THERMAL STRESS ANALYSIS OF A FIBER-EPOXY COMPOSITE MATERIAL

by

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Paper presents stress calculation in the stationary temperature domain for a hybrid structure with standard epoxy-carbon fiber composite and metallic part, showing different thermal behaviour. The thermal stress analysis is based on the finite element method. A sample problem involving such a composite plate and metallic part of structure or mould at two different temperature levels, corresponding to curing process (co-curing) and operating temperatures, respectively, is calculated and discussed. The specific properties of composite are emphasized particularly for thermal coefficients, especially if the shear coefficient is different than zero, causing completely different behaviour compared to isotropic materials.

Key words: *hybrid structure, epoxy-carbon fiber composite, mould, thermal coefficients, thermal stress analysis, reserve of elasticity*

Introduction

The curing of components, as the most important phase during manufacturing of epoxy prepregs (for example, based on the epoxy resin as matrix of prepreg), includes complex behaviour of the hybrid structure, consisting of typically laminate composite and metallic parts or mould. The curing parameters (pressure, temperature, time), as well as mechanical and thermal properties of the constituents cause complex behaviour of coupled pair, laminate composite/metallic tool or mould. The difference between the curing and operating temperature causes thermal residual stresses in a laminate composite. In order to investigate possible thermal effects the thermal stress analysis of epoxy-carbon fiber composite (CFC) and mould should be performed, using the finite element method (FEM) as the most suitable for such an analysis.

Basic theoretical relations

Analysis of mechanical stresses of reinforced laminate cured at temperatures higher than the operating temperature, should be complemented by the analysis of thermal stresses,

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in the scope of design of fiber composites. Toward this aim a relation between stress and strain for each individual lamella can be expressed as matrix equation:

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{21} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_1 & -\alpha_1 & \Delta T\\ \varepsilon_2 & -\alpha_2 & \Delta T\\ & \gamma_{12} & \end{bmatrix} \tag{1}$$

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}=\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x & -\alpha_x & \Delta T\\ \varepsilon_y & -\alpha_y & \Delta T\\ \gamma_{xy} & -\alpha_{xy} & \Delta T\end{bmatrix} \tag{2}$$

where σ_i, and τ_{ij} are stresses in a lamella, Q_{ij} – members of the orthotropic lamella stiffness matrix, $\bar{Q}_{ij}$ – members of the generalized stiffness matrix of orthotropic lamella, which are obtained according to the relation given in [1]. ε_i, ε_{ij} – strains due to mechanical loading, α_i, α_{ij} – coefficients of linear thermal expansion, and ΔT is the difference between the curing and operating temperature.

Since the eq. (1) does not include coefficients of thermal shear α_{12}, the thermal expansion coefficients do not affect shear strain in an orthotropic lamella. Stresses in a lamella whose geometric axes do not coincide with the main material directions, eq. (2), implies the corresponding transformation, as defined in [1], and hence there exists α_{xy} (coefficient of thermal shear). Functional dependence of the coefficients α_x, α_y, α_{xy}, and the rotation angle θ of the main directions of orthotropic material axes 1 and 2, are given by the relations [2]:

$$\begin{aligned}\alpha_x &= \alpha_1\cos^2\theta + \alpha_2\sin^2\theta\\ \alpha_y &= \alpha_1\sin^2\theta + \alpha_2\cos^2\theta\\ \alpha_{xy} &= 2(\alpha_1-\alpha_2)\sin\theta\cos\theta\end{aligned} \tag{3}$$

A graphical representation of the coefficients α_x, α_y, and α_{xy} for specific values of coefficients α_1 and α_2 is shown in fig. 1.

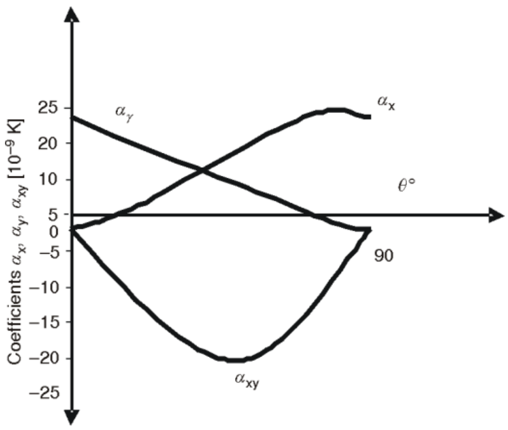

Figure 1. Changes of α_x, α_y, and α_{xy} depending on the θ

From the active and reactive forces equilibrium and the linear variation of strain across the thickness, following relation can be derived, [1, 3]:

$$[\mathrm{N}]=[\mathrm{A}][\varepsilon°]+[\mathrm{B}][\mathrm{k}]-[\mathrm{N}^\mathrm{T}] \tag{4}$$

$$[\mathrm{M}]=[\mathrm{B}][\varepsilon°]+[\mathrm{D}][\mathrm{k}]-[\mathrm{M}^\mathrm{T}] \tag{5}$$

where [A], [B], and [D] – the corresponding stiffness matrix of tension, coupling, and bending, [ε°] and [k] – the corresponding strain vectors, [N] and [M] – vectors of resultant of forces and momentum, [N^T] and [M^T], vectors of thermal forces and momentum given in eqs. (6) and (7), according to [4].

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \int_h \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix} \Delta T \mathrm{d}z \tag{6}$$

$$\begin{bmatrix} M_x^T \\ M_y^T \\ M_{xy}^T \end{bmatrix} = \int_h \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix} \Delta T \mathrm{d}z \tag{7}$$

where h is the total thickness of laminate. The relations (1)-(7), specific for fiber composites, will be incorporated in the corresponding finite element matrix, eqs. (8)-(11), [3]:

$$[\mathrm{F}] = [\mathrm{K}][\mathrm{u}] + [\mathrm{N}] \tag{8}$$

$$[K]\int_V [B]^T [\bar{Q}][B]\mathrm{d}V \tag{9}$$

$$[N] = \int_V [B]^T [\bar{Q}][\alpha]T\mathrm{d}V \tag{10}$$

where [F] is the external loads (forces and momentum) vector, [K] – the stiffness matrix, [u] – the displacement vector, [B] – the interpolation matrix, and $T(x,y,z)$ – the stationary distribution of temperature.

Part of the system based on the isotropic materials (*e. g.* mould) is treated as a special case of the orthotropic material, [3]. Analysis of thermal stress without the influence of external mechanical forces [F] will be conducted using eqs. (2), (11), and (12), the latter two defining displacements strains, respectively, [3]:

$$[\mathrm{u}] = -[\mathrm{K}]^{-1}[\mathrm{N}] \tag{11}$$

$$[\varepsilon] = [\mathrm{B}][\delta] \tag{12}$$

where $[\delta]$ is the displacement vector of the structure derived from [u], $[\varepsilon]$ – the strain vector, and [K] – the stiffness matrix.

Example

For the composite structure shown in fig. 2, stresses due to the different curing and operating temperatures are calculated. As the curing has been done at uniform temperature distribution, every region of structure will be exposed to the same thermal effects, so the laminate configuration and boundary conditions determine the strain distribution in the laminate and mould. Simplified model of the structure is shown in fig. 3. Basic mechanical and thermal properties of materials of mould and laminates are given in tab. 1.

Table 1. Some properties of the structure of mould and laminate

Property	Units	Material	
		Al-alloy	CFC-lamellae
Modulus of elasticity, E_1	Nmm^{-2}	$73\cdot10^3$	$98\cdot10^3$
Modulus of elasticity, E_2	Nmm^{-2}	$73\cdot10^3$	$15.7\cdot10^3$
Poisson coefficient, υ_{12}	–	0.3	0.3
Shear modulus, G_{12}	Nmm^{-2}	$28.3\cdot10^3$	$4.5\cdot10^3$
Coefficient of thermal expansion, α_1	K^{-1}	$23.7\cdot10^{-6}$	$215\cdot10^{-11}$
Coefficient of thermal expansion, α_2	K^{-1}	$23.7\cdot10^{-6}$	$215\cdot10^{-11}$

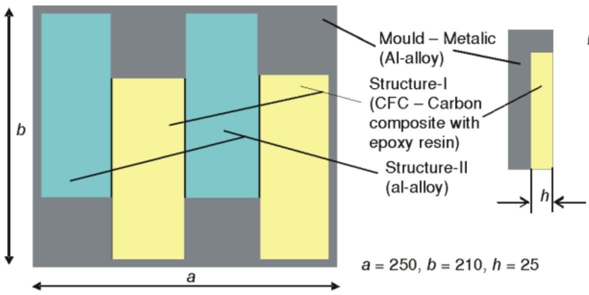

Figure 2. Representative model of the structure of the laminate plate and mould

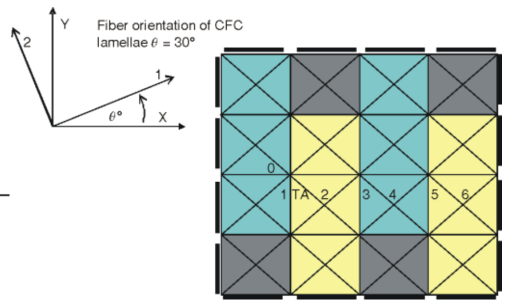

Figure 3. Discretized model of representative structure

Using the relations from the *Basic theoretical relations* section it is possible to calculate the displacements, stresses, and strains. The results (obtained on the basis of the own software solutions) are given in tabs. 2 and 3 (displacement of characteristic nodes for two different curing temperature) and as a diagram (fig. 4).

Table 2. Displacement of the nodes of structure for curing temperature T = 140 °C

Node number		1	2	3	4	5	6
Displacement [10^{-5} m]	x	–78	–15	47	141	271	310
	y	280	234	192	124	43	21

Table 3. Displacement of the nodes of structure for curing temperature T = 180 °C

Node number		1	2	3	4	5	6
Displacement [10^{-5} m]	x	–112	–21	61	183	352	419
	y	372	311	255	165	57	30

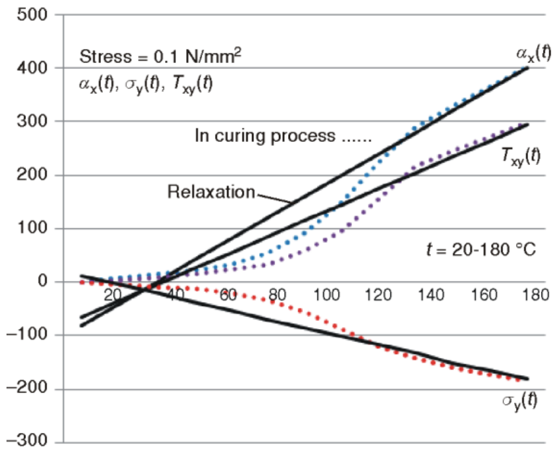

Figure 4. Functional dependence of stress in triangle TA and curing temperature

The results show that the level of thermal stress of the composite laminate is relatively low compared to the strength of lamella (X = 1100 N/mm^2, Y = 100 N/mm^2, S = 75 N/mm^2, [2]). However, when it comes to slanted composite laminates, non-symmetric and unbalanced, due to the presence of coupling stiffness, [1], or the low critical stress of thin laminates, thermal stresses cause the deviation of the global geometry of the structure and it needs to have the particularly attention in design process, [5, 6].

Conclusions

The analysis of thermal effects and residual stresses in a laminated composite due to curing process of epoxy matrix is given, based on the concept of classical laminate theory of thin plates, specific properties of fiber reinforced epoxy composites, and the corresponding theoretical relations.

In accordance with the diagram, shown in fig. 4, it is evident that residual stresses occur after curing of the structure and its cooling to the ambient temperature.

According to the presented method, a thermal analysis (displacements, strain and stress) can be carried out of laminated structures of complex geometry such as wings with negative sweep of the airplane or heat shields. Thus, it is necessary to perform analysis of thermal stress of mould and composite systems (in the cycle of forming the structure) as the first step in the design process.

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