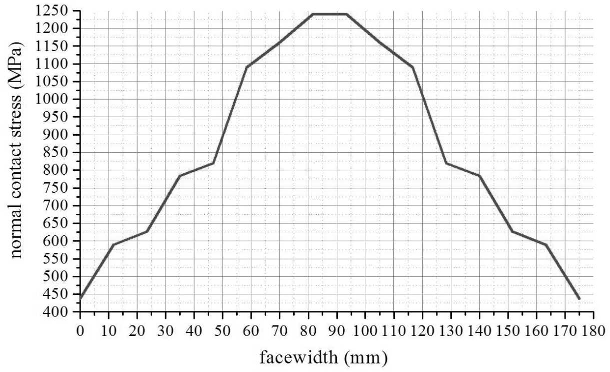


Fig.5. Diagram of normal contact stresses along facewidth for pinion tooth

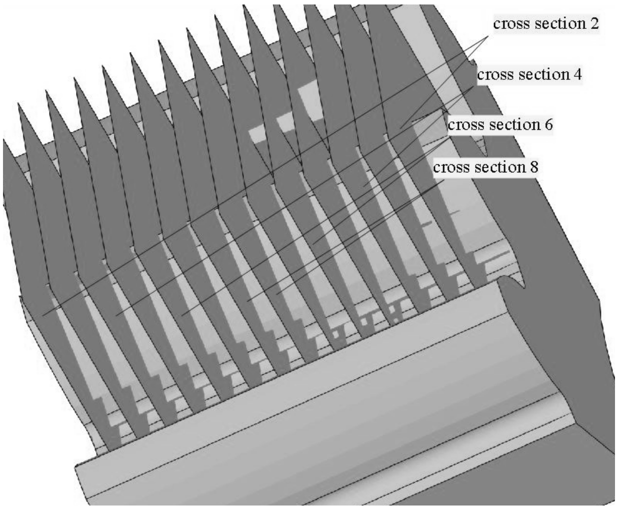
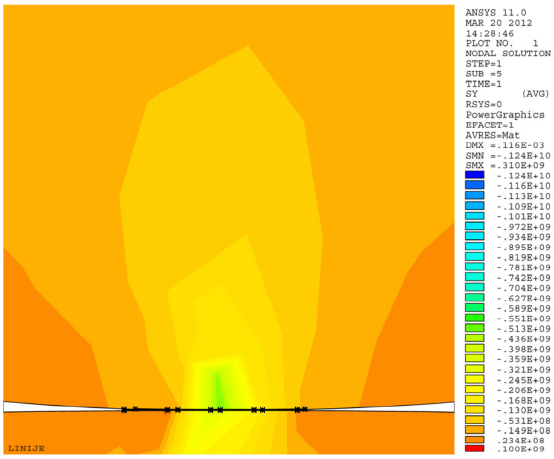
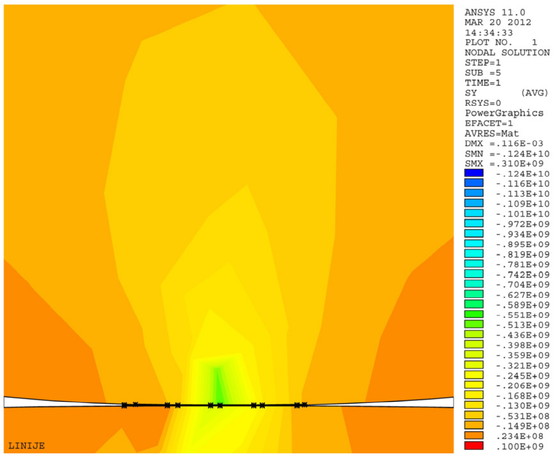
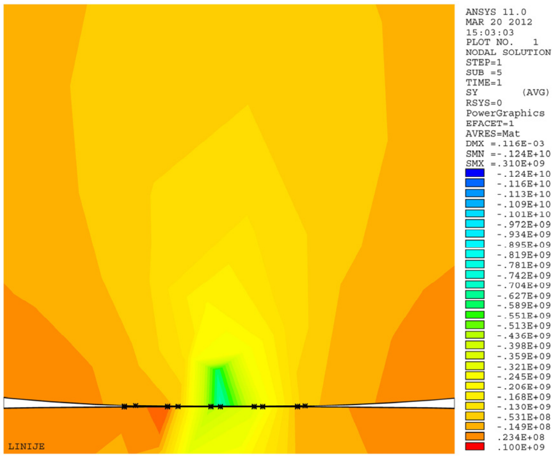


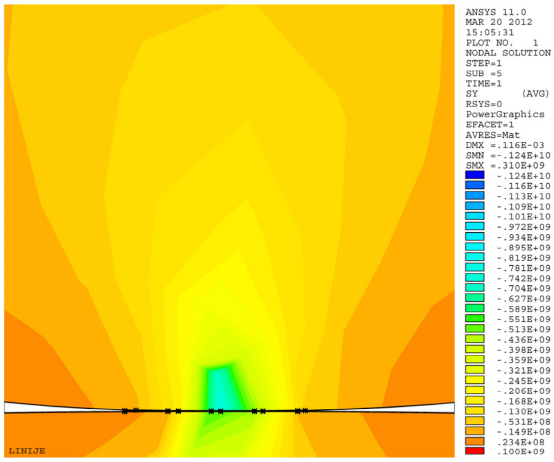
Fig.6. The positions of cutting planes that create pinion cross sections

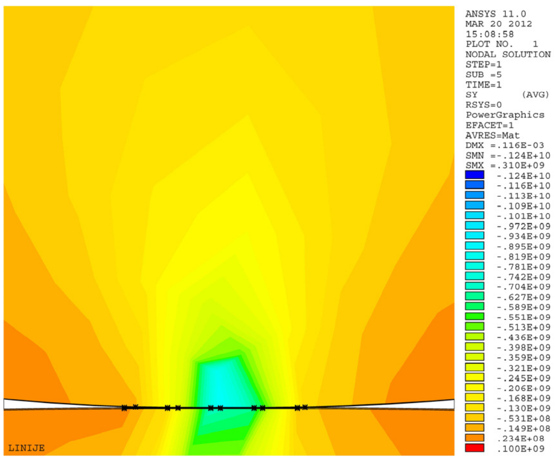
Eight different contour plots for normal compressive stresses distribution in cross section are obtained in equal distances from both of pinion faces, fig.7. At the fig.8 – fig.15 the diagrams of appropriate normal compressive stress gradients are drawn for same eight cross sections.











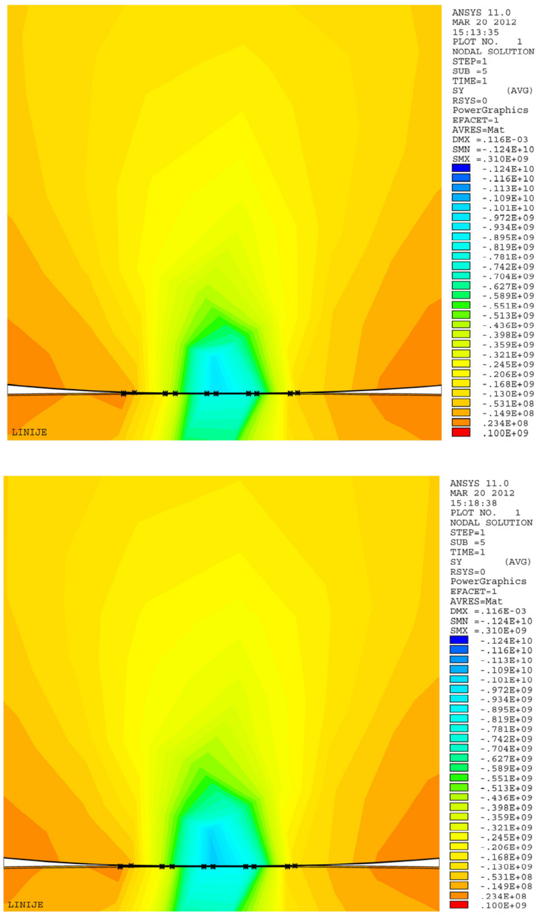


Fig.7. The contours of normal compressive stresses in cross sections from gears face to gears middle at equal distances

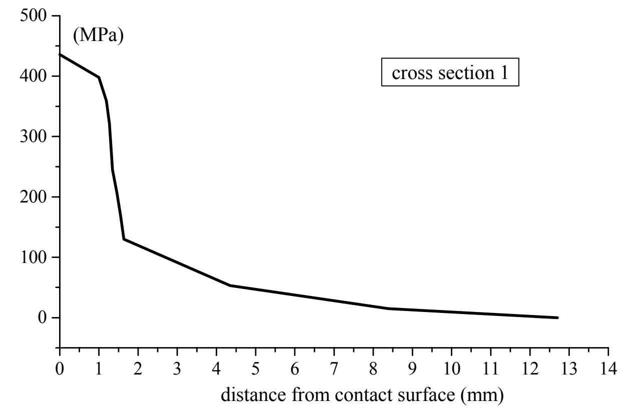


Fig.8. The normal compressive stresses at gears face

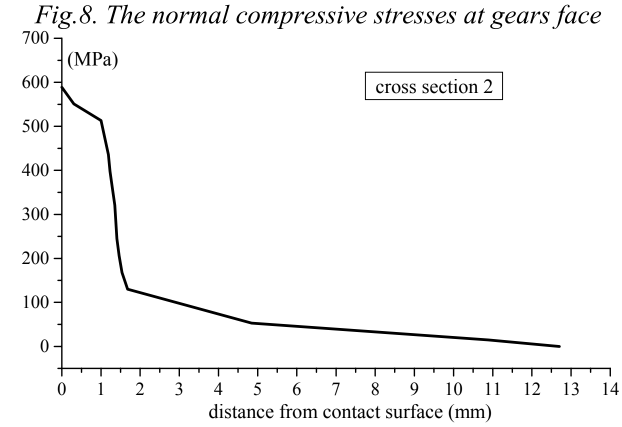


Fig.9. The normal compressive stresses at distance of 11.667 mm from gears face

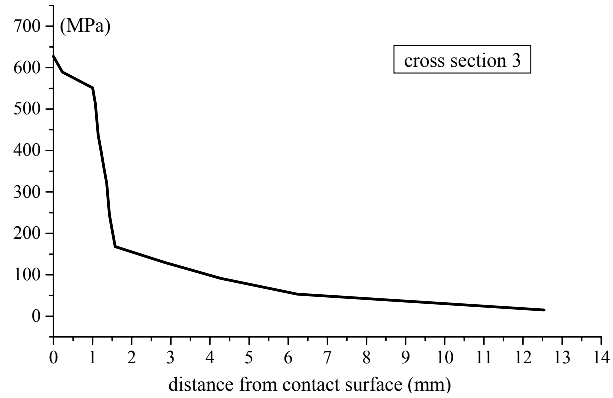


Fig.10. The normal compressive stresses at distance of 23.334 mm from gears face

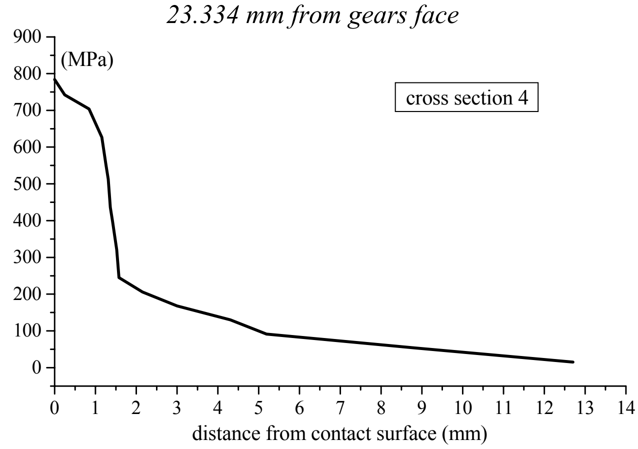


Fig.11. The normal compressive stresses at distance of 35 mm from gears face

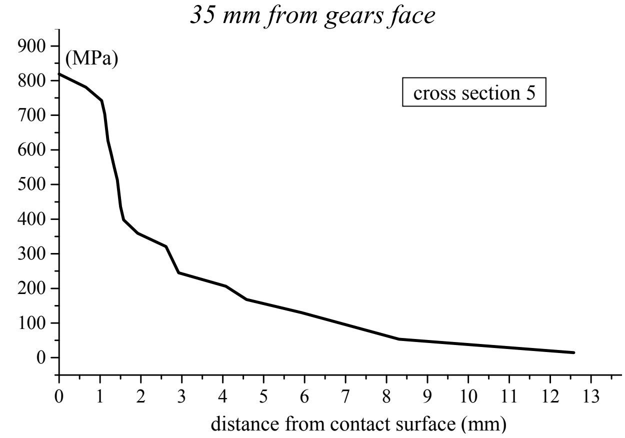


Fig.12. The normal compressive stresses at distance of 46.667 mm from gears face

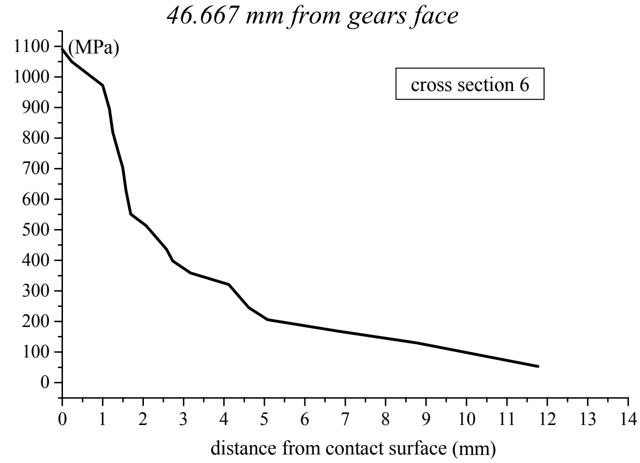


Fig.13. The normal compressive stresses at distance of 58.334 mm from gears face

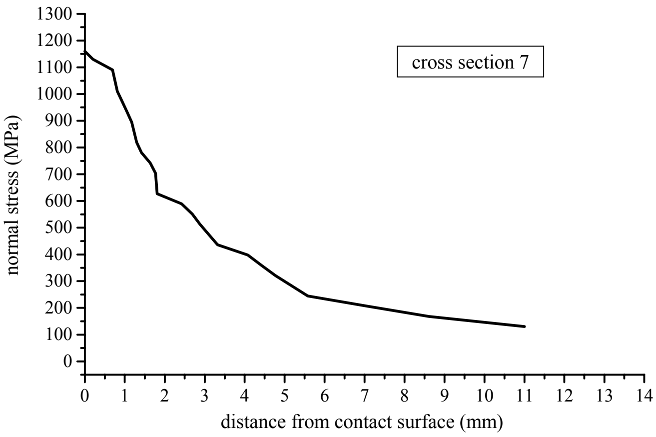


Fig.14. The normal compressive stresses at distance of 70 mm from gears face

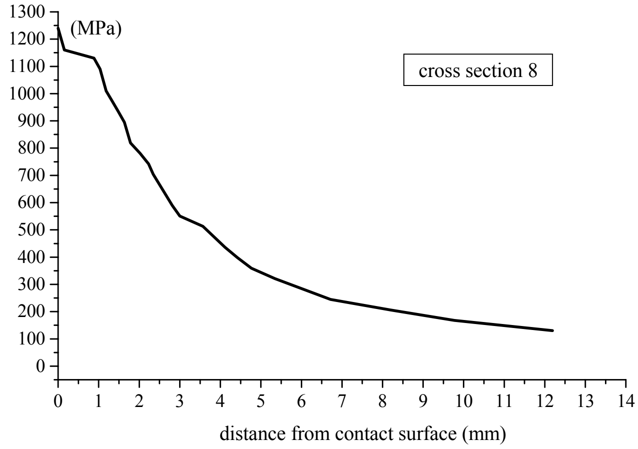


Fig.15. The normal compressive stresses at distance of 81.667 mm from gears face

These eight different normal compressive stress gradients for 16 cross sections of pinion are given in comparative diagram in fig.16. This comparative diagram is the base for TCD point method application, fig.3, on crack initiation prediction in pinion contact zone for the investigated gear pair.

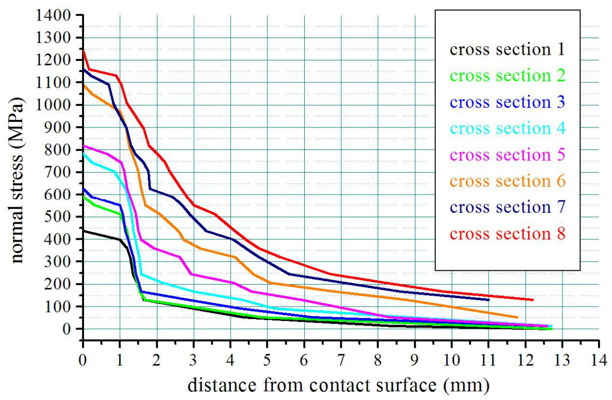


Fig.16. Comparative diagram for normal compressive stress gradients along pinion facewidth

A distance of contact surface on which normal pressures become equal along the pinion tooth could be read from this diagram. This distance in the direction normal to contact surface is 14 mm. When compare to the pinion tooth width in the same pinion radius, which is 45 mm, the complete information for distribution of normal stresses on and under the contact surface is obtained.

Application of the presented methodology, fig.2, in next step requires calculation of the critical distance L (or $L/2$ by TCD point method). For this particular case the critical distance has value of:

$$L = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_0}\right)^2 = \frac{1}{\pi}\left(\frac{18}{612}\right)^2 = 0.000275\,m = 0.275\,mm \quad (2)$$

The detail view of diagram shown in fig.16 is presented in fig.17. This scale of magnitude is appropriate for TCD point method application in investigated case study.

Obtained value of L corresponds to material by available literature data for similar steels, [22]. Also the combination of high stresses at the $L/2$ distance, fig.17, shows that the conditions for crack exist in narrow zone on tooth flank.

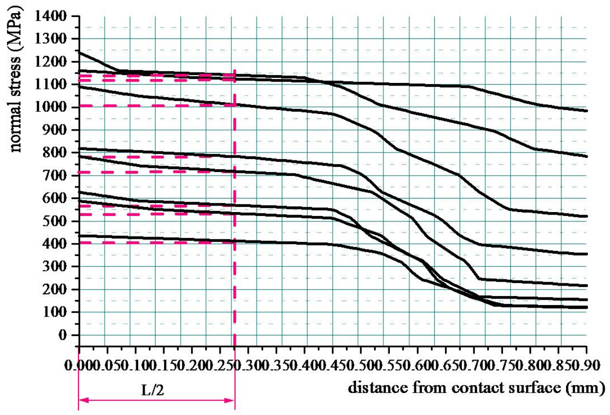


Fig.17. TCD point method in crack prediction in pinion contact zone

6. CONCLUSIONS

Contemporary research in applied mechanics is based on the use of new methods enabled by enormous rise in computing power and on the multidisciplinary approach. One of them is the presented new methodology that used Finite Element Analysis for stress and strain calculations and for prediction of the crack location at gears tooth contact zone, and a new method for crack initiation prediction. The contact at spur gear teeth mesh is studying by the real gear pair research.

The Finite Element Analysis (FEA) of 3D gear contact model calculated stresses and located the critical location with aspect of fatigue cracks appearance. Also, the contact stress gradient curves are made for a pinion tooth in different cross sections along gear facewidth by FEA. The usage of Theory of Critical Distances is demonstrated on these stress gradients in order to give the crack initiation assessment.

The analysis of the obtained results shows that methodology used in this research has significant potentials for estimation of crack initiation in case of contact fatigue. Also, the presented application of TCD approach on contact fatigue problem shows that further work is necessary. The facts that real situation involves surface hardening of gears and different lubrication solutions, keep space for further research.

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