

## EXPERIMENTAL DETERMINATION OF THERMAL CONDUCTIVITY OF SOIL WITH A THERMAL RESPONSE TEST

by

**Miloš J. BANJAC**<sup>a\*</sup>, **Maja N. TODORVIĆ**<sup>b</sup>, **Milan R. RISTANOVIĆ**<sup>c</sup>,  
**and Radoslav D. GALIĆ**<sup>b</sup>

<sup>a</sup> Department of Thermodynamics, Faculty of Mechanical Engineering, University of Belgrade, Belgrade, Serbia

<sup>b</sup> Department of Thermal Science, Faculty of Mechanical Engineering, University of Belgrade, Belgrade, Serbia

<sup>c</sup> Department for Automatic Control, Faculty of Mechanical Engineering, University of Belgrade, Belgrade, Serbia

Original scientific paper  
DOI: 10.2298/TSC1100627156B

*Optimal design of a borehole heat exchanger, as the outer part of a ground source heat pump heating system, requires information on the thermal properties of the soil. Those data, the effective thermal conductivity of the soil  $\lambda_{eff}$  and the average temperature of the soil  $T_0$ , enable us to determine the necessary number and depth of boreholes. The determination of thermal conductivity of the soil in laboratory experiments does not usually coincide with the data under in situ conditions. Therefore, an in-situ method of experimental determination of these parameters, the so-called thermal response test, is presented in this paper. In addition to the description of the experimental procedure and installation overview, the paper describes methods based on theory and presents their basic limitations, through the presentation of experimental data.*

**Key words:** *thermal response test, thermal conductivity of soil, dimensioning of borehole heat exchanger*

### Introduction

When designing a ground source heat pump (GSHP) heating system, the least reliable but the most important calculations are dimensioning of the vertical borehole heat exchanger (VBHE), preceded by determination of the temperature and the thermal properties of the soil (effective thermal conductivity and thermal diffusivity of soil).

The main cause of this uncertainty lies in the fact that the processes of heat transfer, throughout the ground around the VBHE, are directly related to the temperature and the thermal properties of the anisotropic soil that are generally unknown in advance. Even if geological maps for some areas exist, the data which they provide on the soil composition are incomplete, from the standpoint of determining the actual thermal properties of the ground. This is due to the fact that geological maps contain only information on the soil composition, and not on its moisture, local groundwater flows, and volume fraction of air.

---

\* Corresponding author; e-mail: mbanjac@mas.bg.ac.rs

Therefore, from the 90s of the past century, simultaneously with the growing use of GSHP, appropriate experimental methods have been developed, which could be used to determine the required thermal properties of the soil in a relatively easy and sufficiently reliable manner, *i. e.*, to determine the heat flow that a borehole can provide. Thus, the method of the so-called thermal response tests (TRT) was singled out as sufficiently precise and, at the same time, easy to implement. This method, which is based on the use of analytical solution for heat conduction in an infinite body with a line heat source, and the analogy between theory of flow of incompressible fluid in porous media and theory of heat conduction in solid material, was first proposed by Morgenson [1], who was, at the same time, the first to experimentally confirm its applicability. Based on the same theoretical basis, in 1995, Elkof *et al.* [2] made the first mobile apparatus, which enabled them to determine the average temperature and the so-called effective thermal conductivity of the soil\* at various boreholes. Since then, this method has been checked and improved experimentally [3-5] and numerically [6] many times and under different conditions [7, 8]. Because of the growing use of GSHP heating systems, and increasingly pronounced need for precise knowledge of the thermal properties of the soil, this paper describes the theoretical bases of the method, while its practical application is shown through the presentation of the performed experimental procedure and processing of obtained experimental data.

## Theoretical bases

### Linear theory

Design of VBHE is conditioned by the manner of placing it in the soil. It usually consists of two very long plastic U-pipes which are inserted in previously drilled boreholes (fig. 1). For the reason of decreasing of thermal resistance between the pipes and the borehole wall, the space between is mandatory filled by some infill. Usually this infill is the bentonite. In that way, it is possible to assume VBHE as one very long cylinder, with two U-pipes inside (figs. 2 and 4).



Figure 1. Insertion of the heat exchanger in the vertical borehole

In respect to the heat transfer, very long length compared to the other two dimensions, provides that the effect of work of VBHE on soil can be treated as a case of heat transfer from line heat source to an infinite medium. This means that in case when VBHE permanently trans-

\* Average integral value of the thermal conductivity of all layers of the soil which are in contact with the buried heat exchanger



Figure 2. Upper end of (VBHE) before filled by bentonite

fers same heat flux to the soil, a temperature field in the soil can be described with the equation for heat conduction in a homogeneous infinite isotropic medium with a line heat source with constant rate of heat generation.

The analytical equation that describes that problem can be written as [2]:

$$T(r) - T_0 = \frac{\varphi_l}{4\pi\lambda} \int_{\frac{r^2}{4at}}^{\infty} \frac{e^{-u}}{u} du \quad (1)$$

where  $T(r)$  is the temperature of homogeneous infinite isotropic medium on radius  $r$ ,  $T_0$  – the temperature of thermally undisturbed soil, and  $\varphi_l$  – the heat rate per unit length of exchanger.

By introducing the term  $\tau = at/r^2$ , the eq. (1) can be rewritten as:

$$T - T_0 = \frac{\varphi_l}{4\pi\lambda} \int_{\frac{r^2}{4\tau}}^{\infty} \frac{e^{-u}}{u} du = \frac{\varphi_l}{4\pi\lambda} E_1\left(\frac{1}{4\tau}\right) \quad (2)$$

where with  $E_1$  is marked the exponential integral. Analytical solution of exponential integrals is a sum of infinite order:

$$E_1(x) = \int_{-x}^{\infty} \frac{e^{-u}}{u} du = -\gamma - \ln x + \sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^k}{kk!}$$

where  $\gamma$  is the Euler-Mascheroni's constant

$$\gamma = \lim_{n \rightarrow \infty} \left[ \sum_{k=1}^n \frac{1}{k} - \ln n \right]$$

whose approximate numerical value is  $\gamma = 0.577\ 215\ 664\ 9\dots$

By development the exponential integral  $E_1[1/(4\tau)]$  into Taylor series, with error less than 1%, keeping it only on the first two terms, it is possible to get the approximate solution [9]:

$$E_1\left(\frac{1}{4\tau}\right) \approx G(\tau) = \ln(4\tau) - \gamma - \frac{1}{4\tau} + \frac{1}{64\tau^2} \quad (3)$$

More practical and much simpler approximate solution of the exponential integral is possible to get by taking only the basic part of equation:

$$E_1\left(\frac{1}{4\tau}\right) \approx \ln(4\tau) - \gamma \quad (4)$$

The error made with this approximate solution is slightly larger, but under condition that  $\tau \geq 5$ , the difference between the approximate solution given by eq. (3) and this solution is less than 2% (fig. 3).

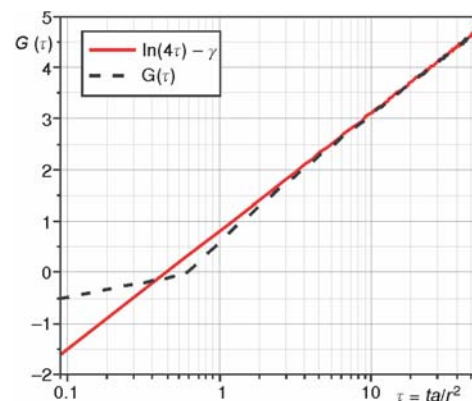


Figure 3. Graphical presentation of approximate solutions of exponential integrals described by eqs. 3  $G(\tau)$  and 4  $\ln(4\tau) - \gamma$

In other words, the approximate solution for the heat conduction in a homogeneous infinite isotropic medium with a line heat source with constant rate of heat generation (eq. 2), under time condition  $t \geq 5r^2/a$  and with quite acceptable error less than 2%, is possible to write in form of:

$$T - T_0 = \frac{\varphi_l}{4\pi\lambda} \left[ \ln\left(\frac{4at}{r^2}\right) - \gamma \right] \quad (4)$$

Transforming eq. (5) to the form:

$$T(r) - T_0 = \frac{\varphi_l}{4\pi\lambda} \ln t + \left[ \frac{\varphi_l}{4\pi\lambda} \ln\left(\frac{4a}{r^2}\right) - \frac{\varphi_l}{4\pi\lambda} \gamma \right] \quad (5)$$

it can be noticed that the temperature change of the infinite medium on a defined radial distance from the source, is only a time function.

The influence of inhomogeneity, caused by the differences of boreholes infill material – bentonite and soil, on heat transfer is possible to include in calculation using so called thermal resistance  $R_b(t)$  of borehole. For this purpose, it is necessary to assume that previous line heat source with the constant rate of heat generation  $\varphi_l$ , lies in the axis of borehole – a homogeneous and anisotropic, infinite cylinder with radius  $r_b$ , which is located in the ground. If the borehole has thermal resistance  $R_b(t)$  and if temperature change of line heat source in their axis  $T_f(t)$  is known, the approximate analytical solution of that problem can be written as:

$$\Phi = \frac{T_f(t) - T_0}{R_b + \frac{1}{4\pi\lambda l} \left[ \ln\left(\frac{4at}{r_b^2}\right) - \gamma \right]} \quad (6)$$

Now it is possible to express that temperature difference by the rate of heat generation of the line heat source  $\varphi_l$  and elapsed time  $t$ :

$$T_f(t) - T_0 = \frac{\varphi_l}{4\pi\lambda} \ln t \left[ \frac{\varphi_l}{4\pi\lambda} \ln\left(\frac{4a}{r_b^2}\right) - \frac{\varphi_l}{4\pi\lambda} \gamma \right] + \varphi_l R_{l,b} \quad (7)$$

where  $R_{l,b} = R_b l$  is thermal resistance of borehole reduced to 1 meter.

By introducing the constant  $k$  as:

$$k = \frac{\varphi_l}{4\pi\lambda} \quad (8)$$

and  $C_1$  as:

$$C_1 = \left[ \frac{\varphi_l}{4\pi\lambda} \ln\left(\frac{4a}{r_b^2}\right) - \frac{\varphi_l}{4\pi\lambda} \gamma \right] + \varphi_l R_{l,b} + T_0 \quad (9)$$

the eq. (4) can be written as:

$$T_f(t) = k \ln t + C_1 \quad (10)$$

Based on the form of eq. (7), it can be concluded that if it were presented in a semi logarithmic co-ordinate system ( $\ln t - T_f$ ), it would represent a straight line. In addition, the constant  $k$  would represent the direction of this line and the constant  $C_1$  the segment of the ordinate.

Also, it can be concluded that, it is possible to determine the thermal conductivity of a homogeneous and anisotropic infinite medium, with the known initial temperature  $T_0$ , and with a line heat source with the constant and known rate of heat generation  $\varphi_l$ , which is located in the axis of an borehole by radius  $r_b$  located in that infinite medium, by only monitoring the temperature changes of that line heat source over time  $T_f(t)$ .

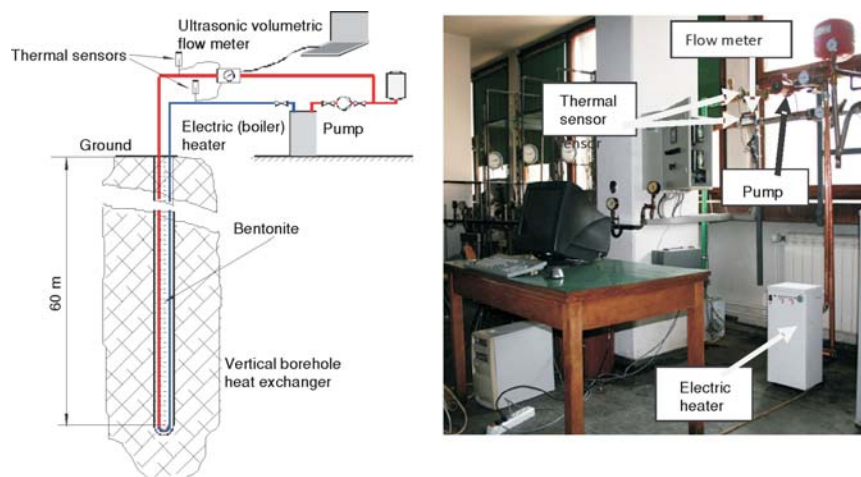
## Experimental procedure and results

### *Description of the experimental installation*

The experimental installation which was used to perform the controlled heating of the earth, and to monitor its thermal “response” to determine the effective thermal conductivity of the soil, is shown in fig. 4. A vertical heat exchanger, 60 m long, buried in a vertical, 16.5 cm diameter borehole, subsequently filled with bentonite, was located in the courtyard of the Mechanical Engineering Faculty of the University of Belgrade. The experimental installation for determination of heat response was located in its vicinity – in the Laboratory of Thermodynamics of this faculty.

For the sake of accuracy and reliability of measurement, water, rather than glycol, was used as a working fluid. The water, flowing through the buried VBHE, transmitted the heat received from the boilers to the earth. The flow of water was provided by Grunfos UP-Basic 25-4 water pumps, while Termomont’s electric boiler ETK(E)-9, with the heating power of 4.5 kW, was used for heating the water.

Danfoss’s ultrasonic heat meter – SONOMETER TM 1000 was built in the installation as the only measuring equipment. Two temperature sensors – thermocouple PT 100 that contain the ultrasonic volumetric flow meter, were used for measuring water temperature at the entrance and at the exit from borehole. In addition, ultrasonic heat meter SONOMETER TM-1000 is equipped with a microprocessor, and Electrically Erasable Programmable Read-Only Memory\* (EEPROM). The memory was used during all the experiments for saving all the measured data every 2 minutes. In order to access the stored data, ultrasonic heat meter was connected through the so-called M-BUS module\*\* with a PC.



**Figure 4. Scheme and photo of experimental installation for determining the effective thermal conductivity of the soil by TRT**

\* This type of non-volatile memory is used in computers and other electronic devices to store small amounts of data that must be saved when power is removed, *e. g.* calibration tables or device configuration

\*\* Mbus (Meter-Bus) module is subsystem for high speed communication and transfer data between components inside and outside computer system which work in accordance with a European standard(EN 13757-2 physical and link layer, EN 13757-3 application layer)

### Measuring procedure

As already mentioned, to determine the effective thermal conductivity of the soil by TRT, it is necessary to know the undisturbed ground temperature  $T_0$ , the rate of heat generation of the heat source  $\dot{q}_1$  and, finally, to monitor the temperature changes of the heat source  $T_j(t)$ .

The undisturbed ground temperature is determined in the so-called previous phase. In this phase, the water pump was the only one that was working and only the changes of the water temperature at the entrance and at the exit from the VBHE were measured. Although this phase lasted longer than 12 hours, it was noticed, that only after 20 minutes, the value of temperature in both water flows already became equal and stabilized at the average temperature of undisturbed ground, *i. e.*  $T_0 = 17.3$  °C.

The rate of heat generation of the heat source  $\dot{q}_1$  was determined in the soil heating phase. This phase started with the electric boiler being turned on, and followed right after the previous phase. The heating phase lasted for 5 days. In fact, out of the total of 6 electric heaters, only 3 were turned on, providing about  $3 \times 1.5 = 4.5$  kW of thermal power. At the same time, the actual value of the realized heat flow to the soil – the rate of heat generation of the heat source  $\dot{q}_1$  – which was measured and recorded using the ultrasonic flow heat meter, had a somewhat lower average value which amounted to 4.489 kW (figs. 5 and 6).

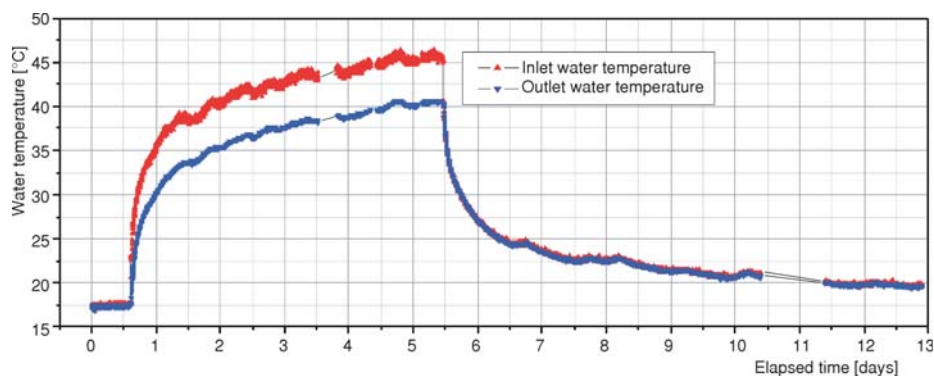


Figure 5. Change of the water temperature at the entrance to the buried heat exchanger and at the exit from it during the preparatory, heating and recovery phases

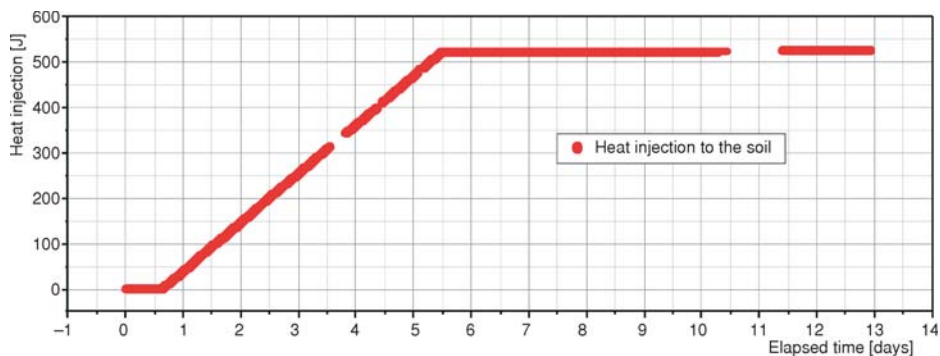


Figure 6. Heat energy delivered to the ground by the buried heat exchanger measured by Danfoss's ultrasonic heat meter – SONOMETER TM-1000

In order to obtain information about the change of the temperature of the line heat source during the heating phase, water temperatures at the entrance of VBHE (incoming fluid temperature  $T_{in}$ ) and at the exit from it (outgoing fluid temperature  $T_{out}$ ) were measured and recorded. The measured temperature values are shown in fig. 7. The same figure also shows the water temperature measured after the heating phase – after the boiler was turned off, in the so-called recovery phase – phase of recovering to the initial state.

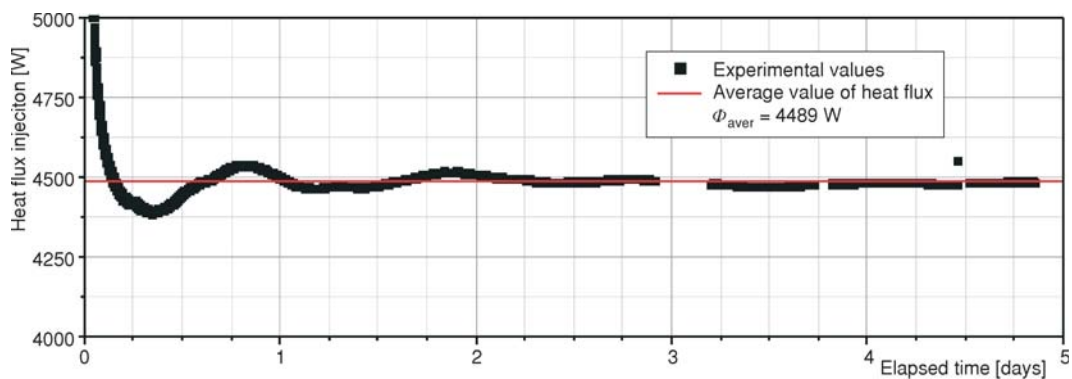


Figure 7. Change of heat flow to the ground and its average value during the “heating” phase

#### Processing of experimental data

In order to correlate collected experimental data to the described theory, only the data collected during the heating phase is analyzed. The change in the mean temperature of the water circulating through the buried exchanger  $T_f$  was determined as the mean arithmetic value of the previously selected temperature data at the entrance to the VBHE ( $T_{in}$ ) and at the exit from it ( $T_{out}$ ) (fig. 8)

Then, those data were transferred in a semi logarithmic coordinate system  $\ln t - T$  (fig. 9). Finally, the commercial program Origin Pro 6.1 was used to determine the equation of the straight line  $T = k \ln t + C_1$ , which most appropriately displays experimental data. In accordance with the presented theory, only experimental data collected after  $t \geq 5r_b^2/a$  were matched. With the correlation coefficient  $r_{xy} = 0.99526$  and standard deviation  $\sigma = 0.39658$ , the value of determined direction of this line was  $k = 3.97924$ , and the value of the segment of the ordinate was  $C_1 = -8.4596$ .

The changing of the fluid’s temperature showed a significant deviation from logarithmic linear dependency, during the initial period of implementation of TRT (fig. 9).

Those deviations can be explained by the fact that logarithmic linear dependency actually represents the very simplified and approxi-

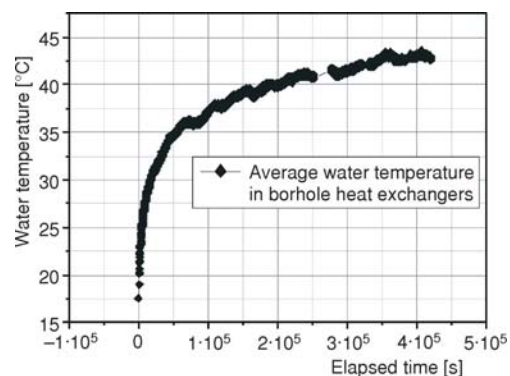
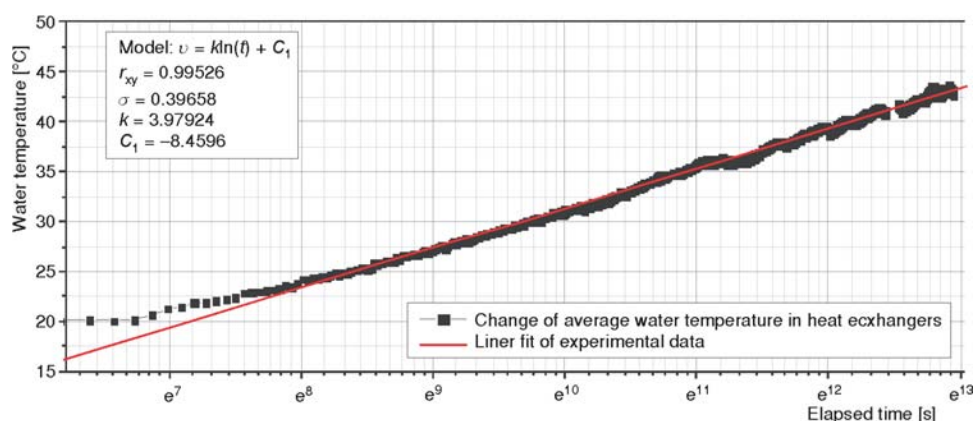


Figure 8. Change of the average temperature of water in the buried heat exchanger during the heating phase



**Figure 9.** Change of the average temperature of water in the buried exchanger during the heating phase and its approximation by function

mated solution of eq. (3), obtained with neglecting all terms of Taylor series (4). The confirmation of the analytical approach and quality of theoretically derived expressions can be seen by comparing the diagrams on fig. 3 and fig. 9. From those diagrams it can be seen that the analytical solution could have excellent matching with the experimental results in the initial period of TRT, only with involvement of at least two members of Taylor's series in the expression. These graphs also show excellent matching between simplified analytical expressions (3) with experimental results in second part of TRT process for  $\tau \geq 5$  (or  $\tau \geq 5r_b^2/a$ ), and confirm the necessity of fulfilling this time condition.

Under the assumption that the tubes of the buried exchanger, together with the bentonite filling, make a homogenous isotropic and infinite cylinder, with the radius  $r_b$ , in whose middle axis is the linear heat source, the previously determined constant  $k$  is, at the same time, the constant with the same name in the function described by eq. 10. Based on that, and by using eq. (8), the effective thermal conductivity of the soil around the examined well was determined:

$$\lambda_{\text{eff}} = \frac{\varphi_1}{4\pi k} = 1.496 \text{ W/mL}$$

In order to verify the accuracy of the obtained result and reliability of the method itself, TRT was repeated three times in the same borehole at 60-day intervals. The obtained results are shown in tab. 1.

Since the experimentally determined values of thermal conductivity differ by less than 3.2%, and since they are within the range of values for the thermal conductivity of clay and bentonite [10], it can be concluded that TRT can determine the thermal conductivity of the soil with very high reliability.

Also, it is interesting to note that contrary to expectations, in each successive measurement the temperature of soil  $T_0$  became slightly lower. Possible explanation for this unexpected result is coming from the fact that TRT does not take into account either the temperature change in depth of the ground, neither the change of the tempera-

**Table 1.** Experimentally obtained value of the effective thermal conductivity of the soil in the same borehole in three different TRT

No. of measurements	$\lambda_{\text{eff}}$	$T_0$
1	1.469	17.3
2	1.488	16.9
3	1.442	16.4



ture of surface layers of the ground during different seasons. This latter reason obviously had a influence on the measured temperature  $T_0$ , because the first measurements were made during September, when the temperature of the surface layers of the soil was the highest, the second measurement was made in December, and the third in late February with the lowest temperature of the surface layers. In accordance with obtained very small changes of the temperature, this influence can be considered as unimportant. Also, it can be concluded that the time between two TRT had been long enough that the heat transferred to soil during the previous TRT did not have remarkable influence on  $T_0$ .

## Conclusions

The increasingly widespread use of geothermal heat pumps and use of the soil as a heat source were the reasons for seeking experimental methods that will enable determining the local thermal properties of the soil and heat flow that a soil can provide, sufficiently accurately. Performing the thermal response test on a vertical, 60 m long heat exchanger, buried in a vertical borehole, subsequently filled with bentonite, with a detailed review of the methodology of measuring and processing the collected data, showed that this test is a reliable and a simple method by which the aforementioned soil properties can be determined on already existing borehole wells.

The procedure for deriving the basic equation for heat conduction in a homogeneous infinite isotropic medium with a line heat source – the equation that is the basic hypothesis on which the thermal response test is based – is showed in detail, with all the assumptions and simplification. In this way, with the detailed precise deriving of the basic equation, the substantial deviations of the analytical model from the real physical processes in the soil caused by TRT are highlighted. It demonstrated that effective thermal conductivity of soil determined in this way involves influence of the so-called borehole thermal resistance and that value of this parameter is not necessary to be determined separately.

The assumed simplified analytical expression for changing the temperature of fluid in the exchanger showed excellent matching with the experimental data only during second period of the implementation of the TRT. In that way it was demonstrated the importance and necessity of fulfilling the time condition  $t \geq 5r_b^2/a$  for getting accurate value of effective thermal conductivity of soil.

## Acknowledgment

This paper is made in a scope of the project TR 33047 “Intelligent climate control systems to achieve energy efficient regime in the complex conditions of exploitation” funded by the Ministry of Education, Science and Technological Development of the Republic of Serbia.

## Nomenclature

$a$	– thermal diffusivity, [ $\text{m}^2\text{s}^{-1}$ ]	$R_b$	– thermal resistance of the (borehole) cylinder with radius, [ $\text{KW}^{-1}\text{m}^{-2}$ ]
$C_1$	– constant, [K or $^{\circ}\text{C}$ ]	$R_{l,b}$	– thermal resistance of the (borehole) cylinder with radius reduced to 1 m length, [ $\text{KW}^{-1}\text{m}^{-1}$ ]
$E_i$	– exponential integral, [–]		
$k$	– constant, [K or $^{\circ}\text{C}$ ]		
$R_l$	– thermal resistance of the (borehole) cylinder reduced to 1 m length, [ $\text{KW}^{-1}\text{m}^{-1}$ ]		

$r$	– radial co-ordinate, [m]	$T_{\text{out}}$	– outlet fluid temperature out of the borehole, [K or °C]
$r_b$	– radius of the borehole heat exchanger, [m]	$t$	– time, [s]
$r_{xz}$	– correlation coefficient for Pearson's chi-squared test, [–]	<i>Greek symbols</i>	
$T$	– temperature of the ground, [K or °C]	$\gamma$	– Euler-Mascheroni's constant, $\gamma = 0.577\ 215\ 664\ 9\dots$
$T_0$	– undisturbed ground temperature, before heat injection, [K or °C]	$\lambda$	– thermal conductivity, [ $\text{Wm}^{-1}\text{K}^{-1}$ ]
$T_f$	– average fluid temperature in the borehole heat exchanger, defined as: $T_f = 0.5(T_{\text{in}} + T_{\text{out}})$	$\lambda_{\text{eff}}$	– effective ground thermal conductivity, [ $\text{Wm}^{-1}\text{K}^{-1}$ ]
$T_f(t)$	– temperature change of line heat source, [K or °C]	$s$	– standard deviation of the Gaussian approximation [–]
$T_{\text{in}}$	– inlet fluid temperature into the borehole heat exchanger, [K or °C]	$\Phi$	– total heat rate transferred by the borehole, [W]
		$\varphi_l$	– heat rate per unit length, [ $\text{Wm}^{-1}$ ]

## References

- [1] Morgensen, P., Fluid to Duct Wall Heat Transfer in Duct System Heat Storage, *Proceedings*, International Conference on Surface Heat Storage in Theory and Practice, Stockholm, 1983, pp. 652-657
- [2] Eklof, C., Gehlin, S., TED – A Mobile Equipment for Thermal Response Tests, M. Sc. thesis, 1996:198E, Lulea University of Technology, Lulea, Sweden, 1996
- [3] Mattsson, N., Steinmann, G., Laloui, L., Advanced Compact Device for the *in situ* Determination of Geothermal Characteristics of Soils, *Energy and Buildings*, 40 (2008), 7, pp. 1344-1352
- [4] Pakud, D., Matthey, B., Comparison of the Thermal Performance of Double U-Pipe Borehole Heat Exchangers Measured *in situ*, *Energy and Buildings*, 33 (2001), 5, pp. 503-507
- [5] Esen, H., Inalli, M., *In-situ* Thermal Response Test for Ground Source Heat Pump System in Elazyo, Turkey, *Energy and Buildings*, 41 (2009), 4, pp. 395-401
- [6] Signorelli, S., *et al.*, Numerical Evaluation of Thermal Response Tests, *Geothermics*, 36 (2007), 2, pp. 141-166
- [7] Wang, H., *et al.*, Thermal Performance of Borehole Heat Exchanger Under Groundwater Flow: A Case Study from Baoding, *Energy and Buildings*, 14 (2009), 12, pp. 1368-1373
- [8] Bandyopadhyay, G., Gosnold, W., Mann, M., Analytical and Semi-Analytical Solutions for Short-Time Transient Response of Ground Heat Exchangers, *Energy and Buildings*, 40 (2008), 10, pp. 1816-1824
- [9] Carslaw, H. S., Jaeger, J. C., *Conduction of Heat in Solids*, 2<sup>nd</sup> ed., Oxford University Press, Oxford, UK, 1959
- [10] Kavanauhg, S. P., Rafferty, K., *Ground-Source Heat Pumps: Design of Geothermal System for Commercial and Institutional Buildings*, American Society of Heating, Refrigeration and Air- Conditioning Engineers (ASHRAE), Atlanta, Geo., USA, 1997

Paper submitted: June 27, 2010

Paper revised: May 1, 2012

Paper accepted: May 2, 2012