ESTIMATION OF LASER-DOPPLER ANEMOMETRY MEASURING VOLUME DISPLACEMENT IN CYLINDRICAL PIPE FLOW

by

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Laser-Doppler anemometry application in measurements of the 3-D swirl turbulent flow velocity in the cylindrical pipe, behind the axial fan, have been analysed. This paper presents a brief overview of uncertainty sources in the laser-Doppler anemometry measurements. Special attention is paid to estimation of laser-Doppler anemometry measuring volume positioning in cylindrical pipe flow due to optical aberrations, caused by the pipe wall curvature. The hypothesis, that in the central part of the pipe ($r/R < 0.6$) exists a small, or negligible pipe wall influence on laser-Doppler anemometry measuring position, is investigate. The required corrections, for measurements of axial, tangential, and radial velocity components such: shift of measuring volume and its orientation are analysed and determined for used test rig and for some other pipe geometries.

Key words: *laser-Doppler anemometry, measuring volume shift, cylindrical pipe, aberrations, uncertainty*

Introduction

Laser-Doppler anemometry (LDA) is an optical technique for measurement of velocity and turbulence in gas, liquid, multiphase fluids, chemically reacting flows, in combustion, flame, wave tanks, rotating machinery, wind or water tunnels, micro and macro channels, in biomedical applications, atmospheres, oceanography and in other scientific, and industrial research, where the conventional techniques can not be performed successfully [1-11]. This optical method of local instantaneous velocity measurement is non-contact, non-intrusive and can be employed where the physical sensors are difficult or impossible to use. It offers a very high accuracy without calibration, because LDA measures the absolute velocity components. The LDA can measure one, two or three instantaneous and time-averaged velocity components, simultaneously with velocities ranging from zero to supersonic. LDA is very suitable for applications with reversing flow, or flows of unknown direction.

Laser anemometers are very complex instruments and offer unique advantages in comparison with others fluid flow testing equipments [1-7].

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The difficulties in a research of pipe flow, especially swirl pipe flow are, both, of experimental and theoretical nature. Swirl flows are 3-D and have complex streamlines. Turbulent swirl flow in technical system can be generated in numerous ways, by guided vanes, swirl generators with different geometries, hydraulic machines pumps, turbines, fans, *etc*. These flows are highly sensitive to the presence of small obstacles in the flow, such as, hot wire anemometry probes. The validity of any experimental investigation based on intrusive measurement systems is questionable. On the other hand, LDA has been used successfully in the number of swirl flow studies [12-16].

A high spatial and temporal resolution is necessary to obtain reliable data for the measurement of turbulent flows. The flow with the fast velocity fluctuations over a short time and a strong dependency of the position is considered. Besides time-averaged information, such as mean velocity or turbulence intensity, temporal and spatial correlation functions are very important for the flow characterization.

A complete analysis of uncertainty, for a test that includes LDA measurements in pipe flow is very extensive and complex. In this paper, the attention is given to the estimation of LDA measuring volume displacement in cylindrical pipe flow, as very important sources of uncertainty caused by geometry of pipe walls.

Characteristics of LDA

LDA is a well-established diagnostic for fluid flow measurements in different scientific, technique, and industrial areas. Some advantages of this technique are listed below.

- *Non-contact, optical measurement*. LDA probe (focused laser beams) can measure the velocity without disturbing the flow. The only necessary conditions are: a transparent medium, a suitable concentration of tracer particles (or seeding) and optical access to the flow through windows, or via a submerged optical probe.
- *No calibration – no drift*. The laser anemometer has a linearly response to fluid velocity, independent of other flow physical parameters such as temperature and pressure.
- *Well-defined directional response*. It is defined by the projection of the velocity vector on the measuring direction, determined by the LDA optical components.
- *High spatial and temporal resolution*. The optics of laser anemometer is able to define a very small measuring volume. The excellent temporal resolution is possible thanks to the small measuring volume and fast signal processing electronics, which permits high bandwidth, time-resolved measurements of fluctuating velocities.
- *Multi-component and multi-directional measurements*. The combinations of laser anemometer modules with component separation, based on colour, polarization or frequency shift, allow one, two or three-component LDA systems.

Opto acoustical frequency shift allows measurement of reversing flow velocities. LDA is very suitable for applications with reversing flow, or flows of unknown direction and it can give accurate measurements in unsteady and turbulent flows, where the velocity is fluctuating with time.

In practice, there are some difficulties in LDA applications. LDA-equipments are expensive, they need an optically transparent flow, they do not give continuous velocity signals and they require the use of light-scattering tracer particles suspended in the flow (seeding). LDA measures the particles velocity and because of that, the relationship between the particle velocity and that of the fluid must be known. The LDA measurements give statistical properties of the flow obtained during a limited amount of time. This introduces a difference between the true statistical property and estimate of it, as derived from LDA measurements.

As pointed in ref. [10], LDA provides flow velocity data with a high quality and therefore, it still remains the preferred measuring technique for the complex turbulent flows study. Nevertheless, LDA data suffer of some disadvantages and uncertainty caused by different sources: geometrical (position of measuring volume), calibration (typically 0.05 to 0.3%), data acquisition, fringe bias, velocity bias, velocity gradient bias, filtering bias, sampling uncertainty, tracer slip uncertainty, *etc*.

Overview of uncertainty sources in LDA measurement

The evaluation of LDA uncertainty depends on the studied flow and used anemometer performances. Careful consideration of each measurement involved in the test is required to identify and list all the factors that contribute to overall uncertainty. This is a very important step and requires a good understanding of measuring equipment, the principles and practice of the test. In general, no measurement or test is perfect and the imperfections give rise to error in measurement. The errors of measurement may have two components, random, statistical errors and systematic one. Uncertainties arise from random effects and imperfect correction for systematic effects.

A number of sources may contribute to fluctuations of velocity flow measurements, and their influence may be continually changing. The random errors, in swirl flow measurement, cannot be eliminated but uncertainty due to their effect may be reduced, by increasing the number of measurements and applying statistical analysis. Systematic errors arise from systematic effects and influence the results of flow measurements. These errors remain unchanged when a measurement is repeated under the same conditions and cannot be eliminated but could be reduced; *e. g.* a correction could be made for the curved pipe wall influence. If no corrections are applied, the difference between true value and measured value can be considered as an uncertainty component.

The uncertainty of the result in flow test needs to be taken into account when interpreting the flow. In some cases, the uncertainty in measurement results may be considered to be so small and not worth of formal evaluation. However, a formal estimate has to be made. Systematic assessment of the factors, influencing the results, based on understanding the principles of the method and practical experience of its application, can be a key for method validation. For uncertainty component estimations, this information may be included: previous measurement data, manufacturer's specifications, data provided in calibration certificates, experience with or general knowledge of the behaviour and properties of relevant flows and instruments.

Many sources of uncertainties are inherent to LDA practices and must be recognized to obtain the good experimental results. The main group of uncertainty sources is determined by the optical components arrangement, data acquisition and data processing. [1, 4, 10]. Such uncertainties can be categorized into two groups: an uncertainty of the actual quantity being measured and then the spatial location of where it is being measured. Clearly, the latter is determined by specific alignment techniques of the measuring volume with respect to the apparatus being studied. With the appropriate application of LDA systems, many of these uncertainties may become insignificant. However, they may also become critical to the quality of measurements. Here are some of uncertainties.

- The alignment of the test section with the LDA probe and uncertainty of real position of laser beam crossing co-ordinates determination, because specific alignment techniques of the measuring volume with respect to the apparatus have to be applied.
- The other instruments, instrument biases, facility variability and human error.

- Variations in LDA sampling when processing LDA data. There is an uncertainty, caused by the random and irregular passages of tracer particles through the measuring volume. A particular number of samples is required to insure a required level of accuracy for a turbulent velocity component. The sampling rate is strongly correlated to velocity.
- Data processing, relating with the photo multiplier section, burst spectrum analyser, setting parameters during collection and reduction of signals and so on.
- The accuracy with which the seeding particles actually follow the flow, especially the turbulent flow and the flow with supersonic velocities, because LDA cannot operate without tracer seeding particles. The particles have to be much smaller than the micro scale of turbulence, but larger enough to avoid any noticeable influence of Brownian motion and to scatter sufficient light to obtain a good signal-to-noise ratio at the photo-detector output. The optical demands also determine the optimum value of the particles concentration in the fluid: because the problem associated with the presence of particles in the flow, is the loss of coherence in the laser beams due to the diffraction of the light waves around the tracer particles. This leads to increased noise levels of the Doppler signal.
- The LDA beams alignment. The optics is calibrated by the manufacturer and errors associated with these are often considered negligible. These groups consist of uncertainties contributed by the fringe spacing and the transmitting beams angle.
- The lack of ability for LDA systems to provide accurate velocity measurements near the walls of pipes, because of boundary layer refractive index gradient. For non-dimensional radial distance within 0.15 of the wall, results in increasing the RMS of the measured frequency fluctuations appear.
- The refraction of the laser beams in the pipe wall, with different refractive index of the material in relation to the tested fluid index. Such refraction effects appear for the path length that the beam transmits through a glass window into the flow.
- The total reflections of the laser light from the pipe wall due to curvature of the pipe wall. The horizontal and vertical laser beams are most sensitive to such disturbances when measuring in the top/bottom regions of the pipe respectively. Many authors point out that the closest measurement to the wall was $r/R = 0.92$, so in this region, the near wall effects were limited.
- The turbulence fluctuations in the regions where laser beams pass.
- The medium random index of refraction fluctuations. If this exists, the ray paths are no longer straight, but will have random fluctuating curvature and detected signals will be with changeable phase and amplitude, like the flow in which heat transfer, chemical reactions or combustion are taking place, the flow with velocity fluctuations over 70 m/s, temperature fluctuation of 2 K, and mixing two or more liquids of different indices that result in index gradient of order 10^{-1}.

In estimation of the LDA measurement uncertainty, some of mentioned uncertainty sources could be neglected, depending of measurement conditions, but generally, it is neccessary to take into account all of them.

LDA measurement in swirl pipe flow

Experimental facility, test rig, where the experimental tests have been performed consists of axial fan positioned in a straight circular pipe for investigating swirl turbulent flow behind it, fig. 1(a), 1 – axial fan in profiled inlet, 2 – first LDA measuring section, 3 – second LDA measuring secton, 4 – classical probe measuring section, 5 – smoke generator probe position, 6 –

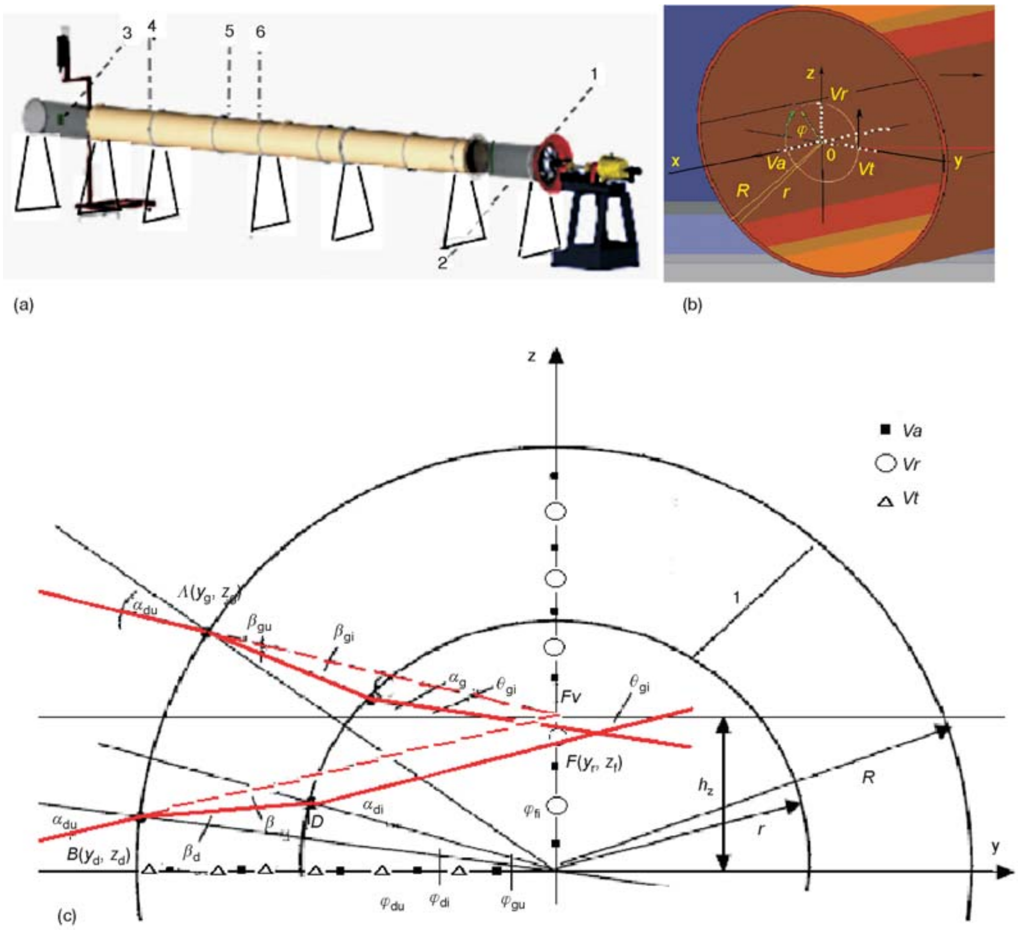


Figure 1. (a) Experimental facility, (b) Pipe co-ordinate system and (c) position of measuring volume for axial *Va*, tangential *Vt*, and radial *Vr* velocity components

fog generator probe position for LDA measurement. The pipe has inner diameter $D = 0.4$ m and the pipe wall is made of 5 mm polymethylmetacrylate (PMMA), with index of refraction $n = 1.491$. The experimental rig is described in refs. [12-14].

The commercial laser anemometer equipment, made by Dantec (one component model Flow Explorer Mini LDA) was used. It works in back-scattered mode, with BSA F30 signal processor model. The focal length of the optical lens is 300 mm, power 35 mW, and maximum velocity 27 m/s. All the three velocity components had to be measured separately. Velocity measurement uncertainty, declared by manufacturer, is lower than 0.1%. Traversing system was designed to measure velocity components in the points marked in fig. 1(c).

The LDA probe head was first mounted on the 3-D computer-controlled traverse system and proper aligned with respect to the test section, to reduce the uncertainty in the experimental LDA data, *i. e.* in LDA measuring volume positioning. During measurements, the laser head was rotated by 90°. The laser beams alignment is carried out using the upper and bottom point of the rig.

The used pipe co-ordinate system is presented in fig.1(b). The position of measuring volume positions for axial *Va*, tangential *Vt*, and radial *Vr* velocity components, when the LDA optics is horizontal is shown in fig. 1(c). The same measurements can be performed using vertically positioned LDA head.

When the axial (*Va*) velocity component is measured in centre line, the two laser beams enter the pipe wall horizontally, along its horizontal middle plane. These beams are known as the "horizontal beams". Since the beam diameter is much smaller than the pipe radius, then it can be approximately considered that the curvature of the pipe wall does not change the horizontal beams path. On the other hand, when the tangential and radial velocity components were measured, the *Vt* and *Vr* velocity components, the laser beams enter the pipe in a vertically plane, perpendicular to the pipe axis. The paths of these "vertical beams" are affected by the curvature of the pipe wall.

The LDA measuring volume parameters, using the well known formula [1-4], are presented below: focal length f is 300 mm, wavelight $\lambda = 660$ nm, beams diameter $d = 2{,}5$mm, coefficient f expansion $E = 1$, and the dimensions of the measuring volume $0.1013 \times 0.1008 \times \times 1.013$ μm. The number of interferometric fringes in it is $N_\mathrm{f} = 30$. The distance between them is $\delta_\mathrm{f} = 3.277$ mm. The laser beams intersection angle in air is $\theta = 5.71°$. The air index of refraction is $n_\mathrm{air} = 1.0$ (for normal conditions, Pa = 1013 mbar, $T = 273$ K). Measurement uncertainty of the whole system is 0.1%.

LDA application, in pipe flow measurement, needs analysis of some possible uncertainty sources related with measuring volume positioning conditions. The geometry of the laser beams (the beam waist, the intersection angles) will not change if the wall is flat and plane-parallel and the refractive index of the fluids on either side of the wall is equal. When the refractive indexes are not equal, the beam geometry inside the flow can be determined from the geometry outside the flow and the known indexes.

If the transparent wall is not flat, the beam geometry will be distorted, in a way that depends on the refractive indexes and on the incidence angle of the beams with the wall. The distortion of the beam geometry may be very large and disturb the beams crossing. This is the effect of astigmatism, which is associated with the off-axis alignment of LDA [17,18] and with the curved interface.

The simultaneous two components velocity measurements, which can be taken by using two pairs of laser beams, can not be carried out, because the four laser beams do not intersect at a unique point in the flow due to optical aberrations. For measuring the tangential velocity component the crossing angle, between the two refracted laser beams, depends on the local position of the measuring volume in the flow.

The unwanted displacement, rotation, and misalignment of the laser beams, lead to the loosing of the Doppler signal. Finding the correction factor for the position and velocity magnitudes could be based on geometrical considerations and Snell's law. The signal quality can be improved by matching the refractive index of the fluid to that one of the pipe wall. Researchers tried to overcome this problem with different methods. For example [19, 20]. They derived a correction factor to adjust the location of the measuring volume and the velocity by ray tracing method for circular pipes in air, with the small angle approximations. In paper [21], authors derived a series of equations, using the ray tracing method, to determine the position of the laser beam intersection and the spacing of the fringes, without making small angle approximations. A box with flat walls around the pipe that was filled with matching fluid was used in [22]. In paper [23] author tried to improve the optical performance by making the outside wall of the pipe planar, without using refractive index matching. He performed an analysis of ray tracing with the small-angle approximation and presented a detailed operating guideline, with respect to the shift of the measuring volume, their optical properties and the beam waist dislocation.

As it is pointed out, in ref. [24], the available signal qualities and thus signal rate, could be achieved only within a depth of about 1/3 of pipe diameter. Beyond that depth, both, the signal strengths and qualities decrease, so that the measurements can't be carried out further. The reason for such a disturbance in the respective measurement is the optical aberration in the receiving optics. It means that the flow in the central area of the circular pipe could hardly be directly measured without matching the refractive index of the flow to the pipe wall.

Exact positioning of the LDA probe is general problems in pipe flow measurements, especially near the wall. For example, the paper [22] describes the methods which find the centre location of the measuring control volume of LDA relative to test section walls. It is demonstrated that the velocity profile, measured by the LDA system, shows a sudden change in the slope of the profile when the measuring control volume enters the pipe walls. Using this information and the known dimensions of the measuring control volume, the centre of the volume can be preciously located relative to the wall.

In paper [25], authors point out that LDA measurements could be taken as close as $r/R = 0.92$ in the y-direction and $r/R = 0.88$ in the z-direction. Erroneous data tended to occur close to the pipe wall in the high and low z regions due to the high curvature of the glass pipe wall.

In order to avoid the difficulty of calculation the complex laser beams refractions on the curved interface, the method of matching the refractive index of the test fluid to that of the pipe wall has been applied often in small laboratory measurements. The benefit of the refractive index matching flow technique is that it permits the optical measurements, such as LDA and particle image velocimetry (PIV), to determine flow characteristics near surfaces around objects having complicated geometries without introducing an intrusive probe into the flow field and without distortion of the optical paths [26]. With a transparent pipe model of different refractive index than the working fluid, the light rays of optical measuring instruments can be refracted in such a manner that measurements are either impossible or require extensive, difficult calibrations. Without refractive index matching LDA beams may not cross and form the measuring volume at the desired focal length. Frequently used matching fluid in systems with glass walls, are mineral oils. This method is often employed, to eliminate the effects of transparent test section walls on the light beams directions. In the paper [27] authors give the review of recent advances in visualization and measuring techniques suited to concentrated particle suspensions. They point out that the refracting index matching (RIM) flow visualization has great potential to provide velocity profiles in a wide range of applications. It has been used for flow measurement within complex geometries, porous media, density-stratified flows and colloidal and non-colloidal concentrated particle suspensions. As exemplified in this paper, different flow measurement techniques, such as PIV, particle tracking velocimetry (PTV), laser induced fluorescence (LDV), or LIF can be used in combination with RIM.

The method, however, is not applicable to the flow in most industrial applications, where the refractive index matching is impossible or the gaseous flows are in use.

The problem of particle number density measurements with a LDA is addressed in [28]. Analytical expressions for the instrument measuring volume are given. An automatic calibration method, for determining unknown scattering parameters, which promises good accuracy in changeable optical conditions, is described. Estimates of the measurement uncertainty are derived and the method is extended to be use in 2-D flow fields.

The two limitations of LDA, inherent to the measurement principle are described in paper [29] They point out that the spatial resolution is limited by the finite size of the measuring volume given by the intersection region of the employed Gaussian beams. In this case, the fringes of the interference system are not exactly parallel, but they show spacing variation along the optical axis which is due to the curvature of the Gaussian beams. This effect leads to decreased accuracy of the velocity measurement and hence to an inaccurately high measured value of the turbulence degree (virtual turbulence). The both effects are complementary to each other, since a strongly focused beam would yield a high spatial resolution but – due to the high wave front curvature – a high virtual turbulence also.

In the ref. [30] the sources of positioning uncertainties of two component LDA system (λ = 660 nm and λ = 785 nm) are considered, in the case of swirl fluid flow in simple cylindrical pipe and in cylindrical pipe with flat external surface of the wall. Geometrical optics laws are applied to the central lines of laser beams. Measuring volumes dislocations, calibration angles and distances of the centre of measuring volumes from the centre of field of view of photo-detector are determined for axial and radial velocity components. The expressions for laser beams intersection points derived in this paper were applied to test rig with swirl flow. That revealed several advantages of use of a simple cylindrical pipe over nowadays favoured use of cylindrical tube with flat external surface. Those results suggest that current avoiding of LDA measurements with simple cylindrical tubes should be reconsidered.

Having in mind all the above mentioned analyses and the geometry of used swirl pipe, the inner radius r = 200 mm and wall thickness d = 5 mm, measurements were planned without matching the refractive index of the flow to the pipe wall, but with corrections of measuring volume positions for all three components.

Measurement of the velocity vectors in the swirl pipe flow (fig.1a) was performed in three steps:

(1) measuring the axial velocity component is performed by horizontal laser beams, which lie in a plane parallel to the pipe axis, changing the distance along the y-axis, z = 0 mm, x = 0 mm, fig. 1(c)

(2) measurement of the tangential velocity component is made by vertical laser beams, which lie in a plane normal to the pipe axis (rotation of laser head by 90º), changing the distance along the y-axis, z = 0 mm, x = 0 mm, fig. 1(c), and

(3) measurement of the radial velocity component is performed by vertical laser beams, changing the distance along the z-axis, x = 0 mm, y = 0 mm.

LDA measurements were performed along the vertical and horizontal diameter in pipe section at the distance L = 3-D, (the distance between measuring points was 10 mm. The first points, in boundary layer, were omitted. The laser head was placed in a horizontal position, during all measurements, fig. 1(c).

This paper presents the results of the LDA measuring volume displacement estimations. The beam intersection angle and the measuring volume orientation of LDA must be known in order to correct systematic measurement errors, too. These results were used for avoiding the position uncertainty and for making correction of the measured values of the velocity components spatial distribution.

Results and discussion

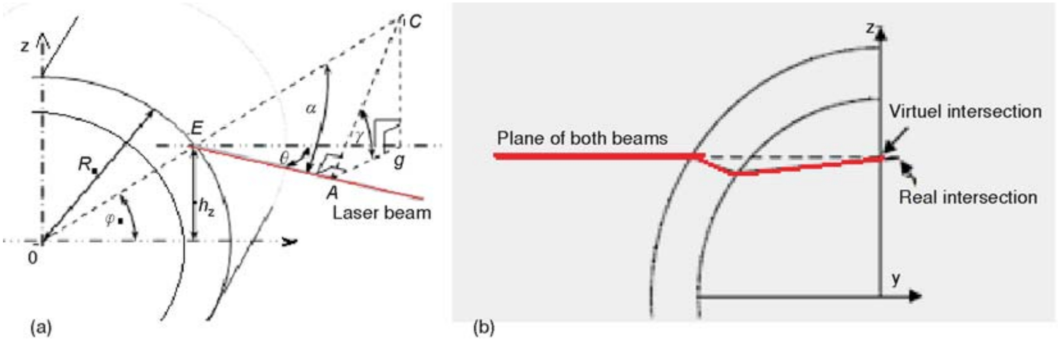
The analysis of the swirl flow parameters will be omitted in this article, because the main aim is to estimate LDA measuring volume displacements for test rig and for the pipes with dimensions often used in industrial devices, where LDA with focus lens of f = 300 mm, can be used. (radius r = 100, 200, and 290 m, as pipe wall thickness d_w = 2, 5, 10 mm). The engineering approximation, based of Snell's law, that gives the good results for $z/R < 0.6$ is used for estimation of measuring volume displacements. This approximation is acceptable because the swirl flow is situated in the region between $-0.6 < z/R < 0.6$.

As it is pointed out, in the section *Characteristics of LDA*, the test rig, is consisted of a straight circular pipe with inner diameter $D = 0.4$ m and the pipe wall made of 5 mm PMMA, with index of refraction $n = 1.49$. Horizontal projection of wall thickness changes, when the distance between pipe center line and laser beams entrance point is changed. The pipe is designed and constructed in a way to minimize the wall influence on the laser beams intersection position. The entering laser beam is bent both upon entering the surface and upon exiting one. The laws of reflection and refraction (Snell's law) have been used to examine the change of light through the pipe wall.

Axial velocity components Va measurement

The exact geometrical presentation of the spatial relation between incidence lasers beams and normal incidence plane is shown in fig. 2(a). Detailed description is given in ref. [30]. Application of the exact geometrical relations for spatial location of the laser beams gives the displacement of measuring volume in y- and z-directions.

The approximation based on preposition that laser beam diameter is enough small than diameter of pipe ($d_l < D$) make possible to consider that the radius of cylindrical pipe is not "visible" for laser beam. In that case the axial velocity measurements were made with the minimum optical aberration, because for this measurement the best available optical condition was obtained. The two laser beams approximately remain in the plane parallel to the pipe axis. The crossing angle between the two refracted laser beams remain constant, too, even if the laser beams do not cross the pipe axis. Disposition of lasers transmitting lens (L), pipe wall (W), and pipe centre line (PC) for measurements of axial velocity component in the pipe axis ($z = 0$ mm, $x = 0$ mm) are presented in fig. 2(b) and 2(c).



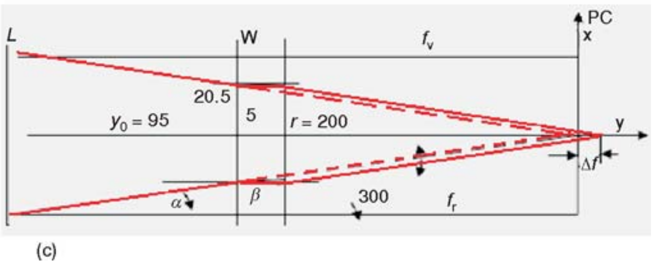


Figure 2. Schematic presentations of laser beams in axial velocity measurements, (a) spatial position of laser beams and radius of pipe, (b) laser beams in vertical plane, (c) laser beams in horizontal plane

The LDA measuring volume of *Va*, (along y-axis) has the constant displacement along that axis, because the thickness of the wall, where the laser beams pass, remained the same for all positions, fig. 3(a). A light ray traversing a wall of transparent material with parallel (entrance and exit) surfaces is displaced laterally with no change in direction. This displacement caused displacement of LDA beam crossing, *i. e*. displacement of measuring volume. The shift of the measuring volume in the flow is proportional to the shift of the LDA optics. Figure 3(b) show the results *Va/Vam* measured in section *L* = 4 D, for *n* = 1000 min.$^{-1}$. *Va/Vam* in real and virtual position around pipe centre is presented in fig 3(c). It is obvious that the source of uncertainty in the LDA, related to displacement measuring volume, can be eliminated by correction of positioning, presented in the computer controlled traversing mechanism.

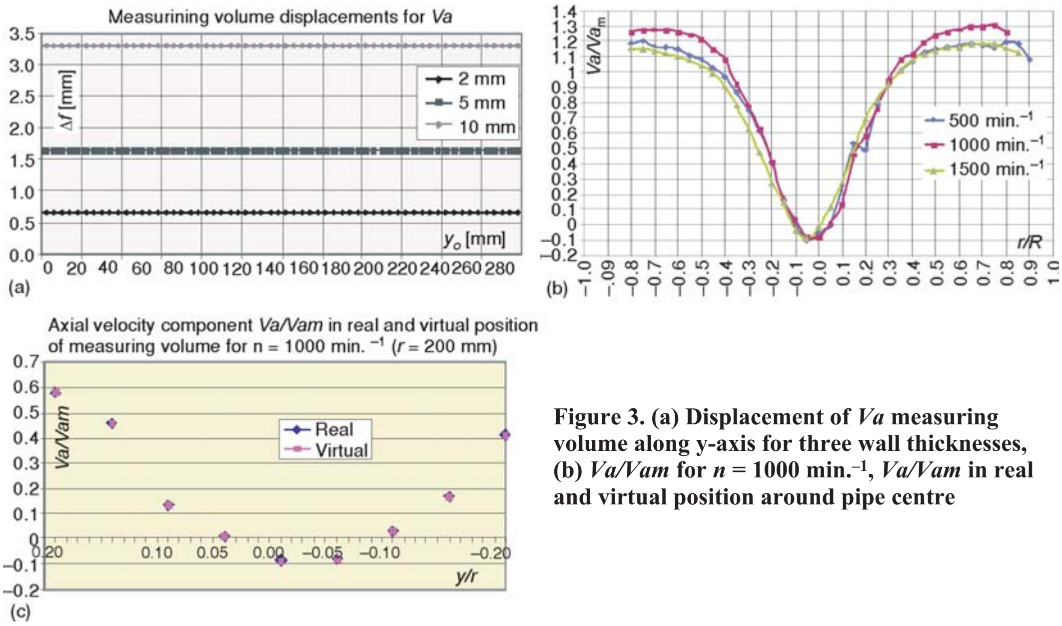


Figure 3. (a) Displacement of *Va* measuring volume along y-axis for three wall thicknesses, (b) *Va/Vam* for *n* = 1000 min.$^{-1}$, *Va/Vam* in real and virtual position around pipe centre

If the *Va* components have to be measured along z-axis, then the measuring conditions are different comparing to the previously mentioned. Figures 4(a), 4(c), and 4(e) show that laser geometrical path lengths *l*" trough pipe wall depends on z-co-ordinate of LDA optical axis (*hz*) and on pipe radius *r*. In that case, the displacement of measuring volume is not constant, as in previously measuring set, fig. 3(a). Figures 4(b), 4(d), and 4(f) show the displacements of laser beams, after passing through pipe wall, versus z-co-ordinate for three wall thicknesses and three different radius vectors. The consequences of these displacements are the LDA measuring volume displacements along y-axis in pipe flow. The displacement in LDA axis for *r* = 200 mm and *l* = 5 mm, in position *hz* = 0 mm is Δ*f* = 1.65 mm, but on *hz* = 195 mm (*hz/r* = 0.975) is Δ*f* = 5.355 mm along y-axis (tab. 1). The shift of the measuring volume in the flow is no longer proportional to the shift of the LDA unit fig. 4(d). Figures 4 show that the displacements of LDA measuring volume, for axial velocity component, are a function of pipe radius *r* and pipe wall thickness *l* (tab.1).

In practice, measuring the axial velocity component is commonly performed along the y axis (not along the z-axis) because the astigmatism is minimal in horizontal pipe axis plane.

Table 1. Displacements of measuring volumes of velocity components in LDA applications

Pipe radius		$r = 100\cdot10^{-3}$ [m]			$r = 200\cdot10^{-3}$ [m]			$r = 290\cdot10^{-3}$ [m]		
Pipe wall thickness	$l\cdot10^{-3}$ [m]	2	5	10	2	5	10	2	5	10
Displacement of axial velocity Va, measuring volume $\cdot10^{-3}$ [m] along z-axis	min	0.6609	1.6523	3.3047	0.6609	1.6523	3.3047	0.6609	1.6523	3.3047
	max	1.5518	3.8793	7.759	2.326	5.3558	8.9034	3.0366	6.4004	10.583
Displacement of tangential velocity Vt measuring volume $\cdot10^{-3}$ [m] along y-axis	min	0	0	0	0	0	0	0	0	0
	max	0.6571	1.595	3.3217	0.6638	1.6347	3.1899	0.68437	1.66715	3.23876
Displacement of radial velocity Vr measuring volume $\cdot10^{-3}$ [m] on ($Z/r = 0.9$)	Δz	0.8799	0.9162	1.0232	1.9447	1.9611	2.0179	2.8441	2.8557	2.8843
	Δy	–0.2887	–1.4794	–2.8641	–0.0309	–3.1156	–7.0329	1.0012	–2.6375	–7.5545
	Bisector angle [°]	–1.0243	–2.379	–4.2671	–0.9482	–2.2267	–4.0677	–0.7086	–1.6884	–3.1453

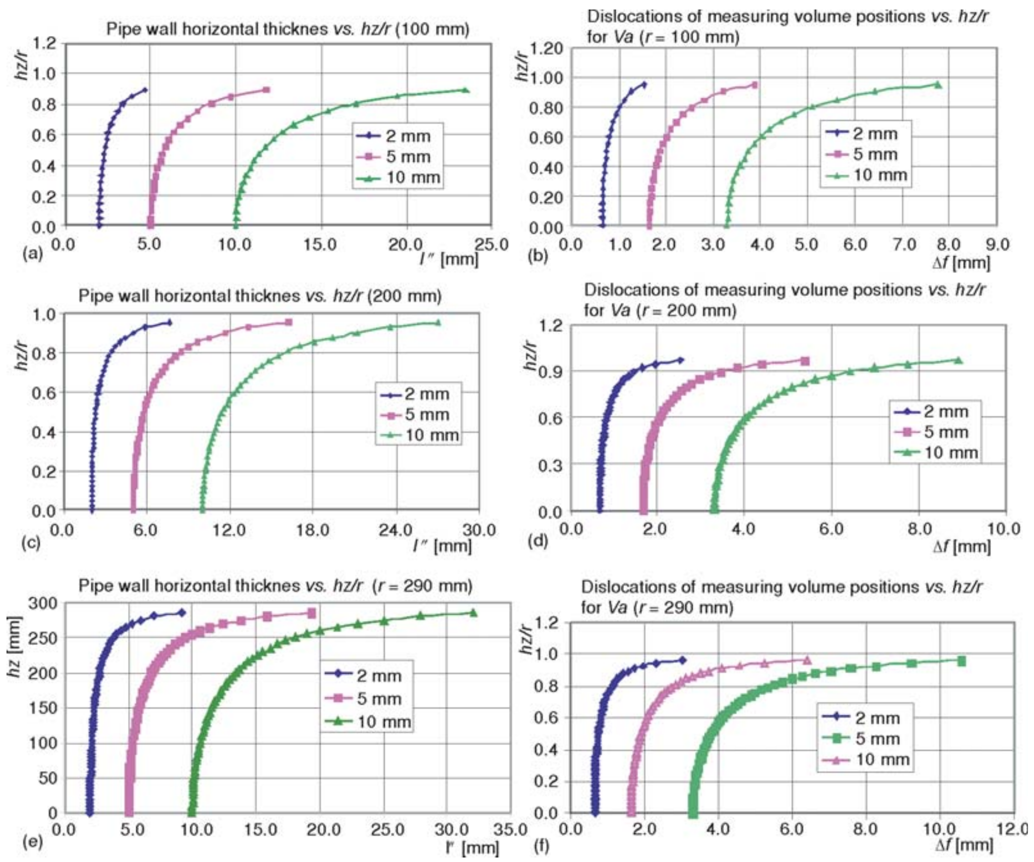


Figure 4. (a, c, e) Horizontal thickness of pipe wall, (b, d, f) Measuring volume displacements along y-axis *vs.* z-coordinate for axial velocity component for different pipe radius

Tangential velocity components Vt measurement

Measurement of the velocity vectors in the swirl pipe flow, performed in the second step, included determinations of tangential velocity components *Vt* along y-axis, for $z = 0$ mm, $x = 0$ mm, and $y = 0$ to $y = -199$ mm. For that purpose, laser head was rotated by 90° and the LDA optical plane is in *yz* plane, perpendicular to the pipe axis *x*. Disposition of lasers transmitting lens (L), pipe wall (W) and pipe center line (PC) for measurements of tangential velocity component along the pipe axis y ($z = 0$ mm, $x = 0$ mm) are presented in fig. 2(c) (instead x-axis, now is z-axis). Measurements of *Vt* by shifting measuring volume along y-axis are correct just outside the boundary layer, *i. e.* $y_o = -195.0$ mm.

The differences between virtual and real measuring volume positions of tangential velocity components are presented in figs. 5(a), 5(c), and 5(e), as a function of y-co-ordinate. Simultaneous measurements of axial and tangential velocity components (*Va* and *Vt*) by 2-D LDA are practically impossible, fig. 5(b), 5(d), and 5(f). Instead, common expected focus (*Fv*), the two real focuses appear for *Va* and *Vt* components. Minimal difference between these focuses is obtained when the measurement is performed in point near the pipe wall, fig. 5(b), 5(d), and 5(f). For example, if $r = 200$ mm, $d = 5$ mm, the difference is $\Delta(Fa - Ft) = 0.09$ mm in the position

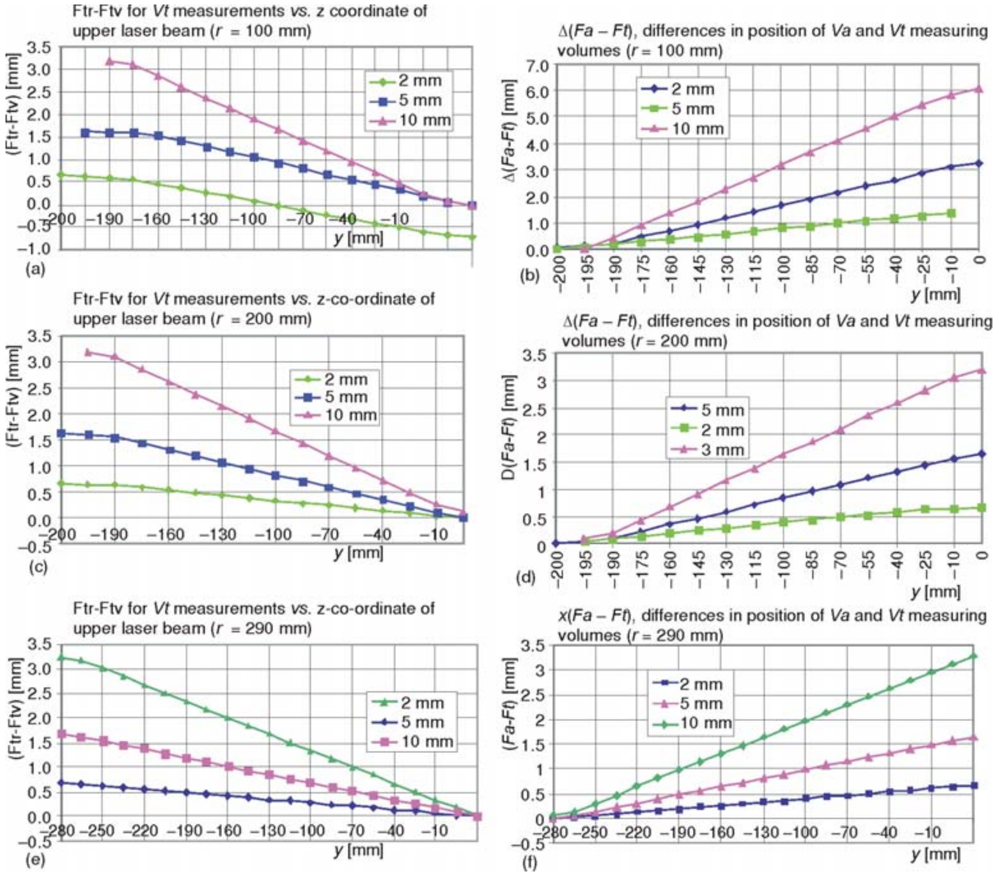


Figure 5. Difference between measuring volume positions for (a), (c), (e) Vtv and Vtr components and (b), (d), (f) *Va* and *Vt* components in 2-D LDA for *r* = 100, 200, and 290 mm *vs. y*

$y = -199$ mm. When the velocity measurements are performed in the point with $y = -145$ mm, than the value $\Delta(Fa - Ft) = 0.5$ mm. After that position, towards the pipe axis, in all other measuring points the distances between volume centres are greater than the differences of length and width of measuring volumes and there aren't no the overlap of the volumes, and there is impossible to measure simultaneous those two components.

Radial velocity components Vr measurement

Measurement of the radial velocity component is performed along the z-axis, at points indicated in fig. 1(b). Laser beams lie in a vertical plane. These are the most complex measurements, since the change of z, changes the incidence angle of the beams entering the pipe wall, and the optical path length through the pipe wall. The optical aberration related to the individual laser beam leads to a deformation of the beam waist, causing a distortion of LDA measuring volume. Determination of beam intersection displacement in Vr measurement shows that there are changes in co-ordinates, y and z. With the shift of the LDA transmitting optics parallel to the y axis, the measuring volume would travel along a 2-D path, figs. 6(a), 6(f).

Besides the changes of the measuring volume location, the change of interferometric fringes orientation appears. The orientation of the measuring volume, *i. e.* the inclination of the

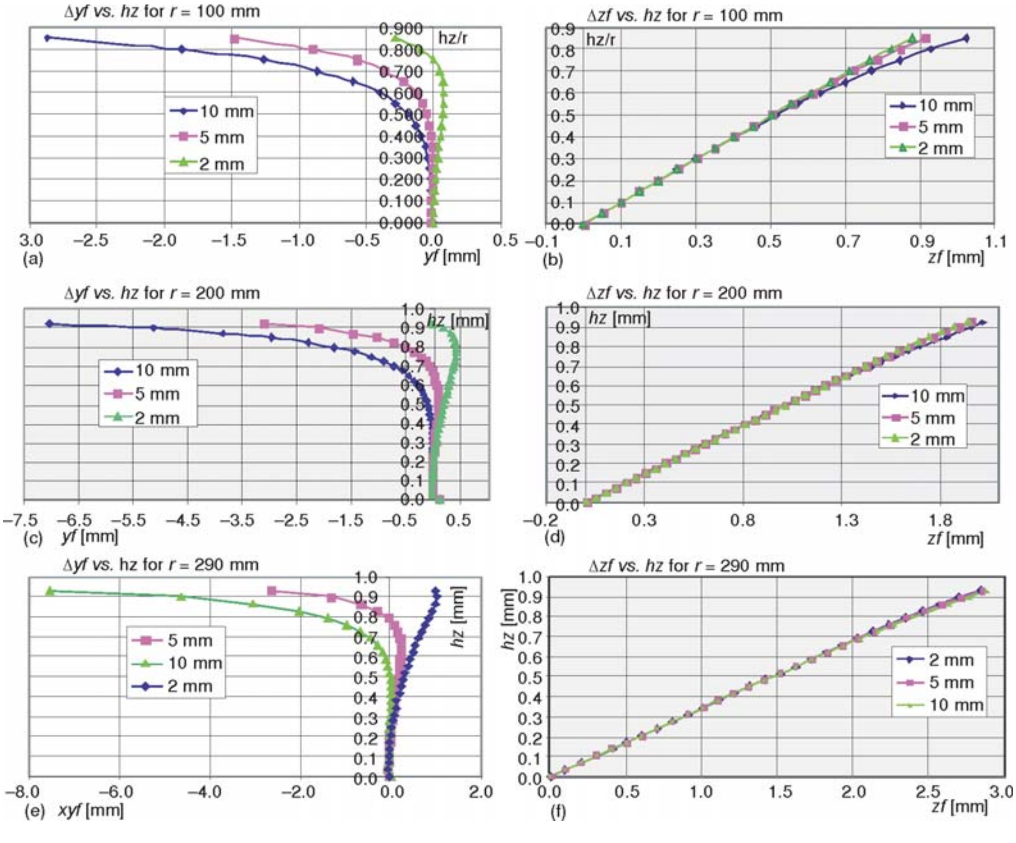


Figure 6. The results of measuring volume displacement for radial velocity component Vr along y and z axis, (a), (b) for $r = 0.1$ m, (c), (d) for $r = 0.2$ m, and (e), (f) for $r = 0.29$ m

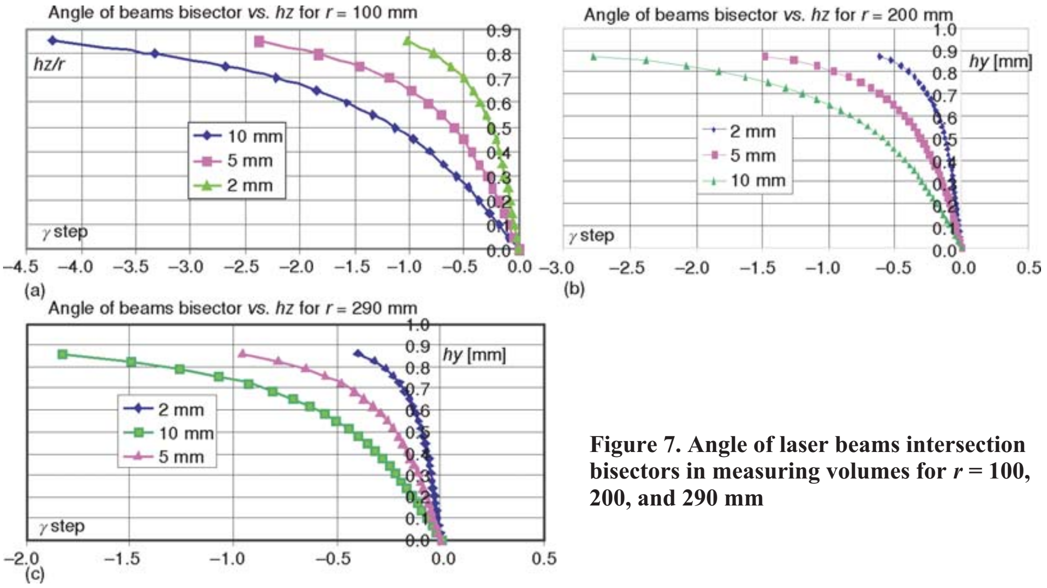


Figure 7. Angle of laser beams intersection bisectors in measuring volumes for *r* = 100, 200, and 290 mm

bisector of two refracted laser beams determines the orientation of measured velocity component which generally differs from the radial velocity. The estimated values of bisector angle for three different pipe radius and three pipe wall thickness are shown in fig. 7(a) to 7(c).

The change of the intersection angle increases or decreases the distance of interference fringes, causes a waist mismatching in the measuring volume and modifies therefore the Doppler frequency by a given constant flow velocity.

Analysis of the results shows that the displacements of measuring volume, for radial velocity components, along y-axis for $r = 200$ mm, and $l = 5$ mm are between 0 and –3.12 mm, but along z are 0 to 2 mm (tab. 1 and figs. 6(c) and 6(d). If the measurements are performed in a defined area ratio $z/r < 0.6$ than it is not necessary to introduce corrections of the results of the measuring volumes displacements because the estimation and calculation show that the corrections are less than: $\Delta yf < 0.4$ mm $\Delta zf < 1.3$ mm, $\Delta\gamma < -1°$.

The bisector angle, with respect to the horizontal line, is dependent on pipe wall thickness and radius. With increasing the thickness, at a constant radius, the bisector angle increases, while, for a constant pipe wall thickness, the angle decreases with increasing the radius (fig. 7, tab.1).

Conclusions

In the introduction of this paper, the short review of measuring uncertainty in application of LDA is presented. The special attention is given to one source of measurement uncertainties, displacement and spatial position of LDA measuring volumes in cylindrical pipe flow due to optical aberrations, caused by the pipe wall curvature and thickness. The required corrections, for measurements of axial, tangential and radial velocity components such: shift of measuring volume and its orientation, are analysed and determined for three different pipe geometries, especially for used test rig. Analysis of fringe spacing and distortion will be subject of further research.

The basic hypothesis, that in the central part of the pipe ($r/R < 0{,}6$) exists a small, or negligible pipe wall influence on measuring position, is proved to be true. Analysis of the graphs confirms this hypothesis. The displacements in the central part of the pipe with radius $r = 200$ mm, are: for axial velocity component $Va\ \Delta yf = 1.65$ mm, for tangential $Vt\ \Delta yf = 0.68$ mm, and for radial one $\Delta yf = 0.1$ mm, $\Delta zf = 1.2$ mm. The calculations show that the displacement of axial velocity component measuring volumes is constant along the pipe with r = 200 mm is $\Delta f = 1.65$ mm, along y-axis, or up to 5.35 mm along z-axis.

The shift of the tangential component measuring volume is for test rig between 0 mm to 1.64 mm. Determination of beam intersection displacement in radial velocity component measurement shows that there are changes in two coordinates, y and z. With the shift of the LDA transmitting optics parallel to the y axis, the measuring volume would travel along a 2-D path. The max values of these displacements for test rig are $\Delta z = 2.3$ mm, $\Delta y = -3.0738$ mm.

Simultaneous measurements of axial and tangential velocity components (Va and Vt) by 2-D LDA are practically impossible, because minimal difference between the measuring volumes positioning is 0.09 mm, maximum 1.64 mm. For the purpose of simultaneous measurements, it is necessary to use the additional methods of compensation or correction of the measuring volume positioning.

The results, presented in this paper, confirm that of the simple cylindrical pipes have some advantages, when LDA measurements are performed in the central part of the pipe, over nowadays favoured use of cylindrical ones with flat external surfaces. Those results suggest that if the pipes are designed and constructed in a way to minimize the wall influence on the laser beams intersection position, than it is possible to avoid the complicated matched index of refraction flow facilities.

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