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Parametric frequency analysis of oscillatory behavior of mouse *Zona Pellucida* spherical net model: cases of successful and of unsuccessful fertilization

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The current paper proposes a model for describing mechanical phenomena that occur during the process of mammal fertilization when spermatozoa impact the surface of Zona Pellucida. *Zona pellucida (ZP)* is a dynamical 3D matrix that surrounds the mammalian oocyte. In the process of fertilization, sperm cell has to penetrate this structure. To describe impact of sperm cells with velocities that are effective and those that are ineffective relative to the oscillatory behavior of *ZP*, the discreet continuum model in the form of spherical net model is used. Resultant trajectories of knot mass particles dynamics of mouse *ZP* spherical net model in the form of generalized Lussajous curves are presented. Using generalized Lussajous curves, parametric frequency analysis of oscillatory behavior of knot material particles in the mouse *ZP* spherical net model is conducted. The influence of impact angles of sperm cells on corresponding knot mass particle trajectory is discussed. Favorable and unfavorable trajectories of knot mass particle motions are discussed in the context of successful fertilization.

Key words: discreet oscillatory spherical net model, Zona pellucida, oscillations, sperm kinetic parameters, Lissajous curves

1 Introduction

For successful fertilization in mammals, a healthy mature oocyte and a certain amount of morphologically normal sperm cells with effective velocities that passed through the process of capacitation are required. After capacitation, sperm cells are hyper-activated, and thus characterized by pronounced flagellar movements, marked lateral excursion of the sperm head and a non-linear trajectory [1, 2]. Considering the fertilization as an oscillatory phenomenon, that essentially takes place on the outer membrane of an oocyte -ZP, we use the mouse Zona Pellucida (mZP) spherical net model [3] to describe oscillatory behavior of knot material particles under impact of sperm cells with effective and ineffective velocities. Capacitated spermatozoa are considered to possess effective velocities. Using generalized Lissajus curves, parametric frequency analysis of oscillatory behavior of knot material particles in the mZP spherical net model is realized. Trajectories of resultant motion of knot mass particles in a plane tangentially to the spherical surface net could be in the form of periodical, non-periodical, stochastic like or chaotic -like trajectories [4]. Periodical trajectories of resultant motion of knot mass particles are considered favorable, and non-periodical, stochastic like or chaotic -like trajectories are considered unfavorable trajectories for successful fertilization.

2 Methods

Using discrete continuum method [5] and basic unit of mZP spherical net model [3] that preserves the molar ratio of mZP glycoproteins, we did parametric frequency analysis of oscillatory behavior of mZP spherical net model: for cases of successful and of unsuccessful fertilization. In this paper we will consider cases when four sperm cells simultaneously impact four different knot mass particles in the mZP spherical net model. Knot mass particles in the mZP spherical net model correspond to ZP1 glycoprotein, component that is very important for the process of fertilization. For our numerical experiment we took the data from [6,7] for sperm velocities and mass, from [2] for masses of the mZP glycoproteins and from [8] for Young module of elasticity for mouse ZP. All the data were transformed into standard units. For our numerical experiment, we approximate that coefficient of elasticity is equal for all material particles and calculate it from the experimental data of Sun et al [8] according to the formula (1):

$$C = \frac{E(R^2 - r^2)\pi}{2R} \qquad (1)$$

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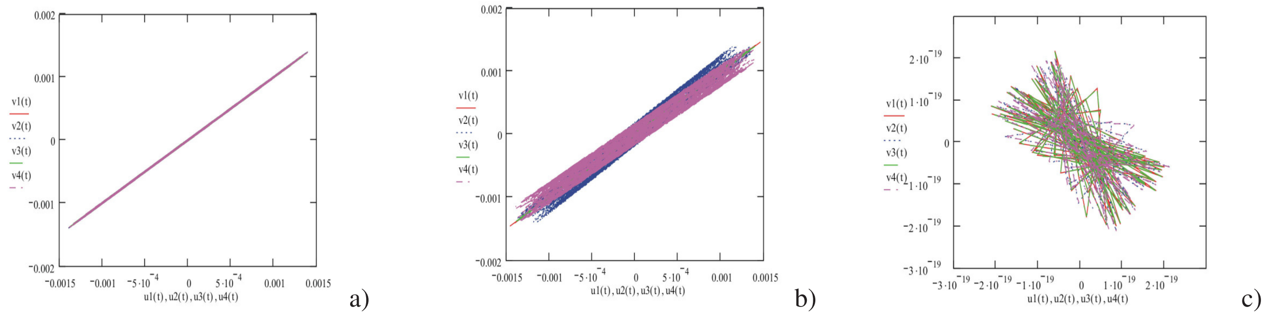

Fig. 1: Trajectories for all 4 knot mass particles when: a) all four sperm cells have maximum velocities and impact angles of $\frac{1}{4}\pi$. b) all four sperm cells have impact angles of$\frac{1}{4}\pi$. and three of them have maximum velocities and: c): all four sperm cells have maximum velocities and impact angles of $\frac{1}{2}\pi$.

E-Young module of elasticity, R is half diameter of the mouse oocyte, r-is half diameter of the oocyte minus approximate thickness of mZP. E=17.5kPa, 2R=56.2μm-average diameter of the oocyte from [8], δ=4.8μm-approximate thickness of the oocyte, C=246.75N/m.

Trajectory of resultant motion of each knot mass particles in a plane tangentially to the spherical surface net in meridional and circular direction were obtained by summing the affect of sperm cell that directly have impact on the knot material particle and affect of two neighboring sperm cells. Sperm cells' velocities were incorporated into equations. Varying sperm impact angles were examined: $\frac{1}{2}\pi$, $\frac{1}{3}\pi$, $\frac{1}{4}\pi$, $\frac{1}{6}\pi$, $\frac{1}{12}\pi$, $\frac{2}{3}\pi$.

3 Results

Although, theoretically, successful fertilization requires a higher percentage of capacitated sperm cells, Lissajous curves of corresponding knot mass particles in the mZP spherical net model depends on impact angles of sperm cells on the tangentially plane of the sphere. Resultant trajectories of knot mass particles arise only under certain angles as $\frac{1}{4}\pi$. See Fig. 1a. and c. If the number of spermatozoa with effective velocities changes, even if the impact angle is favorable, Lissajous curves of corresponding knot mass particles are less favorable for fertilization. See Fig 1b. If two sperm cells with effective velocities impact two diagonally symmetrical knot mass particles, and the other two sperm cells have equal but lower velocities than the first two considered spermatozoa, we obtain Lissajous curves like in Fig 1a.

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