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# Bio-Inspired Control of Redundant Robotic Systems: Optimization Approach

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**The major aim of this paper is to promote a biologically inspired control synergy approach that allows the resolution of redundancy of a given robotized system which can be used for military purposes. It is shown that it is possible to resolve kinematic redundancy using the local optimization method and biological analogues – control synergy approach, introducing hypothetical control and distributed positioning. Also, the possibility of switching synergies within a single trajectory is treated, where the control synergy approach applying logical control is used. The actuator redundancy control problem has been stated and solved using Pontryagin's maximum principle. Control synergy as a class of dynamic synergy is established by the optimization law at the coordination level. Finally, the effectiveness of the suggested biologically inspired optimal control synergy is demonstrated with a suitable robot with three degrees of freedom and four control variables, as an illustrative example.**

*Key words:* **biomechanics, robotics, redundant system, optimal control, optimisation method, redundancy, sinergy.**

## Introduction

IT is well known that biological systems possess significant capabilities perfected in their specialized functions through long evolutionary processes where they developed to optimize themselves under selective pressures for a long time [1]. On the other hand, the system control theory which forms the core foundation for understanding, designing, and operating of many technical systems, is still limited and insufficient to handle complex large-scale systems in real time as biological systems. Meanwhile, it becomes increasingly important for artificial systems to have high flexibility, diversity, reliability, and affinity [2, 3]. Unlike the industrial robotics domain where the workspace of machines and humans can be segmented, applications of intelligent machines that work in contact with humans are increasing, which involves e.g. haptic interfaces and teleoperators, cooperative material-handling, power extenders and such high-volume markets as rehabilitation, bioengineering, as well as security and military applications. In that way, robotic systems are more and more ubiquitous in the field of direct interaction with humans. Recent rapid development of biological science and technologies will further improve the active applications of control engineering by advanced biomimetic and biologically inspired research [4-6].

The control of complex redundant systems is a challenging problem of interest where kinematically redundant, redundant sensors and actuation can be found in many biological and robotic applications [7-11]. It is noticeable that the optimal control theory has been repeatedly and successfully used to account for many aspects of motor control as well as for redundant robotic systems [7-9], [12-14].

In literature, standard optimization methods dealing with the redundancy problem are usually divided into two groups: local and global methods [2] [7-9]. According to the optimal criterion that state/control variables depend on at each time constant, we obtain a local optimization method, or an optimal criterion (given as functional) that depends on the motion as a whole, we recognize as the global optimization method. Different kinematic or dynamic optimization criteria could be introduced to achieve the unique solution of the redundancy problem, such as: the kinetic energy, the sum of squared generalized velocities, total driving power, potential energy, etc. In that way, it is necessary to convert the redundancy resolution problem to an optimal control problem, and thereafter to obtain the locally or globally optimal resolution of redundancy using the necessary conditions of optimal control. Besides, sensorimotor control is best described as being near optimal, where a hierarchical controller can closely approximate an optimal controller, [12, 13] i.e. optimal feedback controllers for redundant systems exhibit a hierarchical organization, even when such an organization is not imposed by design. This imposes the decomposition of the system into several subsystems with a strong coupling between the subsystems.

In addition, in biological systems it is noticed that the redundancy problem is usually solved by using the biologically inspired principle – *synergy* [10]. It was observed in the execution of functional motions that certain trajectories are preferable from the infinite number of

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options [10, 15].The existence of invariant features in the execution of functional motions points out that the central nervous system (CNS) uses *synergy* i.e. rule(s) that can be developed by the CNS based on some principles[10, 15]. Such behavior of organisms can be only explained by the existence of inherent optimization laws in self-organized systems governing the acquisition of motor skills. Moreover, it obeys the optimization at the coordination level of the hierarchical organization of control, where the goal is to minimize efforts in terms of synergy patterns [2]. Therefore, the CNS may simplify the control problem by utilizing a finite set of synergies instead of an infinite set of muscle patterns, where synergies reduce the number of degrees of freedom and that must be independently controlled. Mathematically, the synergy imposes specific constraints on the control variables of joints related to the task dependent functions pertaining to classes of motor acts. Many authors have suggested that for movements that involve multiple body segments, kinematic/kinetic descriptions of the moving segments or joints may be reduced to a small number of variables using the idea of synergy, i.e they are known as dynamic synergies and kinematic synergies, see [16, 17].

Our work focuses on the resolved kinematic redundancy and redundant actuation of a given robotic system applying a biologically inspired synergy approach together with an optimization procedure. First of all, we are going to use the term '*synergy*', as suggested by Bernstein [8]; in order to have a set of rule(s) that unite the central control signal and other control signal(s) to the given redundant bio-robotic system, one may obtain additional equation(s) helping to solve the problem of redundancy. In that way, *control synergy* is obtained using local optimization, where a suitable dynamic criterion based on another biological analogue - distributed positioning - is introduced [7, 8]. On the other hand, in the rest of this paper the actuator redundancy control problem has been established and resolved within the framework of globally optimal control, using Pontryagin's maximum principle, [9]. C*ontrol synergy* established by the optimization law is also obtained at the coordination level, where a central control is also used, now acting upon the joints of the redundant robotic system at the actuator level.

## Robot kinematics and dynamics

*Robot kinematics and dynamics based on the Rodriguez method*

A robotic system is considered as an open linkage consisting of $n+1$ rigid bodies $[V_i]$ interconnected by $n$ one-degree-of–freedom joints formed of kinematic pairs of the fifth class, Fig.1, where the robotic system possesses $n$ degrees of freedom. Here, the Rodriguez method [18, 19], is proposed for modeling the kinematics and dynamics of the robotic system in contrast to Denavit-Hartenberg's method (DH). The configuration of the robot mechanical model can be defined by the vector of joint (internal) generalized coordinates $q$ of the dimension $n$, $(q)=\left(q^1,q^2,...,q^n\right)^T$, where the relative angles of rotation (in the case of revolute joints) and the relative displacements (in the case of prismatic joints). If all the variables $q^i, i=1,2,...n$ are zero, it is said that the robotic system is in the reference configuration (position). The geometry of the system has been defined by unit vectors $\vec{e}_i$, $i=1,2,...,j,..,n$ where unit vectors $\vec{e}_i$ describe the axis of rotation (translation) of the $i$-th segment with respect to the previous segment and as well as vectors $\vec{\rho}_i$ and $\vec{\rho}_{ii}$, usually expressed in local coordinate systems connected with the bodies, $\left(\vec{\rho}_i^{(i)}\right), \left(\vec{\rho}_{ii}^{(i)}\right)$. The parameters $\xi_i, \bar{\xi}_i = 1-\xi_i$ denote the parameters for recognizing joints $\xi_i, \bar{\xi}_i = 1-\xi_i$, $\xi_i =$ (1-prismatic, 0-revolute). For the entire determination of this mechanical system, it is necessary to specify masses $m_i$ and tensors of inertia $J_{Ci}$ expressed in local coordinate systems. In order that the kinematics of the robotic system may be described, the points $O_i, O_i'$ are noticed somewhere at the axis of the corresponding joint $(i)$ such that they coincide in the reference configuration. The point $O_i$ is immobile with respect to the $(i-1)$-th segment and $O_i'$ is immobile with respect to the $i$ – th one; obviously, for the revolute joint $(i)$, the points $O_i$ and $O_i'$ will coincide all the time during robotic motion. For example, the position vector of a point of interest $\vec{r}_H$ can be written as a multiplication of the matrices of the transformation $\left[A_{j-1,j}\right]$ and the position vectors $\vec{\rho}_{ii}$ and $\xi_i q^i \vec{e}_i$ are expressed by

$$\begin{aligned}\vec{r}_H(q) &= \sum_{i=1}^{n}\left(\vec{\rho}_{ii}+\xi_i q^i \vec{e}_i\right) = \\ &= \sum_{i=1}^{n}\left(\prod_{j=1}^{i}\left[A_{j-1,j}\right]\right)\left(\left(\vec{\rho}_{ii}^{(i)}\right)+\xi_i q^i\left(\vec{e}_i^{(i)}\right)\right)\end{aligned} \tag{1}$$

where the appropriate Rodriguez matrices of transformation [18], [19] are

$$\begin{aligned}\left[A_{j-1,j}\right] &= [I]+\left[e_j^{d(j)}\right]^2\left(1-\cos q^j\right)+ \\ &+\left[e_j^{d(j)}\right]\sin\left(q^j\right)\end{aligned} \tag{2}$$

and

$$\begin{aligned}\left(e_j^{(j)}\right) &= \left(e_{\xi j}, e_{\eta j}, e_{\zeta j}\right)^T, \\ \left[e_j^{d(j)}\right] &= \begin{bmatrix} 0 & -e_{\zeta j} & e_{\eta j} \\ e_{\zeta j} & 0 & -e_{\xi j} \\ -e_{\eta j} & e_{\xi j} & 0 \end{bmatrix}\end{aligned} \tag{3}$$

Also, it is shown, [18, 19], regardless of the chosen theoretical approach, that we could start from different theoretical aspects (e.g. general theorems of dynamic, d`Alembert`s principle, Langrange`s equation of second kind, Appell`s equations, etc.) and get equations of motion of the robotic system which can be expressed in the identical covariant form as follows

$$\sum_{\alpha=1}^{n} a_{\alpha i}(q)\ddot{q}^{\alpha} + \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\Gamma_{\alpha\beta,i}(q)\dot{q}^{\alpha}\dot{q}^{\beta} = Q_i \qquad i=1,2,...,n \tag{4}$$

The kinetic energy of the given robotic system is given by

$$E_k = \frac{1}{2}\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} a_{\alpha\beta}\dot{q}^{\alpha}\dot{q}^{\beta} = \frac{1}{2}(\dot{q})^T\left[a_{\alpha\beta}\right](\dot{q}), \quad (5)$$
$$\alpha,\beta = 1,2,...,n,$$

where the coefficients $a_{\alpha\beta}$ are the covariant coordinates of the basic metric tensor $\left[a_{\alpha\beta}\right] \in R^{n\times n}$ and $\Gamma_{\alpha\beta,\gamma}$ $\alpha,\beta,\gamma = 1,2,...,n$ presents the Christoffel symbols of first kind. Generalized forces $Q_i$ can be presented in the following expression (6) where $Q_i^c, Q_i^g, Q_i^\beta, Q_i^w, Q_i^a$ denote the generalized spring forces, gravitational forces, viscous forces, semi-dry friction and generalized control forces, respectively

$$Q_i = Q_i^c + Q_i^g + Q_i^\beta + Q_i^w + Q_i^a,\ \ i = 1,2,...,n \quad (6)$$

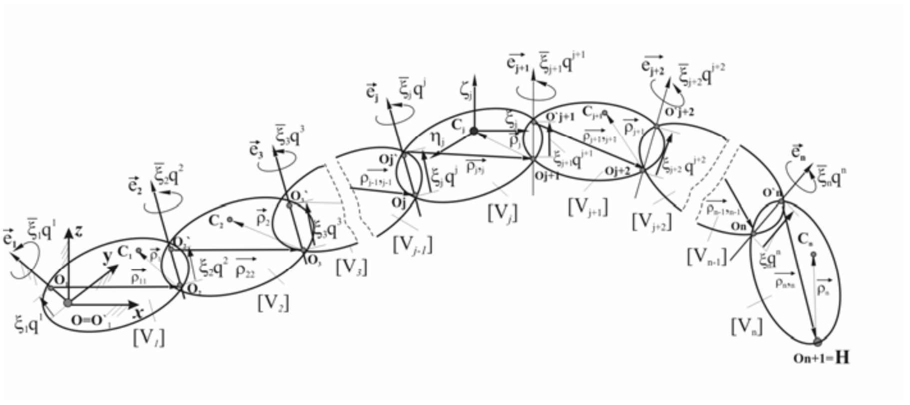

**Figure 1.** Open-chain structure of the mechanical multi-body system

*Illustration of biological analog – the concept of distributed positioning*

Here, the biological analogue-based approach is introduced and illustrated, i.e. the modeling is based on the separation of the prescribed movement into two types of motions: smooth global motions and fast local motions called *distributed positioning (DP)*. The distributed positioning is an inherent property of biological systems. In humans, highly inertial arm joints (shoulders and elbows) provide smooth global motions, and low inertial hand joints (fingers) perform fast and precise local motions [7].

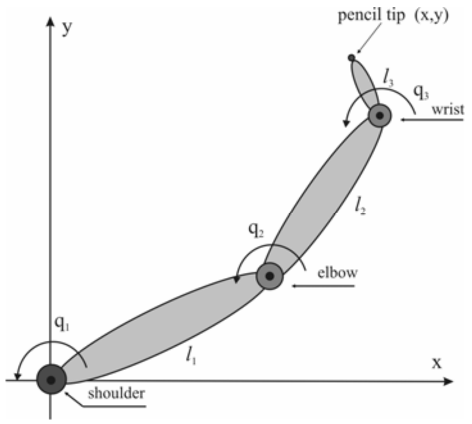


**Figure 2.** Three-DOF arm-hand planar mechanism

Let the position of the arm, Fig.2, be defined by the vector of joint (internal) coordinates of the dimension $n = 3{:}q = \left(q_1, q_2, q_3\right)^T$. The position of the terminal device is defined by the vector of the external coordinates of dimension $n_e = 2{:}\overline{q} = \left(x\ y\right)^T$, where *x, y* define the tip position. The kinematic model of the arm-hand complex i.e. the transformation of coordinates (internal to external, and vice versa) is highly nonlinear

$$\overline{q} = f(q), \quad (7)$$

where *f* is the function: $R^3 \to R^2$. The inverse kinematics (calculation of $q(t)$ for the given $\overline{q}(t)$) has an infinite number of solutions since (7) represents a set of 2 equations with 3 unknowns due to the presence of redundancy. The dimension of redundancy is $n_r = n - n_e = 3 - 2 = 1$. The kinematic model can be written in the Jacobean form of the first or of the second order

$$\dot{\overline{q}} = J(q)\dot{q}\,, \quad \ddot{\overline{q}} = J(q)\ddot{q} + A(q,\dot{q})\,, \quad (8)$$

where $J(q)$ is the $n_e \times n$ (i.e. $2\times 3$) Jacobian matrix and *A* is the $n_e \times 1$ (i. e. 2x1) adjoint vector containing the derivative of the Jacobian. Let $\overline{q}_a$ be the subvector containing the accelerated motions (dimension $n_a$), and $\overline{q}_s$ be the subvector containing the smooth motions ($n_e - n_a$), so one may write $\overline{q} = \left(\overline{q}_a\overline{q}_s\right)^T$. The redundant robot ($n = 3$ DOFs) is now separated into two subsystems. The subsystem with $n_e = 2$ DOFs with the greatest inertia is called *the basic configuration***.** The other subsystem is *the redundancy* having $n_r = 1$ DOFs. It holds that $n = n_e + n_r$. Analyzing the plane writing task, one finds that there are $n_a$ =2 accelerated external motions: *x*(*t*) and *y*(*t*). The basic configuration can be defined as a mechanism $q_b = \left(q_1 q_2\right)^T$. The resting joint- wrist joint ($q_3$) forms the redundancy and $q_r = q_3$ defines the position of the redundancy. So, one may use the DP concept to solve the inverse kinematics of a redundant robot in two steps. The first step is to calculate the motion of the basic configuration ($q_b$) using the kinematic model and the properties of the DP concept $\left(q_r = const\right)$, and the second step is to determine the motion of redundancy ($q_r$) [7]. But here, we are considering the dynamic model of the robotic system Eq(4) with the gravitational forces $Q_i^g(q) = -\frac{\partial \Pi(q)}{\partial q_i}$, ( $\Pi(q)$ is potential energy) and the generalized control forces $Q_i^a = Q_i^a(u)$, partially applying the DP concept to the dynamic model ( $q = (q_b q_r)^T$, where $q_b$ is calculated using the DP concept), which is presented in a condensed form as follows:

$$\sum_{\alpha=1}^{n} a_{\alpha i}(q)\ddot{q}^{\alpha} + \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} \Gamma_{\alpha\beta,i}(q)\dot{q}^{\alpha}\dot{q}^{\beta} = Q_i \quad , \qquad (9a)$$
$$i = 1,2,...,n \Rightarrow a(q) + b(q,\dot{q}) = Q^g + Q^a,$$

$$a(q)\ddot{q} + \left(b(q,\dot{q}) - Q^g(q)\right) = Q^a \Rightarrow$$
$$\Rightarrow \left(a_b(q) \;\; a_r(q)\right)\begin{pmatrix}\ddot{q}_b \\ \ddot{q}_r\end{pmatrix} + D(q_b, q_r, \dot{q}_b, \dot{q}_r) = Q(u), \qquad (9b)$$

In addition, kinematic model (8) can be presented as

$$\ddot{\ddot{q}} = \begin{bmatrix} J_b(q) \\ J_r(q) \end{bmatrix}\begin{pmatrix}\ddot{q}_b \\ \ddot{q}_r\end{pmatrix} + A(q,\dot{q}) \Rightarrow$$
$$\Rightarrow \ddot{\ddot{q}} - A(q,\dot{q}) = J_b(q)\ddot{q}_b + J_r(q)\ddot{q}_r, \qquad (10)$$

## Main results

As we pointed out previously, numerous concepts of synergies have already been discussed in the literature, [2-14]. However, synergies have been subject to different interpretations and have been used differently in different contexts. Recently, biologically inspired synergies have been used for the balance control in humanoid robots [21]. Based on the principle that biological organisms recruit kinematic synergies that manage several joints, a control strategy for the balance of humanoid robots is developed. Also, synergies based on the principles of data reduction and dimensionality reduction, are soon to find place in telesurgery and telerobotics [22].

Our work focuses on resolving kinematic redundancy and the redundant actuation of a given robotic system by applying biologically inspired synergy approach together with the optimization procedure. According to our point of view, it is possible to resolve kinematic redundancy where *control synergy* is obtained using local optimization. Also, the actuator redundancy control problem can be established and resolved within the framework of globally optimal control, using Pontryagin's maximum principle where one may obtain *control synergy* at the coordination level.

*Resolving the redundancy degree of one using local optimization of the dynamic criterion – control synergy*

First of all, we are going to use the term "*synergy*", Bernstein [10], meaning a set of rule (s) that unites the central control signal and belongs to a higher-level hierarchical control (denoted here as logical $u_L$ ), and other control signal (s) applied to the given redundant robotic system, into an equation (or a number of equations) helping to solve the problem of kinematic redundancy. In that way, *control synergy* is obtained using local optimization, where a suitable dynamic criterion based on another biological analogue - distributed positioning - [7] is introduced. Specially, it has been observed that the possibility of switching synergies within a single movement according to task requirements may be an essential component of acquiring motor skills [23]. Similarly, we have considered and discussed the problem of switching synergies within the redundant robotic system. Applying the synergy approach with the local optimization of a suitable dynamic criterion in respect to logical control, one can solve the problem of redundancy with the possibility of obtaining two-synergy control within a single movement. Here, generalized forces are introduced such that

$$Q(t) = u(t) + Q_L(t) = u(t) + th(\alpha(t)u_L)\cdot th(\beta(t)(u_L - 1)) \quad (11)$$

where $u_L \in (0,1)$ is logical control and $th(u) = \frac{e^u - e^{-u}}{e^u + e^{-u}}$.

Also, it is assumed that the vectors $\alpha(t)$, $\beta(t)$ are obtained (through training) at a higher level of control, (see Fig.3).

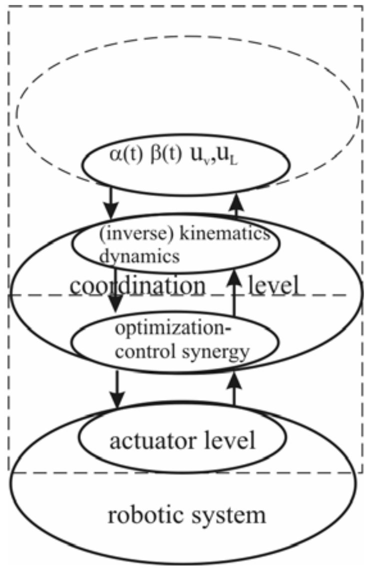


**Figure 3.** Hierarchical organization of control

The following dynamic criterion is suggested (see Eq.12), which is appropriate for the on-line use in robotics,

$$I_l = \frac{1}{2}\ddot{q}_r^T\left(a_r(q) - a_b^{-1}(q)J_r(q)\right)^T$$
$$\left(a_r(q) - a_b^{-1}(q)J_r(q)\right)\ddot{q}_r \to \underset{u_L}{ext} \qquad (12)$$

where one needs to take into account equality constraints, (9), (10). Now, equation (9) can be rewritten as

$$\underbrace{\left(a_r(q) - a_b(q)J_b^{-1}(q)J_r(q)\right)}_{a_r^*}\ddot{q}_r =$$
$$= Q - \underbrace{D(q,\dot{q}) - a_b(q)J_b^{-1}(q)\left(\ddot{\ddot{q}} - A(q,\dot{q})\right)}_{D^*} \qquad (13)$$

so, one can obtain the augmented objective cost function as

$$I_l = \frac{1}{2}(Q - D^*)^T(Q - D^*) \to \min_{u_L} \qquad (14)$$

The necessary conditions for optimality are

$$\frac{\partial I_l}{\partial u_L} = 0 \Rightarrow \frac{\partial Q^T}{\partial u_L}(Q - D^*) = 0 \qquad (15)$$

or

$$\left[\frac{\alpha}{ch^2(\alpha u_L)}th(\beta(u_L - 1)) + \frac{\beta}{ch^2(\beta(u_L - 1))}th(\alpha u_L)\right]\cdot(Q - D^*) = 0 \quad (16)$$

where $ch(u)=\frac{e^{u}+e^{-u}}{2}$. In the same manner, it yields for the case $u_L=0$

$$u_{|u_L=0}=\begin{bmatrix} J(q)a^{-1}(q) \\ \alpha\, th(-\beta) \end{bmatrix}^{-1}\begin{pmatrix} J(q)a^{-1}(a)(D(q,\dot q)-A(q,\dot q) \\ \alpha\, th(-\beta)D(q,\dot q) \end{pmatrix},$$

$$\det\begin{bmatrix} J(q)a^{-1}(q) \\ \alpha\, th(-\beta) \end{bmatrix}^{-1}\neq 0 \tag{17}$$

and for the case $u_L=1$

$$u_{|u_L=1}=\begin{bmatrix} J(q)a^{-1}(q) \\ \beta th(\alpha) \end{bmatrix}^{-1}\begin{pmatrix} J(q)a^{-1}(q)(D(q,\dot q)-A(q,\dot q) \\ \beta th(\alpha)D(q,\dot q) \end{pmatrix},$$

$$\det\begin{bmatrix} J(q)a^{-1}(q) \\ \beta th(\alpha) \end{bmatrix}^{-1}\neq 0 \tag{18}$$

*Resolving actuator redundancy degree of one using global optimization of the dynamic criterion – synergy control*

Using control policies or actuator synergies have attracted great interest in robotics research. For example, the authors in [24] suggested that synergistic control may not mean dimensionality reduction or simplification, but might imply task optimization. This opinion was consistent with the view in [12], where the authors applied optimal feedback control as a theory of motor coordination.

Here, the actuator redundancy control problem has been established and resolved within the framework of globally optimal control, using Pontryagin's maximum principle, (see [9]). This work presents how one may obtain *control synergy* established by the optimization law at the coordination level. We suggest using a central control $u_c\neq 0$ which acts on the joints of the redundant robotic system at the actuator level. All control signals are introduced as a superposition of two control signals – the central control $u_c$ and the corresponding additive control $u_i$. The redundancy control problem has been resolved and discussed within the framework of globally optimal control, using Pontryagin's maximum principle. Here, the dynamic model of the robotic system is described with the application set of the $2n$ Hamiltonian equations with respect to the Hamiltonian phase variables $q_i, p_i$, [19, 24], where $p_i$ denotes conjugate (canonical) momenta

$$\dot q_i=\frac{\partial H}{\partial p_i}\quad \dot p_i=-\frac{\partial H}{\partial q_i}+Q_i(u),\ i=1,2,...,n \tag{19}$$

$Q_i,\ i=1,2,..,n$ are non-conservative control forces, $\Pi(q)$ is potential energy and $H(q,p)$ is the Hamiltonian presented as

$$\begin{aligned} H(q,p)&=E_k(q,\dot q)+\Pi(q)= \\ &=\frac{1}{2}\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}a^{\alpha\beta}(q)p_\alpha p_\beta+\Pi(q) \end{aligned} \tag{20}$$

where the contravariant coordinates $a^{\alpha\beta}(q)$ of the basic metric tensor $[a_{\alpha\beta}]\in R^{n\times n}$ can be obtained from the following relation

$$\left[a^{\alpha\beta}\right]=\left[a_{\alpha\beta}\right]^{-1} \tag{21}$$

With Eq.(19), the Hamiltonian equations are

$$\dot q^i=\sum_{\alpha=1}^{n}a^{i\alpha}p_\alpha \tag{22}$$

$$\begin{aligned} &\dot p_i=-\frac{1}{2}\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\frac{\partial a^{\alpha\beta}}{\partial q^i}p_\alpha p_\beta-\frac{\partial\Pi}{\partial q^i}+Q_i(u), \\ &i=1,2,...,n \end{aligned} \tag{23}$$

Also, in this case, it is assumed that a hierarchical control structure is introduced, see Fig.3. First, one can be interested in the possibility to optimize the robotic motions, and the optimization is used to resolve the redundancy control problem at the coordination level. For a global optimization, the problem is set up as follows

$$J=\int_{t_0}^{t_k}f_o(q,p,u)dt\rightarrow min \tag{24}$$

The goal is to find $u(t),\ t_0\le t\le t_k$ which drives the system from a given initial state $(q_0,p_0)$ to a final state $(q_k,p_k)$ under the condition that the whole trajectory minimizes the performance criterion. The performance criterion is introduced at the coordination level as the energy criterion which is, in our case, the functional sum of the weighted controls of the robot

$$f_o(u)=\frac{1}{2}u^TRu \tag{25}$$

Alternatively, the control can be smoothed by minimizing the energy function, quadratic in control, in addition to time. Here, $t_0t_k$ are the initial and final times, known and fixed, of an end-effector movement. The control weighting matrix $R=diag(r_1,r_2,...,r_m)$ is a symmetric positive definite matrix; $u(t)$ must be the entry of a given subset $U$ of admissible controls of the *m*-dimensional Euclidean space: $u(t)\in U\subset R^m$. It is also assumed that the optimal control problem has a solution. Pontryagin's maximum principle which yields the optimal control law will be used. So, Pontryagin's Function [25] is introduced as

$$\aleph=\mu_o f_o+\mu^T(\frac{\partial H}{\partial p})+\lambda^T(-\frac{\partial H}{\partial q}+Q) \tag{26}$$

or, after taking into account Eq.(26),(27) as

$$\begin{aligned} \aleph&=\mu_0 f_0+\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\mu_\beta a^{\alpha\beta}p_\alpha+ \\ &+\sum_{\gamma=1}^{n}\lambda^\gamma\left(-\frac{1}{2}\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\frac{\partial a^{\alpha\beta}}{\partial q^\gamma}p_\alpha p_\beta-\frac{\partial\Pi}{\partial q^\gamma}+Q_\gamma(u)\right) \end{aligned} \tag{27}$$

where $\mu_o\in R^1,\mu\in R^n,\lambda\in R^n$ are Langrange multipliers. Now, the necessary conditions can be obtained by using the following theorem

**Theorem 1***:* If there is $u^*(t),\ t\in[t_0t_k]$ optimal control of the given optimal control problem and if $q^*(t),p^*(t)$ are the corresponding optimal trajectories, then the continuous $2n+1$ dimension vector $(\mu_0, \mu(t), \lambda(t))$ exists, which is different from zero and it satisfies the following expressions:

a) $q^*, p^*, u^*, \mu^*, \lambda^*$, on interval $t \in [t_0, t_k]$ satisfying the following system of equations

$$\dot{q}^i = \sum_{\alpha=1}^{n} a^{i\alpha} p_\alpha \tag{28}$$

$$\dot{p}_i = -\frac{1}{2}\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} \frac{\partial a^{\alpha\beta}}{\partial q^i} p_\alpha p_\beta - \frac{\partial \Pi}{\partial q^i} + Q_i(u), \quad i = 1,2,...,n \tag{29}$$

$$\dot{\mu}_i = -\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} \mu_\beta \frac{\partial a^{\alpha\beta}}{\partial q^i} p_\beta + \frac{1}{2}\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\sum_{\gamma=1}^{n} \lambda^\gamma \frac{\partial^2 a^{\alpha\beta}}{\partial q^\gamma \partial q^i} p_\alpha p_\beta + \sum_{\gamma=1}^{n} \lambda^\gamma \frac{\partial^2 \Pi}{\partial q^\gamma \partial q^i} \tag{30}$$

$$\dot{\lambda}^i = -\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} \mu_\beta a^{i\beta} + \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\sum_{\gamma=1}^{n} \lambda^\alpha \frac{\partial a^{\beta i}}{\partial q^i} p_\beta \tag{31}$$

b) $\aleph(\mu_0^*, \mu^*, \lambda^*, q^*, p^*, u)$ in a point $u = u^*$ has a maximum:

$$\aleph(\mu_0^*, \mu^*, \lambda^*, q^*, p^*, u) \le \aleph(\mu_0^*, \mu^*, \lambda^*, q^*, p^*, u^*) \tag{32}$$

c) $$\mu_o^* = const \le 0 \tag{32}$$

In this paper, it is assumed that there is no constraint on the control vector and admissible controls belong to open set, so that the condition (*b*) can be expressed as

$$\left(\frac{\partial \aleph}{\partial u}\right)_{u^*} = 0, \ \left(\frac{\partial^2 \aleph}{\partial u_\alpha \partial u_\beta}\right) u_\alpha u_\beta \le 0, \quad \alpha, \beta = 1,2,...m \tag{34}$$

or, in a more explicit form

$$\mu_0 \frac{\partial f_0}{\partial u} - \frac{\partial Q^T(u)}{\partial u} \lambda \Big|_{u^*} = 0 \tag{35}$$

*Proof*: Applying Pontrygin's maximum principle [25, Theorem 6, p.78,] and taking into account that the phase variables $q(t) \in R^n, p(t) \in R^n$ correspond to the vector $x(t) \in R^{2n}$ and that $\mu_o \in R^1, \mu(t) \in R^n, \lambda(t) \in R^n$ correspond to the vectors $\psi_0 = \mu_0 \in R^1$ and $\psi(t) = (\mu(t)\ \lambda(t)) \in R^{2n}$ (see [25]), one can directly prove the previous Theorem 1. Also, applying the biologically inspired concept of control and introducing the central control $u_c$ as suggested by Bernstein [9, 10], one may introduce (in our example, see below, $n = 3$, $m = 4$ ), the control vector $u = (u_1, u_2, u_3, u_c)^T$ . Here, the generalized forces can be presented as the functions of the components of the control $u$ as

$$Q_i = u_i + \alpha_i u_c, \ i = 1,2,3 \qquad u = (u_1, u_2, u_3, u_c)^T \tag{36}$$

In other words, all control signals are introduced as the superposition of the two control signals – the central control $u_c$ and the corresponding additive control $u_i$ , where the coefficients $\alpha_i = const, \ i = 1,2,3$ are introduced due to different dimensions of control variables. In that way, one of the possible control strategies is established. Taking into account the condition (c) with $\mu_o = -1$ , [25] the condition (35) yields to

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \alpha_1 & \alpha_2 & \alpha_3 \end{bmatrix} \cdot \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \begin{pmatrix} -u_1^* r_1 \\ -u_2^* r_2 \\ -u_3^* r_3 \\ -u_c^* r_0 \end{pmatrix} \tag{37}$$

Applying the matrix theory to solve Eq (36), it implies that the following condition must be fulfilled, so that there is a unique solution for the vector $\lambda$

$$\det \begin{bmatrix} 1 & 0 & 0 & u_1^* r_1 \\ 0 & 1 & 0 & u_2^* r_2 \\ 0 & 0 & 1 & u_3^* r_3 \\ \alpha_1 & \alpha_2 & \alpha_3 & u_c^* r_0 \end{bmatrix} = 0 \tag{38}$$

After some algebraic operations, it yields

$$u_c^* r_0 = \alpha_1 u_1^* r_1 + \alpha_2 u_2^* r_2 + \alpha_3 u_3^* r_3 \tag{39}$$

Eq (39) presents *control synergy* according to control variables. Here, we assumed that all coefficients $\alpha_i = 1, \ i = 1,2,3$ are of appropriate dimensions. Moreover, one may apply another possible strategy presented as follows

$$Q_i = \alpha_i u_c + th(u_i), \quad i = 1,2,3 \tag{40}$$

In the same manner, using the previous procedure, one may obtain the corresponding *control synergy*

$$u_c^* r_0 = r_1 u_1^* ch^2(u_1^*) + r_2 u_2^* ch^2(u_2^*) + r_3 u_3^* ch^2(u_3^*) \tag{41}$$

The obtained control synergy control and our proposed approach differ from the recently suggested rivalling controls introduced in [26] by splitting up one original control into two independent and additive controls, where the joints of the redundant robotic system are driven by weak but fast and strong but slow actuators acting in parallel. Further, in order to obtain finite solutions of the problem mentioned, it is necessary to solve a two-point boundary value problem for a system of ordinary differential equations or, even in particular cases, to solve complex algebraic problems.

## Simulation results

To demonstrate the previous synergy approach, we consider a robotic system connected as an open-loop kinematic chain with three DOF`s and four control variables. In particular, we solve the optimal problem using expression (31) for the given cylindrical robot Automelec

ACR with three DOFs. The geometry of the cylindrical robot is depicted in Fig.4 [27], where $M = 0.65\,\mathrm{kg\,m}^2, B = C = D = 8.3\,\mathrm{kg}$ and the control weighting matrix is $R = diag(1,1,2,3)$. The covariant coordinates of the basic metric tensor $\left[a_{\alpha\beta}\right]_{\alpha,\beta=1,2,3}$ are

$$\left[a_{\alpha\beta}\right] = \begin{bmatrix} M + B\,q_2^2 & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & D \end{bmatrix}. \tag{42}$$

and the potential energy due to gravitational forces is given $\Pi = D\,g q_3 + const$ where $g$ is the gravity acceleration.

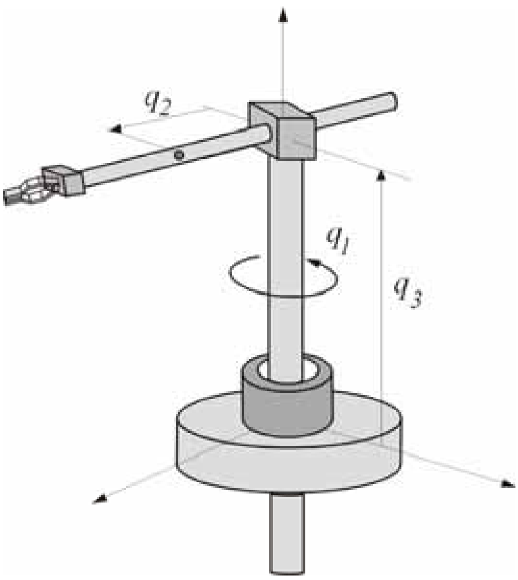


**Figure 4.** Cylindrical robot Automelec ACR with three DOFs

The two-point boundary value problem has been solved for the following boundary conditions, where the initial position is given:

$$\begin{aligned} &t_0 = 0 \text{ s}, \\ &q_1(t_0) = 0 \text{ rad}, \\ &q_2(t_0) = 0.1\,\text{m}, \\ &q_3(t_0) = 0 \text{ m}, \\ &p_1(t_0) = 0 \text{ kgm}^2/\text{s}, \\ &p_2(t_0) = 0 \text{ kgm/s}, \\ &p_3(t_0) = 0 \text{ kgm/s}, \end{aligned} \tag{43}$$

and the final position for three different values $t_1 = 1s, 1.5s, 2s$ is:

$$\begin{aligned} &t_1 = 1\text{s}, 1.5\text{s}, 2\text{s} \\ &q_1(t_1) = \pi/4 \text{ rad}, \\ &q_2(t_1) = 0.25\,\text{m}, \\ &q_3(t_1) = 0.2\,\text{m}, \\ &p_1(t_1) = 0\,\text{kgm}^2/\text{s}, \\ &p_2(t_1) = 0 \text{ kgm/s}, \\ &p_3(t_1) = 0\,\text{kgm/s}. \end{aligned} \tag{44}$$

Using the results of Theorem 1, one can obtain the optimal controls (Figs. 5-8) as follows:

$$\begin{aligned} &u_c^* = (\lambda_1 + \lambda_2 + \lambda_3)/r_0, \quad &u_1^* = \lambda_1/r_1 \\ &u_2^* = \lambda_2/r_2, &u_3^* = \lambda_3/r_3 \end{aligned} \tag{45}$$

Now, substituting the previously obtained optimal controls $u^*(t),\ t \in [t_o, t_k]$ into Eqs. (28-31), one can obtain the system of twelve nonlinear differential equations of the first order as follows:

$$\begin{aligned} \dot{q}_1 &= \left(M + B\,q_2^2\right)^{-1} p_1 \\ \dot{q}_2 &= C^{-1} p_2 \\ \dot{q}_3 &= D^{-1} p_3 \\ \dot{p}_1 &= (\lambda_1 + \lambda_2 + \lambda_3)/r_0 + \lambda_1/r_1 \\ \dot{p}_2 &= (\lambda_1 + \lambda_2 + \lambda_3)/r_0 + \lambda_2/r_2 + \left(M + B\,q_2^2\right)^{-2} B\,q_2 p_1^2 \\ \dot{p}_3 &= (\lambda_1 + \lambda_2 + \lambda_3)/r_0 + \lambda_3/r_3 - D\,g \end{aligned} \tag{46}$$

$$\begin{aligned} \dot{\mu}_1 &= 0 \\ \dot{\mu}_2 &= 2\mu_1 p_1 B\,q_2 \left(M + B q_2^2\right)^{-2} + \\ &\quad + \lambda_2 \Big( -4 \left(B\,q_2 p_1\right)^2 \left(M + B q_2^2\right)^{-3} + \\ &\quad + B p_1^2 \left(M + B q_2^2\right)^{-2} \Big) \\ \dot{\mu}_3 &= 0 \\ \dot{\lambda}_1 &= -\mu_1 \left(M + B q_2^2\right)^{-1} - 2\lambda_2 B\,q_2 p_1 \left(M + B q_2^2\right)^{-2} \\ \dot{\lambda}_2 &= C^{-1} \mu_2 \\ \dot{\lambda}_3 &= D^{-1} \mu_3 \end{aligned} \tag{47}$$

The problem is solved by the use of the program [28] based upon the method of finite differences with the tolerance of a relative error of $10^{-6}$ (see Figs. 5-20). Finally, one can easily check that condition (39) is fulfilled, which presents "*control synergy*".

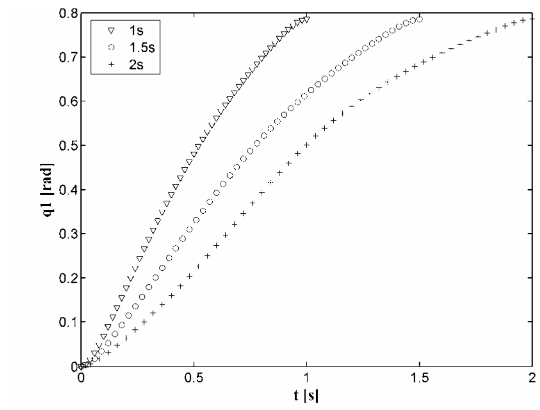


**Figure 5.** Optimal trajectory $q^1$

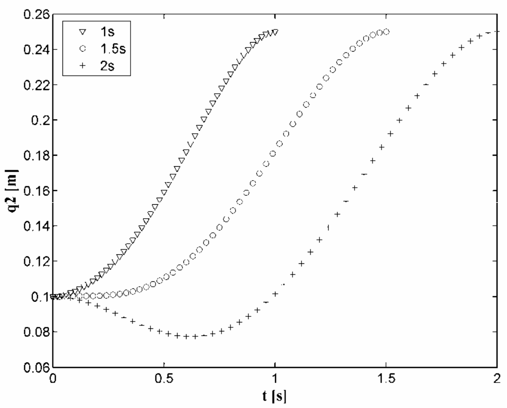


**Figure 6.** Optimal trajectory $q^2$

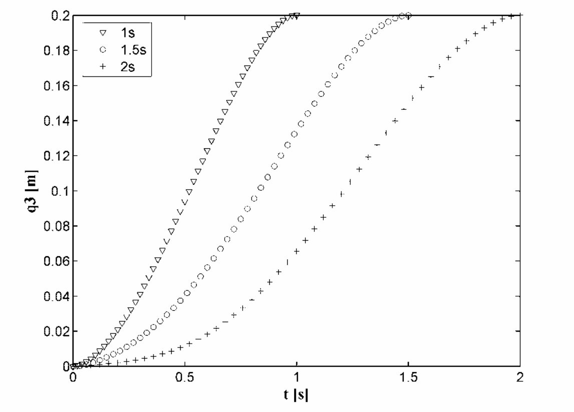


**Figure 7.** Optimal trajectory *q*3

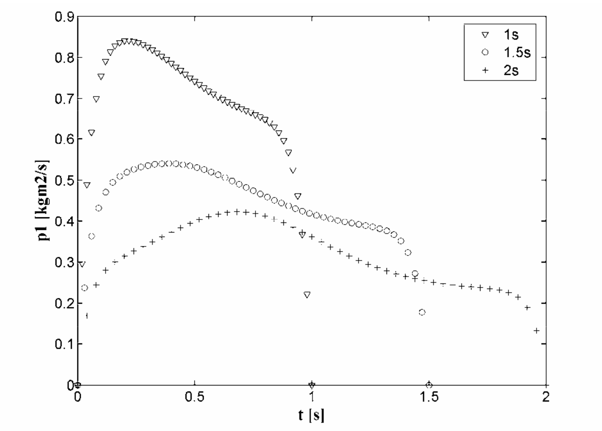


**Figure 8.** Optimal momenta $p^1$

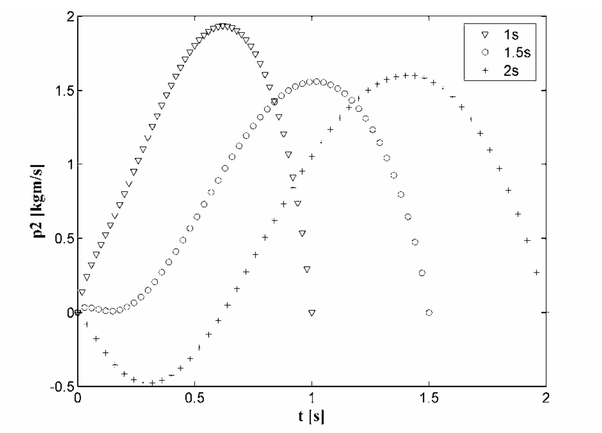


**Figure 9.** Optimal momenta $p^2$

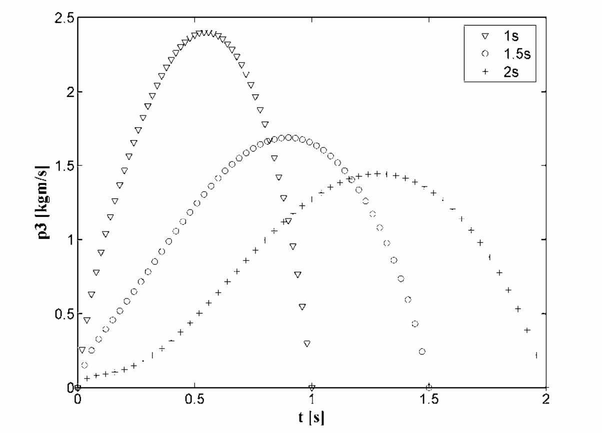


**Figure 10.** Optimal momenta $p^3$

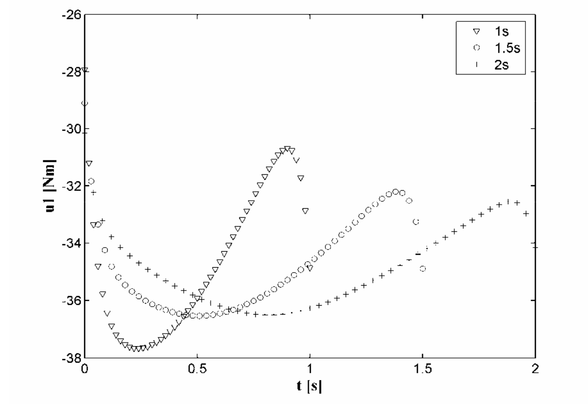


**Figure 11.** Optimal control $u^1$

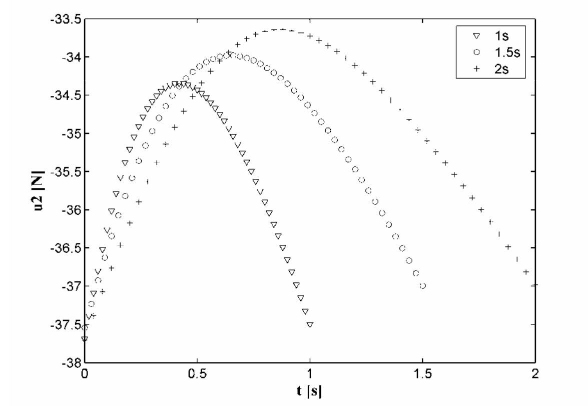


**Figure 12.** Optimal control $u^2$

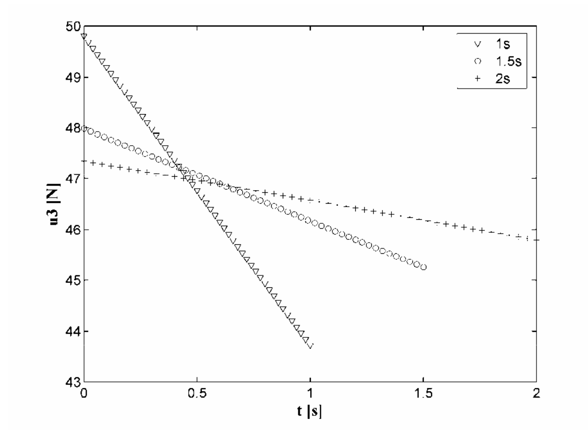


**Figure 13.** Optimal control $u^3$

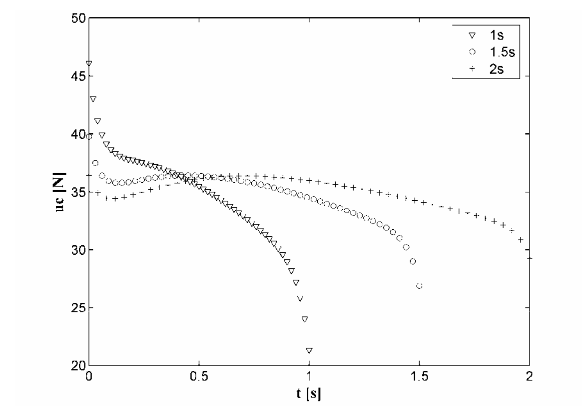


**Figure 14.** Optimal control *uc*

## Conclusions

The present paper proposes a new kind of biologically inspired control of a redundant robotic system which allows resolving actuator and kinematic redundancy. A suggestion is given for obtaining a class of dynamic synergy, or more precisely *control synergy,* arising from a theory of optimal actuator behaviour. Firstly, it is illustrated that it is possible to resolve kinematic redundancy using the local optimization method together with biological analogues – the control synergy approach-hypothetical control and the distributed positioning. In addition, the possibility of switching synergies within a single movement according to task requirements is treated, where the control synergy approach is proposed using a suitable logical control. Finally, the actuator redundancy control problem has been discussed and proved within the framework of the optimal control problem which is solved by Pontryagin's maximum principle. The control synergy approach is suggested, established by the optimization law at the coordination level of the hierarchical control structure, where a central control is used. The usefulness of the idea of biologically inspired control synergy has been tested by the applications to a suitable redundant actuation robot with 3DOF`s and 4 control variables.

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## References

[1] WEBB,B.: *Can robots make good models of biological behaviour*? Behav. Brain Sci. 24, 2001, 24:1033– 1094.

[2] TOMOVIĆ,R., POPOVIĆ,D., STEIN,R.: *Nonanalytical Methods for Motor Control*, World Sc.:London;1995;40-45.

[3] LATHASH,M.: *Control of human movement*, Human Kinetics Publishers,1994.

[4] BAR-COHEN,Y.: *Biomimetics-Biologically Inspired Technologies*, CRC Press, Taylor & Francis Group, Boca Raton:FL, 2006;399-425.

[5] VEPA,R.: *Biomimetic Robotics-mechanisms and control*, Cambridge University Press, 2009.

[6] KLUG,S., LENS,T., VON STRYK,O., MÖHL,B., KARGUTH,A.: *Biologically Inspired Robot Manipulator for New Applications in Automation Engineering*, Proc. of Robotik 2008, Munich, Germany, June 11-12.

[7] POTKONJAK,V., POPOVIĆ,M., LAZAREVIĆ,M., SINANOVIĆ,J.: *Redundancy problem in writing: from human to anthropomorphic robot arm*, IEEE Trans. on Systems, Man and Cybernetics: part B: Cybernetics, 1998; 28(6):790-805.

[8] POTKONJAK,V., KOSTIĆ,D., TZAFESTAS,S., POPOVIĆ,M., LAZAREVIĆ,M., ĐORĐEVIĆ,G.: *Human – like behavior of robot arms: general considerations and the handwriting task-Part II: the robot arm in handwriting*, Robotics and Computer Integrated Manufacturing 17,(2001), pp.317-327.

[9] LAZAREVIC,M., OBRADOVIC,A., JOKA3,M., BUCANOVIĆ,Lj.: *Biologically inspired optimal control of robotic system:synergy approach*, 17th Mediterranean Conference on Control & Automation,Macedonia P.,Thessaloniki, Greece, June 24 - 26, 2009, pp.958-963.

[10] BERNSTEIN,A.: *The Coordination and Regulation of Movements*, Oxford Pergamon, 1967; 77-92.

[11] HARKEGARD,O., GLAD,S.T.: *Resolving actuator redundancy-optimal control vs. control allocation*, Automatica 2005 41 :137-144.

[12] TODOROV,E, JORDAN,M.I.: *Optimal feedback control as a theory of motor coordination*, Nat Neurosci 2002,5: 1226–1235.

[13] TODOROV,E.: *Optimality principles in sensorimotor control*, Nat. Neuroscience 2004;7(9): 907–915.

[14] LAZAREVIĆ,M.: *Fractional Order Control of a Robot System Driven by DC Motors*, Scientific Technical Review ISSN 1820-0206, 2012,Vol.62,No.2,pp.20-29.

[15] LATASH,M.L.: Synergy, Oxford University Press US, 2008, pp.432.

[16] COURTINE,G., SCHIEPPATI,M.: *Tuning of a basic coordination pattern constructs straight-ahead and curved walking in humans*, J Neurophysiol. 2004,91:1524-1535.

[17] GRINYAGIN,I.V., BIRYUKOVA,E.V., MAIER,M.A.: *Kinematic and Dynamic Synergies of Human Precision-Grip Movements*, J Neurophysiol. 2004,94: 2284-2294.

[18] Lazarević,M.: *Optimal control of redundant robots in human-like fashion: general considerations*, FME Transaction, Faculty of Mechanical Engineering, University of Belgrade, Belgrade. 2005; 33(2):53-64.

[19] ČOVIĆ,V., LAZAREVIĆ,M.: *Lectures - Mechanics of robots*, Faculty of Mechanical Engineering, Belgrade (in Serbian),2009.

[20] LURIE,I.: *Analytical Mechanics,* (in Russian), G. Publishing, Moscow, 1961.

[21] HAUSER,H., NEUMANN,G., IJSPEERT,A.J., MAASS,W.: *Biologically inspired kinematic synergies provide a new paradigm for balance control of humanoid robots*, In Proceedings of the 7th IEEE RAS/RSJ Conference on Humanoids Robots (HUMANOIDS07), Pittsburgh, PA, December 2007.

[22] VINJAMURI,R., MAO,Z.H, SCLABASSI,R, M.SUN.: *Time-varying synergies in velocity profiles of finger joints of the hand during reach and grasp*, IEEE International Conference of the Engineering in Medicine and Biology Society, France,2007.

[23] JARIĆ,S., LATASH,M.: *Learning a Motor Task Involving Obstacles by a Multi-Joint*, Redundant Limb: Two Synergies within One Movement, Journal of Electromyography and Kinesiology, 1998; 8,(3),169-176.

[24] TODOROV,E, GHAHRAMANI,Z.: *Analysis of the synergies underlying complex hand manipulation*, Proc. Annu. Conf. IEEE Engineering in Medicine and Biology Society 2, 4637-4640,2004.

[25] PONTRYAGIN,S., BOLTYANSKII,V.G., GAMRELIDZE,R.V., MISHCHENKO,E.F.: *The Mathematical Theory of a Optimal Processes*, (in Russian), Moscow, 1983.

[26] SCHANZER,G, CALLIES,R.: *Multiple constrained rivaling actuators in the optimal control of miniaturized manipulators*, Multibody System Dynamics 2008; 19:21–43.

[27] GERING,H., GUZZELLA,L., HEPENER,S., ONDER,C.: *Time optimal motions of robots in assembly tasks*, IEEE Transactions on Automatic Control, 1986; AC-31,(6):512-518.

[28] PEREYRA,V.: PASVA 3: *An adaptive finite difference fortran program for first order nonlinear, ordinary boundary problems*, Lecture Notes in Computer Science. 1978; 76: 67-88.

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# Bio-inspirisano upravljanje redundantnog robotskog sistema: optimizacioni pristup

**Osnovni cilj ovog rada je da promoviše pristup biološki inspirisanog sinergijskog upravljanja koji omogućava da se razreši redundansa datog robotizovanog sistema koji se može koristiti i za vojne svrhe. Pokazano je da je moguće razrešiti kinematički redundansu primenom metode lokalne optimizacije i bioloških analogona- sinergijsko upravljački pristup sa uvođenjem logičkog upravljanja i distribuiranog pozicioniranja. Takođe, mogućnost prebacivanja između sinegrija u okviru jedne trajektorije je razmatrano. Na kraju, problem aktuatorske redundanse je postavljen i rešen primenom Pontrjaginovog principa maksimuma. Upravljačka sinergija je ustanovljena primenom postupka optimizacije na koordinacionom nivou. Na kraju, efikasnost predložene biološki inspirisane optimalne upravljačke sinergije je demonstriran na pogodno usvojenom robotskom sistemu sa tri stepena slobode i četiri upravljačke promenljive, kao ilustrativnog primera.**

*Ključne reči:* **biomehanika, robotika, redundantni sistem, optimalno upravljanje, metoda optimizacije, redundansa, sinergija.**

# Био-инспирированное управление избыточной робототехнической системы: оптимизационный подход

**Основной целью настоящей работы является содействие подхода к биологически инспирированному синергетическому управлению, позволяющему решить избыточности данной роботизированной системы, которая может быть использована и в военных целях. Показано, что это можно решить кинематической избыточностью с использованием методов местной оптимизации и биологических аналог-синергетических управлений со введением логического контроля доступа и распределённых позиционирований. Кроме того, здесь обсуждается и возможность переключения между синергиями в пределах одной траектории. Наконец, проблема привода резервирования установлена и решена с помощью принципа максимума Понтрягина. Управляющая синергия определяется применением процессов оптимизации на уровне координации. Наконец, эффективность предлагаемой биологически инспирированной оптимальной синергии продемонстрирована на принятой совершенной робототехнической системе с тремя степенями свободы и с четырьмя переменными управления, в качестве иллюстративного примера.**

*Ключевые слова:* **биомеханика, робототехника, дублирующие системы, оптимальное управление, методы оптимизации, резервирования, взаимодействия.**

# Le contrôle bio inspiré du système robotique redondant: approche d'optimisation

**Le but principal de ce travail est de promouvoir l'approche du contrôle synergique inspirée biologiquement qui permet de résoudre la redondance du système robotique donné qu'on peut utiliser dans les fins militaires. On a démontré qu'il était possible de résoudre la redondance cinétique à l'aide de la méthode de l'optimisation locale et les analogies biologiques – approche synergique de contrôle avec l'introduction du contrôle logique et le positionnement distribué. On a considéré aussi la possibilité du transfert parmi les synergies dans le cadre d'une seule trajectoire. A la fin le problème de la redondance actuaire a été posé et résolu par le principe de maximum de Pontryagin . La synergie de contrôle a été établie au moyen du procédé d'optimisation au niveau de la coordination. Pour finir, l'efficacité de la synergie de contrôle inspiré biologiquement a été démontrée chez le système robotique adopté à trois niveaux de liberté et quatre variables de contrôle, comme un exemple d'illustration.**

*Mots clés:* **biomécanique, robotique, système de redondance, contrôle optimale, méthode d'optimisation, redondance, synergie.**