

Tooth Flanks Scoring Resistance of Noninvolute Teeth Profiles In Plane Toothed Cylindrical Gears

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The paper presents results of theoretical and experimental research into the scoring resistance of plane convex-concave gears. There are generalized basic relations of the integral temperature criterion for all kinds of path of contact in defined plane cylindrical gears given. Theoretically obtained relations are illustrated by the results of the scoring tests according to the FZG method (DIN 51354).

Keywords: non-involute teeth profile, convex-concave gearing, integral temperature criterion.

1. INTRODUCTION

The main requirements in the process of designing toothed gears are low weight and small size of the gearbox assembly. Such requirements can cause an increased temperature load of the gear box and consequent temperature scoring of delicate gear teeth surfaces.

The most efficient solution of scoring problems is additional oil cooling or using some special lubricant additives. However, there are some cases where such solutions can not be applied. For example, there is no possibility of using an EP lubricant when there is the same oil in gear boxes using the hydraulic torque converter for the gear box and the converter. In the case of large gearboxes such as those applied in marine engineering, motor oil is often used with insufficient scoring parameters and external oil cooling. This solution is in general uneconomical, which means higher maintaining price. Because of this and due to increased complication and lower reliability this solution is unacceptable. In these peculiar cases there is the possibility of using a tooth of special geometric shape which is resistant to scoring. For involute profiles the scoring resistance can be influenced by changes of the basic geometric parameters only in a limited range. It is therefore necessary to use special, non-involute profiles in all cases where extremely high friction forces and scoring exertions are expected.

Intensive research into scoring problems was carried out at the end of the seventies and at the beginning of the eighties of the last century. The result was the introduction of the Integral Temperature Criterion by Winter and Michaelis [1] for the evaluation of scoring of cylindrical and bevel gears. This criterion has also been successfully used for hypoid and spiral gears [2]. Extensive experimental research of these problems was also made by Schauerhammer [3]. Systematic analysis of scoring resistance from the geometrical teeth-profile point of view has not been carried out yet. The article

concludes with the results of Hlebanja in relation to convex-concave gears [4].

2. CONVEX-CONCAVE GEARING AND ITS SCORING PROPERTIES

The scoring load capacity, from the point of view of the geometric features determined by the Integral Temperature Criterion of scoring quoted by ISO and DIN is principally a function of the relative radius of curvature of the teeth profiles and the difference between the square roots of the tangential velocities at particular points along the contact path. This can be expressed by the relation valid for the flash temperature according to Blok:

$$\begin{aligned} \mathcal{G}_{Bl} &= 0,62\mu w_{bt}^{0,75} \left(\frac{E_r}{\rho_r} \right)^{0,25} \frac{|v_{t1} - v_{t2}|}{\sqrt{B_{M1}v_{t1}} + \sqrt{B_{M2}v_{t2}}} = \\ &= 0,62\mu w_{bt}^{0,75} \left(\frac{1}{\rho_r} \right)^{0,25} X_M \left(\sqrt{v_{t1x}} - \sqrt{v_{t2x}} \right). \quad (1) \end{aligned}$$

For the examined case (1) can be expressed in the suitable form

$$\mathcal{G}_{Bl} = k \frac{|v_{t1} - v_{t2}|}{(\rho_r)^{0,25}}. \quad (2)$$

The coefficient k expresses the impact of the non-geometric parameters of the gear drive, μ is the coefficient of friction for the concrete mesh point. To simplify the relation, we can replace the local value of the coefficient of friction μ by its mean value μ_m which is dependent on the geometric features of the gears.

In terms of the relation for the calculation of \mathcal{G}_{Bl} it is obvious that gears with convex-concave gear profiles have good resistance against scoring damage. Problems concerning the design of plane convex-concave gears have been elaborated by Hlebanja [4] and his work has proved this type of gear has its corresponding contact path in the shape of an S sign (Fig. 1).

During the design process of the gearing with convex-concave flank, additional problem is achieving a correct mating gear toothed system between two profiles designed according to the given contact path. It

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is practical to define the given path of contact in the shape of two circular arcs. The centers of these arcs (S_1, S_2) are not located on the line connecting the turning centers (O_1, O_2) of the two teeth gears. In this case it is accordingly [5] possible to express the parameter equation of two correctly mating gear tooth-profiles in the way shown by (3), (4) and (5).

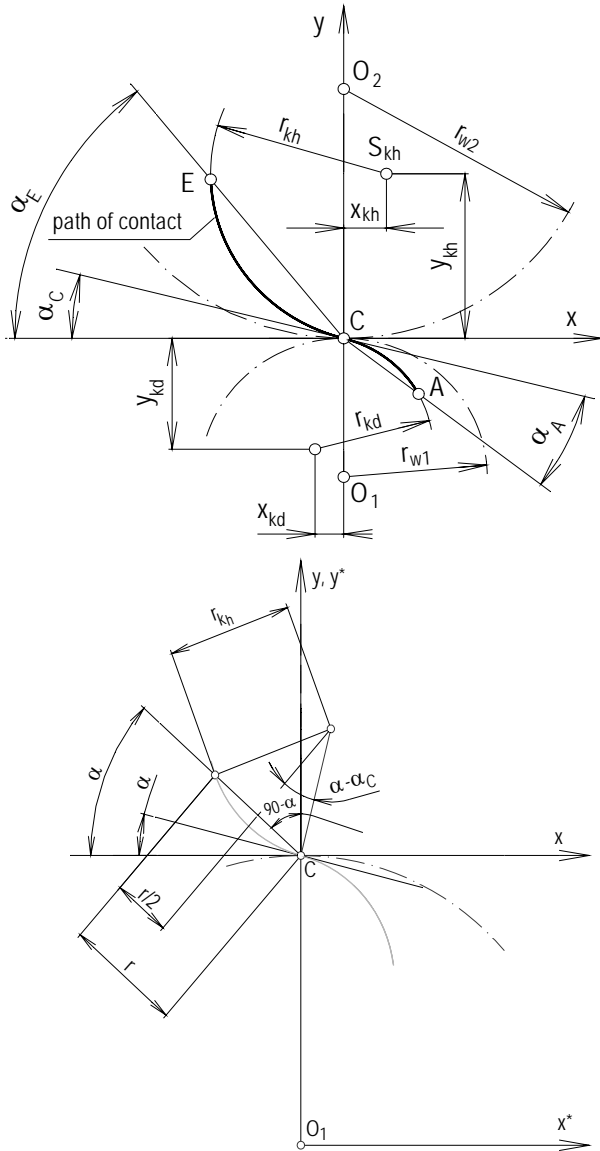


Figure 1. Definition of convex-concave cylindrical gear flanks

$$x^* = \mp 2r_{kh,d} \sin(\alpha - \alpha_c) \cdot \cos[\alpha + \varphi_r(\alpha)] + r_1 \sin \varphi_r(\alpha) \quad (3)$$

$$y^* = \pm 2r_{kh,d} \sin(\alpha - \alpha_c) \cdot \sin[\alpha + \varphi_r(\alpha)] + r_1 \cos \varphi_r(\alpha) \quad (4)$$

where:

$$\varphi_r = \pm \frac{2r_{kh,d}}{r_1} \left[(\alpha - \alpha_c) \cos \alpha_c + \sin \alpha_c \lg \frac{\cos \alpha_c}{\cos \alpha} \right] \quad (5)$$

The pressure angle α of an arbitrary point on the path of contact is used as a parameter. Design and production of this gearing cannot be achieved by common methods. Figure 2 displays the profile of the

convex-concave tooth, while Figure 3 shows 3D model of the wheel which was produced by a special CAD/CAM system described in [6]. Right side of Figure 3 shows manufactured wheel and pinion with convex-concave profile.

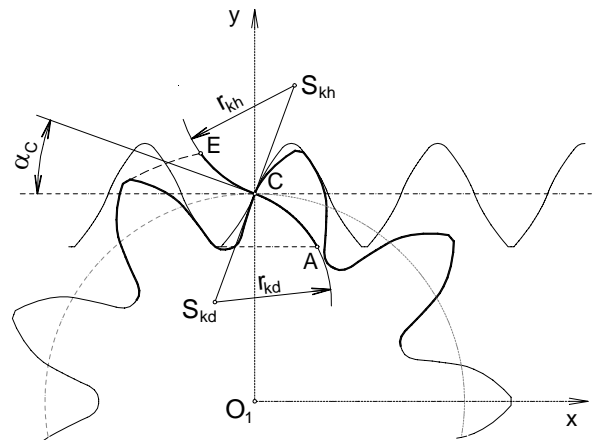


Figure 2. Shape of the convex-concave gear-tooth profile

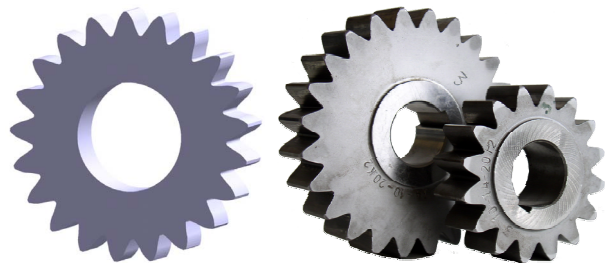


Figure 3. 3D model and manufactured cylindrical gears with the convex-concave teeth profiles

The graph of the relative radius of curvature of teeth profiles and the difference between the square roots of the tangential velocities are shown in Figure 4.

In order to make a comparison with involute flanks the courses of these flanks are plotted in the same figure (dashed line). It is obvious that the courses of the convex-concave gears are much more advantageous from the tooth flank scoring point of view than the courses of the involute gears.

3. GENERALIZING THE INTEGRAL TEMPERATURE CRITERION FOR CYLINDRICAL GEARS IF THE PATH OF CONTACT IS GIVEN

The evaluation of a large number of experiments has shown that the Integral Temperature Criterion is the most appropriate criterion for assessing the scoring resistance of tooth flanks. According to [5], possibly the most common form of this criterion valid for all types of plane gears is derived where the path of contact can be expressed exactly enough in circular arcs. The following relation was derived to classify this kind of gear which defines m , the contact path shape coefficient, where m_n represents the normal module of gearing and β is the helix angle of the pitch cylinder.

$$\chi = \frac{\pi m_n}{2r_{kh} \cos \beta \cos \alpha_c} \quad (6)$$

For involute gears $\chi = 0$, for all cycloidal tooth forms $\alpha_c = 0$ and for convex-concave gears it is accepted that

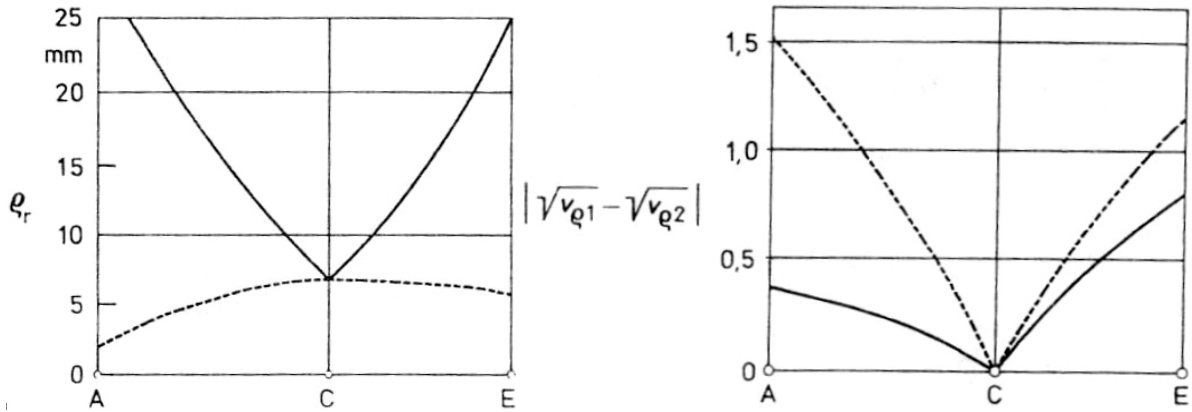


Figure 4. Mean curvature radius and the difference of square roots of the tangential speeds along the path of contact for convex-concave and involute gear-teeth

$\chi \neq 0$ and $\alpha_C \neq 0$. The resulting relations which generalize the Integral Temperature Criterion can be expressed as a function of the parameter χ . In this way it is possible to express clearly the impact of tooth profiles on scoring resistance. The value of the integral temperature can be calculated using the following equation [1]:

$$\vartheta_i = \vartheta_{ol} + C \mu_{mc} w_t^{0,75} v^{0,5} |a_w|^{-0,25} \frac{X_M X_G X_{\alpha\beta}}{X_Q X_{C_a}} X_\varepsilon \cdot (7)$$

X_G represents a geometrical factor and X_ε a contact ratio factor. Those are direct functions of the teeth profiles shapes and relations (8) and (9) can be derived.

Factor C represents the heat transfer and conductivity from the wheels into the oil and it is simultaneously used as an adjustment of the quantitative difference between the mathematically established mean value of flash temperature along the path of contact and the real value of bulk temperature. It is possible to state the value of factor C only by measuring the real temperature on the teeth surfaces. The results of such measurements are plotted in Figure 5.

4. EXPERIMENTAL INVESTIGATIONS

It is necessary to respect the values of factors C and μ_m in relation to teeth profiles in a more exact assessment of the impact of geometrical parameters of convex-concave gears on scoring resistance.

The measurement was carried out on a gearing machine at a pitch circle velocity of $v = 12.5$ m/s and with an initial oil temperature of $\vartheta_{ol} = 90$ °C. During the experimental investigation the temperature of the oil increased by about 24 °C with involute tooth flank gears, but the oil temperature in a gear box which used

convex-concave tooth flank gears stayed constant throughout the whole test.

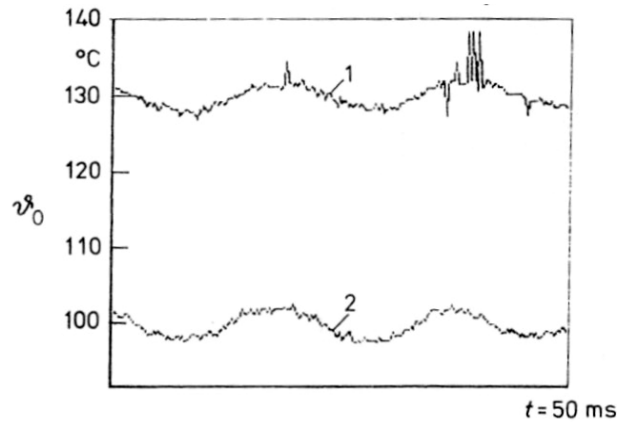


Figure 5. Oil temperature for convex-concave (2) and involute (1) teeth form measured in the FZG gear-rig machine (without additional oil heating)

Further experiments were carried out without the additional heating of oil. Using gears with convex-concave tooth flanks the oil temperature increased from an initial value $\vartheta_{ol} = 25$ °C (equal to the surrounding air temperature) to $\vartheta_{ol} = 57$ °C. This is presented in Figure 6. This figure shows also the mean values of the coefficient of friction μ_m along the contact path calculated from the measured power loss P_s in a test gearbox which has input power P . The calculations were conducted using the equation:

$$\mu_m = \frac{P_s}{PH_s} \cdot (10)$$

The following relation (11) was derived for H_s – tooth loss factor for convex-concave gears:

$$X_G = 0,62\sqrt{1+i} \frac{\sqrt{1 + \frac{2\pi \cos \beta \sin(\chi\varepsilon_1)}{z_1 \chi \cos \alpha_c \sin(\alpha_c + \chi\varepsilon_1)}} - \sqrt{1 - \frac{2\pi \cos \beta \sin(\chi\varepsilon_1)}{z_1 \chi i \cos \alpha_c \sin(\alpha_c + \chi\varepsilon_1)}}}{\sqrt[4]{\left[i - \frac{\pi \sin(2\chi\varepsilon_1)}{z_1 \chi \cos \alpha_c \sin(\alpha_c + 2\chi\varepsilon_1)} \right]} - \sqrt[4]{\left[1 + \frac{\pi \sin(2\chi\varepsilon_1)}{z_1 \chi \cos \alpha_c \sin(\alpha_c + 2\chi\varepsilon_1)} \right]}} \sqrt{\frac{\tan(\alpha_c + \chi\varepsilon_1) \cos(\chi\varepsilon_1)}{\sin(\alpha_c + 2\chi\varepsilon_1)}} \sqrt[4]{\tan(\alpha_c + \chi\varepsilon_1)} \quad (8)$$

$$X_\varepsilon = \frac{1}{(\chi + 2)\varepsilon \cdot \varepsilon_1^{\chi+1}} \left\{ (1 - \varepsilon_1)^{\chi+2} + (1 - \varepsilon_2)^{\chi+2} + (\chi + 2)(\varepsilon - 1) \left[0,22 \left(\varepsilon_1^{\chi+1} + \varepsilon_1^{\chi+2} \right) + 0,37 \left((1 - \varepsilon_1)^{\chi+1} + (1 - \varepsilon_2)^{\chi+1} \right) \right] \right\} \quad (9)$$

$$H_s = \frac{2(1+i)r_k \cos \alpha_C \cos \beta}{i z_1 m_n (\alpha_A + \alpha_D - 2\alpha_C)} \left[\ln \frac{\cos \alpha_A \cos \alpha_B \cos \alpha_D \cos \alpha_E}{\cos^4 \alpha_C} + (\alpha_A + \alpha_B + \alpha_D + \alpha_E) \tan \alpha_C \right] \quad (11)$$

where the angles α_A , α_B , α_C , α_D and α_E are pressure angles of corresponding points on the contact path. It is possible to prove that the well-known expression for the tooth loss factor of involute gears is a special case of this relation.

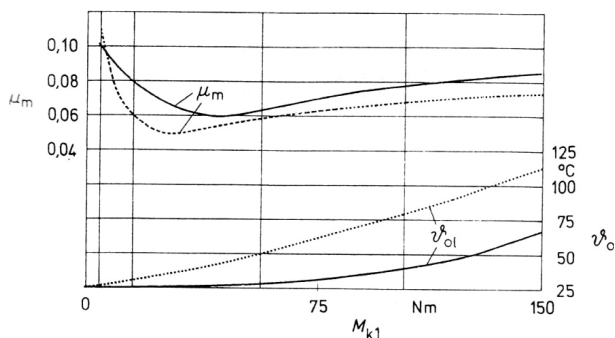


Figure 6. Measured bulk temperature on test gears for initial oil temperature $T_{ol} = 20^\circ\text{C}$

5. CONCLUSION

The scoring resistance of tooth flanks can be significantly raised by using special convex-concave profiles in plain tooth gears. There is also the possibility of reliably determining the value of loading or the scoring safety factor for such profiles using the Generalized Integral Temperature Criterion. It is possible to demonstrate that the equations concerning X_G and X_E which were derived for convex-concave profiles of teeth are also valid for involute and cycloidal profiles as a special type of Generalized Integral Temperature Criterion. Compared with involute gears and same values of slide-roll ratios at the beginning and at the end of contact, convex-concave gearing (with $\varepsilon_1 > 0$) has a great advantage in that – it can be loaded two to three times higher. This means that scoring damage in the case of extremely loaded tooth flanks could be avoided by the use of this profile.

ACKNOWLEDGEMENT

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REFERENCES

- [1] Winter, H. and Michaelis, K.: Scoring load capacity of gears lubricated with EP oils, AGMA Technical Paper No. P219.17, 1983.
- [2] Winter, H. and Michaelis, K.: Berechnung der fresstragfähigkeit von hypoidgetrieben, Antriebstechnik, Vol. 21, 1982.
- [3] Schauerhammer, K.: *Untersuchungen zur Fresstragfähigkeit Mineralölgeschmierter Stirnradverzahnungen*, Thesis, Technische Universität Dresden, Dresden, 1978.
- [4] Hlebanja, J.: Konkav-konvexe Verzahnung, Ermittlung der Zahnflanken und einige Grenzfälle, Antriebstechnik, Vol. 15, No. 6, 1976.
- [5] Vereš, M.: Odolnost ozubenia voči zadieraniu z hľadiska jeho tvaru, Thesis, Slovak University of Technology in Bratislava, Bratislava, 1987.
- [6] Vereš, M., Petrák, L. and Bošanský, M.: Komplexná optimalizácia geometrie čelného evolventného ozubenia z hľadiska termického zadierania, in: *Proceedings of the Theory, Design and Exploitation of Gear and Gearboxes Conference – ICESA 94*, 1994, Ostrava, Czech Republic, pp. 136-141.

ОТПОРНОСТ НА ЗАРИБАВАЊЕ ЦИЛИНДРИЧНИХ ПРАВОЗУБИХ ЗУПЧАНИКА СА НЕЕВОЛВЕНТНИМ ПРОФИЛИМА БОКОВА ЗУБАЦА

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Александар Маринковић

Рад приказује резултате теоријских и експерименталних истраживања отпорности на заривавање бокова зубаца цилиндричних зупчаника са конвексно-конкавним облицима профила зубаца. Постоје општи основни изрази, засновани на температурном критеријуму, који су применљиви за све задате облике контактних површина бокова зубаца цилиндричних зупчаника. Изрази који су добијени на основу теорије су илустровани резултатима испитивања на заривавање према ФЗГ методи испитивања зупчаника на уређају са затвореним колом снаге (DIN 51354).